

- 1) Absolute Instrument
- 2) Transfer instrument.

Transfer instrument :- is constructed that its response to alternating (a.c.) currents is the same as d.c. response. Such an instrument can be calibrated on d.c. & then brought to the same setting on a.c.

Eg :- electrodynamometer milli-ammeter,  
Thermocouple type.

## PMMC

principle of operation	Lorentz Force eqn
Used for	DC only
Type of control	Spring
Damping method	Eddy current
Formulae (a) scale-type	$T_d = BINA$ $T_c = K\theta$
Flux density value	0.1 to 1 wb/m <sup>2</sup>
Flux density	0.1 to 1 wb/m <sup>2</sup>
Sensitivity	$(20,000 - 30,000) \text{ r/v}$



# Electrical & Electronics Measurements

①

## Syllabus

Basics of measuring system

Error analysis

Analog instruments

- PMMC
- EMMC
- MI
- ESU
- Thermal instruments
- Rectifier type

Measurement of Resistance

- VA, AV methods
- \* DC Bridges
- \* AC Bridges  
(L, C, m).

Measurement of power

↓                      ↓  
DC power          AC power

Reactive power measurement.

Measurement of Energy

[1- $\phi$  EM, 3- $\phi$  EM].

Potentiometers

Q-meter

power factor meter

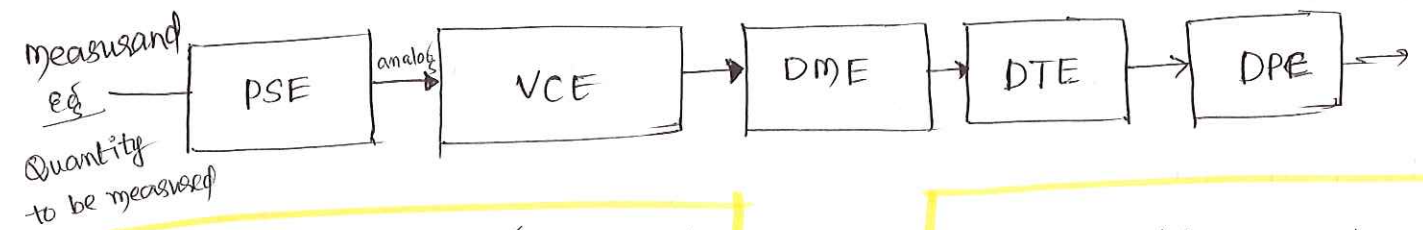
Flux meter

CRO

DVM

TRANSUDUCERS.

## Generalized building blocks of measuring system :-



**PSE** - primary sensing Element.  
(sensor)  
eg: Thermocouple (temperature)  
i.e.  $E_{mf} \propto (\Delta T)$ .  
(Seebeck effect)

**VCE** - variable conversion element  
eg: ADC, DAC converters  
V to I, I to V, V to f. convert.

**DME** - Data manipulation element  
eg: Amplifier, Modulator, attenuator

**DTE** - Data transmission Element.  
eg: Any transm. channel (optical fibre)  
two-wire co-axial channel.

**DPE** - Data present elements  
eg: CRO, digital displays.  
Analog pointer-scale indicators,  
xy-recorders, LCD, LED display.

The purpose of measurement system is to present an observer with a numerical value. So that the observer can understand in easy way. Usually, the type of display preferred is BCD seven segment display.

In a gen. meas. system the following building blocks are given

- as.
- 1) VCE
  - 2) DPE
  - 3) DTE
  - 4) DME
  - 5) PSE

correct order is .. PSE, VCE, DME, DTE, DPE for the purpose of measurement.

### Signal conditioning Elements : VCE, DME, DTE.

#### Types of measuring methods

(mass, length, time...)

- 1. Direct methods.  $\Rightarrow$  Unknown quantity compared with known standard qua.
- 2. Indirect Methods.

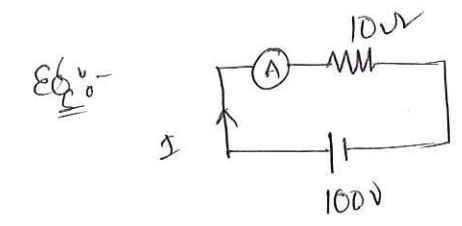
$\frac{\text{neutrons (n)}}{\text{protons (p)}} > 1.54 \Rightarrow$  unstable atoms ; <sup>(or)</sup> atomic number  $> 84$ .

1 meter = 1,650,763,73 no. of times of wavelength of radiation emitted by "KRYPTON-86" element

Time (1sec) = 9,19,26,31,770 times of no. of periods of radiation corresponding to hyper-transition levels of "cesium-136" element.

Mass (1kg) = prototype unit of mass  
 1 kg = 2.2 pounds.

2. Indirect method of measurement (Eg: Amp, volts b/c we dont have the standard current b/c we cannot see or hold).



(i)  $I_{\text{true}} = \frac{100}{10} = 10 \text{ A}$

$I_{\text{meas}} = 9.5$  (say)

Errors =  $I_{\text{meas}} - I_{\text{true}}$

error =  $9.5 - 10 = -0.5 \text{ A}$

(ii)  $I_{\text{meas}} = 10.5$  (say)

error =  $10.5 - 10 = +0.5 \text{ A}$



### Correction Factor :- (CF)

The value that you are added (or) subtracted from the measured value is known as correction factor.

The error may be either positive (or) negative.

$$\boxed{C.F. = -(\text{Error})}$$

**Static error:** If error is independent of time is known as static error.

**Dynamic error:** If error is changing w.r.t. time is known as dynamic error.

$$\text{absolute error} = \delta A = A_m - A_t$$

$$\% \text{ RSE} = \left\{ \begin{array}{l} \% \text{ relative static error} \\ \text{(or)} \\ \% \text{ Limiting error} \end{array} \right\} = \frac{A_m - A_t}{A_t} \times 100 = \frac{\delta A}{A_t} \times 100.$$

Ex 1:

$$A \\ \delta A = 1 A$$

$$B \\ \delta B = 10 A$$

Good instrument.

- a) only A
- b) only B
- c) A & B
- d) None

Ex 2:

$$A \\ \delta A = 1 A$$

$$B \\ \delta B = 10 A$$

$$A_t = 2 A$$

$$A_t = 1000 A$$

$$\% \text{ error} = \frac{1}{2} \times 100 > \% \text{ error} = \frac{10}{1000} \times 100$$

- a)
- b) only B
- c)
- d)

The quality of the instrument is given by the % relative static error, the error is always expressed in terms of true value of the instrument. If the accuracy of the instrument is mentioned by manufacturer known as **guaranteed accuracy error (GAE)**.

GAE (Guaranteed Accuracy Error) is always calculated w.r.t. Full scale deflection ③

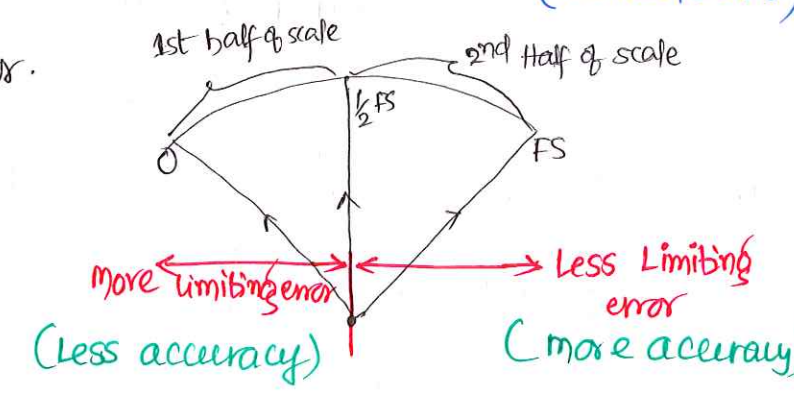
Ex: - (0-10)A meter ;  $GAE = \pm 1 \text{ FSD} = \pm \frac{1}{100} \times 10 = \pm 0.1 \text{ A}$ .  
 (is irrespective of load).  $\therefore$  constant error.

(Load or DC m/c brake test)	(I)			
1 kg	1 A $\pm 0.1 \text{ A}$	$\Rightarrow$	$1 \times \frac{1}{100} = 0.1$	$\Rightarrow (1 \pm 10\%)$
2 kg	2 A $\pm 0.1 \text{ A}$	$\Rightarrow$	$2 \times \frac{1}{100} = 0.1$	$\Rightarrow (1 \pm 5\%)$
3 kg	3 A $\pm 0.1 \text{ A}$	$\Rightarrow$	$3 \times \frac{1}{100} = 0.1$	$\Rightarrow (1 \pm 3.3\%)$
...				
10 kg	10 A $\pm 0.1 \text{ A}$	$\Rightarrow$	$10 \times \frac{1}{100} = 0.1$	$\Rightarrow (1 \pm 1\%)$

(Initial value)  $\downarrow$   
 (Full scale deflection)  $\downarrow$   
 GAE is constant error.  
 % Limiting error (variable error)

$x \rightarrow$  percentage limiting error.

$$\left( \text{True value} \right) \left( \frac{x}{100} \right) = \left( \text{GAE value} \right)$$



GAE is always (measured) mentioned by manufacturer, it is always calculated w.r.t. full-scale value, it is also known as constant error, whereas the % limiting error always calculated w.r.t. true value, it decreases in magnitude when the pointer is moving from initial position to full scale value. so it is also called as variable error.

Q:- A (0-200V) is having a GAE of  $\pm 1\%$  of full scale reading. The voltage measured by this instrument is 50V. Find the corresponding limiting error.

Sol:-  $GAE = \pm \frac{1}{100} \times 200 = \pm 2 \text{ Volts.}$

$$A_m = 50 \text{ Volts, } \pm 2$$

$$\% RSE = \frac{2}{50} \times 100 = 4\%$$

Q:-  $A_m = 205.5 \mu F$  Sol:-  $\% RSE = \frac{A_m - A_T}{A_T} \times 100 = \frac{3.1}{202.4} \times 100 = +1.53\%$   
 $A_T = 202.4 \mu F$

### Error Analysis

1.  $y = x^n \Rightarrow \frac{dy}{dx} = ? \Rightarrow \frac{\delta y}{y} = \pm n \frac{\delta x}{x}$

$$\log y = n \log x \Rightarrow \frac{1}{y} \cdot dy = \pm n \cdot \frac{dx}{x}$$

2. composite factors ;  $y = x_1^m \cdot x_2^n$

$$\frac{\delta y}{y} = \pm m \frac{\delta x_1}{x_1} \pm n \frac{\delta x_2}{x_2}$$

$\frac{\delta x_1}{x_1}, \frac{\delta x_2}{x_2} \Rightarrow \%$  limiting error is  $x_1$  &  $x_2$  respectively.

3. Addition (or) Subtraction of 3 variables.

$$x = (x_1 + x_2 + x_3) \quad \text{(or)} \quad x = x_1 - x_2 - x_3$$

$$\left[ \frac{\delta x}{x} = \pm \left[ \frac{\delta x_1}{x_1} \left( \frac{x_1}{x} \right) + \frac{x_2}{x} \frac{\delta x_2}{x_2} + \frac{x_3}{x} \frac{\delta x_3}{x_3} \right] \right]$$



proof :-

$$X = x_1 + x_2 + x_3 \quad \text{or} \quad X = x_1 + x_2 + x_3 \quad (4)$$

diff. partially on both sides.

$$\delta X = \delta x_1 + \delta x_2 + \delta x_3$$

$$\frac{\delta X}{X} = \left(\frac{x_1}{X}\right) \frac{\delta x_1}{x_1} + \left(\frac{x_2}{X}\right) \frac{\delta x_2}{x_2} + \left(\frac{x_3}{X}\right) \frac{\delta x_3}{x_3}$$

$$\frac{\delta X}{X} = \pm \left[ \frac{x_1}{X} \left(\frac{\delta x_1}{x_1}\right) + \frac{x_2}{X} \left(\frac{\delta x_2}{x_2}\right) + \frac{x_3}{X} \left(\frac{\delta x_3}{x_3}\right) \right]$$

4. Multiplication/division of variables.

$$X = x_1 x_2 x_3 \quad \text{or} \quad \frac{1}{x_1 x_2 x_3} \quad \text{or} \quad \frac{x_1}{x_2 x_3}$$

$$\frac{\delta X}{X} = \pm \left[ \frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} + \frac{\delta x_3}{x_3} \right]; \quad \text{(proof: Apply log on both sides, diff... you will get...)}.$$

$$5. \quad y = \frac{x_1^m \cdot x_2^n}{x_3^p}$$

$$\frac{\delta y}{y} = \pm \left[ m \frac{\delta x_1}{x_1} + n \frac{\delta x_2}{x_2} + p \frac{\delta x_3}{x_3} \right].$$

If any error is lying within the limits that means min. value and max. value, it is known as unknown error. It is denoted by " $\pm$ " symbol. Eg:-  $(100 \pm 5) \Omega \Rightarrow (95 \Omega \text{ to } 100 \Omega)$

Question

Two resistors are given as  $R_1 = 100 \pm 4\%$  ( $100 \pm 4 \Omega$ )

$$R_2 = 50 \pm 2\%$$
 ( $50 \pm 4 \Omega$ )

(i) when they are connected in series, Find the equivalent resistance.

(ii) Find  $(R_1 - R_2)$ ; (iii) Find  $R_1 R_2$  (iv)  $\frac{R_1}{R_2}$

Solution:-

$$\begin{aligned} \text{(i)} \quad (R_1 + R_2) &= (100 \pm 4\%) + (50 \pm 2\%) \\ &= (100 \pm 4\Omega) + (50 \pm 2\Omega) \\ &= 150 \pm 6\Omega \\ &= 150 \pm \frac{6}{150} \times 100 \equiv 150 \pm 3.33\% \end{aligned}$$

$$\begin{aligned} \frac{\delta x}{x} &= \pm \left[ \frac{x_1}{x} \frac{\delta x_1}{x_1} + \frac{x_2}{x} \frac{\delta x_2}{x_2} \right] \\ &= \pm \left[ \frac{100}{150} \times 4 + \frac{50}{150} \times 2 \right] \\ &= \pm 3.33\% \end{aligned}$$

$$\therefore R_{eq} = 150 \pm 3.33\% = 150 \pm 5\Omega \quad (145\Omega \text{ to } 155\Omega)$$

$\downarrow$  nominal value       $\downarrow$  % limiting error       $\downarrow$  error in value form.

$$\text{(ii)} \quad (R_1 - R_2) = (100 \pm 4\%) + (50 \pm 2\%)$$

$$x = x_1 - x_2 = 100 - 50 = 50\Omega$$

$$\frac{\delta x}{x} = \pm \sqrt{\left[ \frac{x_1}{x} \frac{\delta x_1}{x_1} + \frac{x_2}{x} \frac{\delta x_2}{x_2} \right]} = \pm \left[ \frac{100}{50} \times 4 + \frac{50}{50} \times 2 \right]$$

$$\frac{\delta x}{x} = \pm 10\%$$

$$R_{eq} = 50 \pm 10\% = 50 \pm 5\Omega \Rightarrow (45 \text{ to } 55)\Omega$$

$\downarrow$  nominal value       $\downarrow$  error in value form

$$\text{(iii)} \quad R_1 R_2 \Rightarrow x = x_1 x_2 \Rightarrow \frac{\delta x}{x} = \frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2}$$

$$\therefore x = 100 \times 50 = 5000\Omega; \quad \frac{\delta x}{x} = 4 + 2 = \pm 6\%$$

$$\therefore R_{eq} = R_1 R_2 = 5000 \pm 6\% = (5000 \pm 300)\Omega$$



$$(iv) \frac{R_1}{R_2} = \frac{(100 \pm 4\%)}{(50 \pm 2\%)} = \frac{100}{50} \pm (4+2)\% = 2 \pm 6\% = (2 \pm 0.12)\Omega \quad (5)$$

In case of multiplication (or) division the % limiting errors are simply added but don't add the error in value form.

Q2 :- Two resistors are given as  $R_1 = 100 \pm 6\Omega = (100 \pm 6\%)$

$$R_2 = (50 \pm 2\Omega) = (50 \pm 4\%)$$

(i)  $R_1 + R_2$    (ii)  $R_1 - R_2$    (iii)  $R_1 R_2$    (iv)  $\frac{R_1}{R_2}$

Sol :-

(i)  $R_{eq} = R_1 + R_2 = 100 + 50 = 150 \Omega$

$$\therefore \frac{\delta x}{x} = \pm \left( \frac{x_1}{x} \frac{\delta x_1}{x_1} + \frac{x_2}{x} \frac{\delta x_2}{x_2} \right) = \pm \left( \frac{100}{150} \times (6) + \frac{50}{150} (4) \right)$$

$$= \cancel{4.667\%} \quad 5.33\%$$

$$\therefore R_{eq} = 150 \pm \cancel{4.667} = 150 \pm 8\Omega = (100 \pm 6\Omega) + (50 \pm 2\Omega) = (150 \pm 8\Omega)$$

$$\left( \text{Nominal value} \right) \left( \frac{x}{100} \right) = \left( \text{Error in value} \right)$$

(ii)  $R_{eq} = R_1 - R_2 = 100 - 50 = 50\Omega = (100 \pm 6) - (50 \pm 2) = 50 \pm 8\Omega$

$$\therefore \frac{\delta x}{x} = \pm \left( \frac{100}{50} (6) + \frac{50}{50} (2) \right) = \pm 16\%$$

$$\therefore R_{eq} = 50 \pm 16\% = (50 \pm 8)\Omega$$

(iii)  $R_{eq} = R_1 R_2 = (100 \pm 6\%)(50 \pm 4\%) = 5000 \pm (10\%) = (5000 \pm 500)\Omega$

(iv)  $R_{eq} = \frac{R_1}{R_2} = \frac{100}{50} \pm (6 \pm 4) = 2 \pm 10\% = (2 \pm 0.2)\Omega$

(v)  $R_{eq} = \frac{R_1 R_2}{(R_1 + R_2)}$

In case of addition (or) subtraction the error in value form is simply added but don't add % limiting errors.

$$* R_1 R_2 = (100 \pm 6\Omega) (50 \pm 2\Omega) = (100 \times 50 \pm 8\Omega) \times$$

$$= (100 \pm 6\%) (50 \pm 4\%) = (100 \times 50 \pm 10\%) \checkmark$$

$$* R_{eq} = \frac{R_1 R_2}{(R_1 + R_2)} = \frac{(100 \pm 6\Omega)(50 \pm 2\Omega)}{(100 \pm 6\Omega) + (50 \pm 2\Omega)} = \frac{(100 \pm 6\%) (50 \pm 4\%)}{(100 + 50) \pm 8\Omega}$$

$$= \frac{(5000 \pm 10\%)}{(150 \pm 8\Omega)} = \frac{(5000 \pm 10\%)}{(150 \pm 5.33\%)}$$

$$= \frac{5000}{150} \pm (10 + 5.33)\%$$

$$R_{eq} = 33.33 \pm 15.33\%$$

$$R_{eq} = 33.33 \pm 5.1099\% \checkmark$$

$$* \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

diff. partially on both sides ...

$$\frac{-1}{R_{eq}^2} \delta R_{eq} = \left( \frac{-1}{R_1^2} \delta R_1 \right) + \left( \frac{-1}{R_2^2} \delta R_2 \right)$$

$$\frac{1}{R_{eq}} \left( \frac{\delta R_{eq}}{R_{eq}} \right) = \frac{1}{R_1} \left( \frac{\delta R_1}{R_1} \right) + \frac{1}{R_2} \left( \frac{\delta R_2}{R_2} \right)$$

$$\frac{\delta R_{eq}}{R_{eq}} = \frac{R_{eq}}{R_1} \left( \frac{\delta R_1}{R_1} \right) + \frac{R_{eq}}{R_2} \left( \frac{\delta R_2}{R_2} \right)$$

$$\frac{\delta R_{eq}}{R_{eq}} = \pm \left[ \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{\delta R_1}{R_1} \right) + \left( \frac{R_1}{R_1 + R_2} \right) \left( \frac{\delta R_2}{R_2} \right) \right]$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = 33.33 \Omega \quad (6)$$

$$\therefore \frac{\Delta R_{eq}}{R_{eq}} = \pm \left[ \frac{33.33}{100} \times (6) + \frac{33.33}{50} \times (4) \right] = \pm \frac{33.33}{5000} (3000 + 4000)$$

$$= \pm 4.667\%$$

$$\therefore R_{eq} = 33.33 \pm 4.667\% = 33.33 \pm 1.5556 \dots$$

short cut  $R_{eq} = \frac{(5000 \pm 10\%)}{(150 \pm 5.33\%)} = \frac{5000}{150} \pm (10 - 5.33)\%$

Q: The input of a electrical m/c is given as  $6500 \pm 3\%$   
output is given as  $5000 \pm 2\%$ . Find the loss of the m/c.  
(ii) Find the efficiency.

Sol: (i) Losses = Input - output =  $(6500 \pm 3\%) - (5000 \pm 2\%)$   
 $= 1500 \pm (195 + 100) = 1500 \pm (295)$   
 Losses =  $1500 \pm 19.6667\%$

(ii)  $\eta = \text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{5000 \pm 2\%}{6500 \pm 3\%}$   
 $= 76.923 \pm 5\%$   
 $= 76.923 \pm 3.846\%$

Q: 3 resistors are given as  $R_1 = 50 \pm 2\%$ ;  $R_2 = 37 \pm 2\%$   
 $R_3 = 75 \pm 2\%$ ; (i) when they are connected in  
 series find the equivalent resistance  
 (ii) if they are connected in parallel,  $R_{eq} = ?$



Sol:

$$\begin{aligned} \text{(c)} \quad R_{eq} &= R_1 + R_2 + R_3 = 50 + 37 + 75 = 162 \pm \\ &= (50 \pm 1\Omega) \pm (37 \pm 0.74) \pm (75 \pm 1.5) \\ &= 162 \pm 3.24 \Omega \\ &= 162 \pm 2\% \end{aligned}$$

$$\begin{aligned} \text{(or)} \\ &= (50 \pm 2\%) + (37 \pm 2\%) \pm (75 \pm 2\%) \\ &= (50 + 37 + 75) \pm 2\% \\ &= 162 \pm 2\% \end{aligned}$$

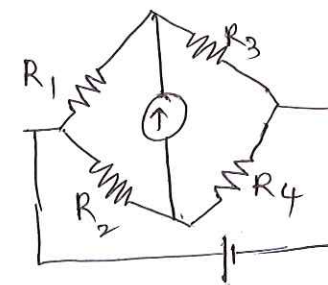
**Note:-** If 'n' different resistors are having same % limiting error (x), when they are connected in series...  
The % limiting error in the equivalent resistor is also x%.

$$R_1 \pm x\%, R_2 \pm x\%, \dots, R_n \pm x\%$$

$$R_{eq} = (R_1 + R_2 + \dots + R_n) \pm x\%$$

Q: In a following wheatstone bridge, the arms of the resistances are given as  $R_1 = 500 \pm 5\%$ ,  $R_2 = 1000 \pm 5\%$ ,  $R_3 = 200 \pm 5\%$ .

find  $R_4$



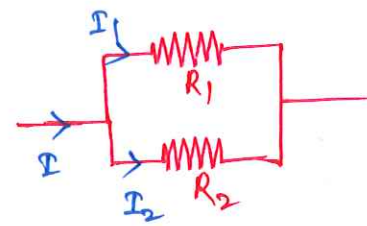
Sol:

$$R_1 R_4 = R_2 R_3$$

$$R_4 = \frac{R_2 R_3}{R_1}$$

$$= \frac{1000 \times 200}{500} \pm (5+5+5) = 400 \pm 15\%$$

Q:- Find the total current from the following parallel ckt. ⑦



$$I_1 = (150 \pm 1) \text{ A}$$

$$I_2 = (250 \pm 2) \text{ A}$$

Sol:-

$$I = I_1 + I_2 = (150 + 250) \pm (1 + 2) = (400 \pm 3) \text{ A} .$$

Q:- In the above problem if the errors are given in the form of standard deviation, Find the total current and also the resultant standard deviation

Sol:-

$$I_1 = (150 \pm 1) \text{ A} \quad \begin{matrix} \downarrow \\ \sigma_1 \end{matrix}$$

$$I_2 = (250 \pm 2) \text{ A} \quad \begin{matrix} \downarrow \\ \sigma_2 \end{matrix}$$

- a)  $(400 \pm 2)$
- b)  $(400 \pm 2.24)$
- c)  $(400 \pm 3)$
- d)  $(400 \pm 3.24)$

$\sigma_1 = 1, \sigma_2 = 2 .$

$I = I_1 + I_2 \Rightarrow \frac{\partial I}{\partial I_1} = 1 ; \frac{\partial I}{\partial I_2} = 1$

$\frac{\partial I}{I} = \sqrt{\left(\frac{\partial I}{\partial I_1}\right)^2 \sigma_1^2 + \left(\frac{\partial I}{\partial I_2}\right)^2 \sigma_2^2} = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{1^2 + 2^2} = \sqrt{5} = 2.24$

$\therefore I = (I_1 + I_2) \pm \frac{\partial I}{I} = 400 \pm 2.24$

$\sigma_{\text{resultant}} = \frac{\partial x}{x} = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 \sigma_2^2 + \dots + \left(\frac{\partial x}{\partial x_n}\right)^2 \sigma_n^2}$

\* If the data is given in variance form.  $\therefore V_{\text{resultant}} .$

$V_{\text{result}} = \left(\sqrt{(1^2 \times 1 + 1^2 \times 2^2)}\right)^2 = \left(\sqrt{5}\right)^2 = (1.732)^2 = 3$

- a)  $400 \pm 2$
  - b)  $400 \pm 1.732$
  - c)  $400 \pm 2.24$
  - d) None.
- $(400 \pm 3)$



# Errors

## GROSS Errors

- \* Due to human mistakes
- \* sight/oversight ...  
26.8  $\Rightarrow$  28.6 ...  
parallax error -  
wrong reading/wrong calc.
- (i) Incorrect adjustment
- (ii) Improper application
- (iii) misleading of instruments.

## 2. Environmental Errors

- \* Error due to temperature, humidity, stray-fields (due to soil,  $\vec{E}$ )

## 3. Observational Errors

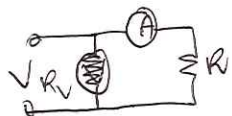
- \* parallax error. (avoided by placing the mirror on the instrument). (scale).

## Systematic Errors

### 1. Constructional (or) Instrumental error

- (1,2,3)
- ex: (i) inherent shortcomings of instruments (or) defective problems (or) defective parts (or) worn parts.
- eg: - weak magnet, weak spring
- (ii) misuse of instruments. Some of the instruments which are gravity control can be suitable for vertical placements only. If they are placed horizontally it may not give proper reading. The customer always expect same results.

(ii) Loading effect. (Ideally,  $R_a = 0$   
 $R_v = \infty$   
practically not possible)



$$R_{th} = \frac{V}{A} = \frac{R_a R}{R_a + R}$$

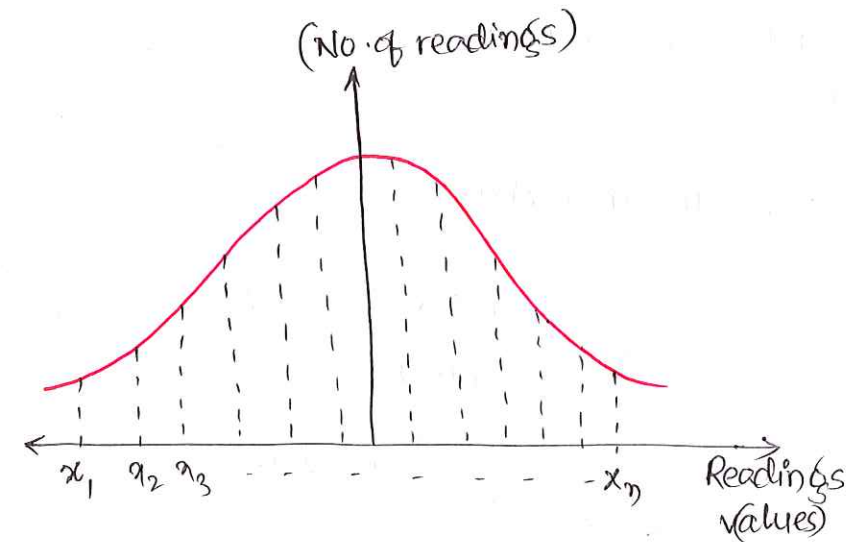
We don't know the reason for Random Errors

- (or) probable errors
- (or) Accidental errors.

These are the errors occurs randomly, These are also called as residual errors. These errors are related confidence interval. These errors can be minimized by using Gaussian statistical analysis. It has been consistently found that experimental results show variation from one reading to another, even after all systematic errors are accounted for. These errors are due to multitude of small factors. The quantity being measured is affected by many happenings in the universe. Some of them we are aware rest are unaware. ... are lumped together called random (or) residual errors. These errors are remain even after systematic errors have been taken care of, we call these are residual errors. Probability can be applied to study the random. There is no other way as the random errors are known & only statistical study can lead us to the best approximation of the true value of the quantity.

## Gaussian statistical Analysis :-

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$$\text{Arithmetic mean (AM)} = \bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n} = \bar{x}$$

$$\text{Deviations (d}_i\text{)} ; d_1 = x_1 - \bar{x} ; d_2 = x_2 - \bar{x} \dots , d_n = x_n - \bar{x}$$

$$d_i = x_i - \bar{x}$$

$$\text{mean deviation} = \bar{D} = \frac{|d_1| + |d_2| + \dots + |d_n|}{n}$$

$$\text{standard deviation } (\sigma) = \sqrt{\frac{d_1^2 + d_2^2 + \dots + d_n^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n |d_i|^2}{n}}$$

for infinite no. of observation.  
i.e.  $n > 20$ .

$$\sigma = \sqrt{\frac{\sum_{i=1}^n |d_i|^2}{(n-1)}}$$

for finite no. of observations  
(or)  $n \leq 20$ .

$$\text{Variance (V)} = \sigma^2 = (\text{standard deviation})^2$$

$$\text{probable error } (\gamma) = 0.6745 \sigma ; \gamma \propto \sigma$$

$$\gamma = 0.6745 \sigma$$

$$\gamma = \frac{0.4765}{h}$$

$h \rightarrow$  precision index



$$\gamma = 0.8453 \bar{D}$$

$$\therefore \gamma = 0.6745\sigma = \frac{0.4765}{h} = 0.8453 \bar{D}$$

$$h = \frac{0.4765}{(0.6745)\sigma}$$

$$h = \frac{0.706}{\sigma}$$

$$h = \frac{1}{\sigma\sqrt{2}}$$

$$\boxed{h\sigma = \frac{1}{\sqrt{2}}}$$

mean probable error =  $\gamma_m$

$$\gamma_m = \frac{0.6745\sigma}{\sqrt{(n-1)}} = \frac{\sigma}{\sqrt{(n-1)}} \text{ for } (n \leq 20)$$

$$\boxed{\gamma_m = \frac{\sigma}{\sqrt{n}}; n > 20.}$$

standard deviation of mean  $\boxed{\sigma_m = \frac{\sigma}{\sqrt{n}}}$

standard deviation of standard deviation

$$\Rightarrow \boxed{\sigma_\sigma = \frac{\sigma}{\sqrt{2n}}}$$

$$\sigma_\sigma = \frac{\sigma}{\sqrt{n}} \times \frac{1}{\sqrt{2}} = \frac{\sigma_m}{\sqrt{2}}$$

$$\therefore \boxed{\sigma_\sigma = \frac{\sigma_m}{\sqrt{2}}}$$

minimum range of error =  $x_{avg} - x_{min}$

maximum range of error =  $x_{max} - x_{avg}$ .

Average range of error =  $\frac{(x_{avg} - x_{min}) + (x_{max} - x_{avg})}{2}$

$$\boxed{\text{Avg. Range of error} = \frac{x_{max} - x_{min}}{2}}$$



Q:- Five students given the current readings by using ammeter and recorded as 10.03 A, 10.11 A, 10.12 A, 10.08 A. (9)

Find the (i) AM

(ii) deviations ( $d_i$ ) (iii) mean deviations ( $\bar{D}$ )

(iv) std. deviation ( $\sigma$ ) ; (v) variance =  $\sigma^2$  ; (vi) probable error

(vii) mean probable error ; (viii)  $\sigma_m$  (std deviation of mean)

(ix)  $\sigma_{\sigma}$  (std. deviation of std. deviation).

(x) min. range of error, max range of error.

Sol:-

$$AM = \bar{x} = 10 + 0.085 \cong 10.08$$

$$d_1 = 10.03 - 10.08 = -0.05$$

$$d_2 = 10.11 - 10.08 = +0.03$$

$$d_3 = 10.12 - 10.08 = +0.04$$

$$d_4 = 10.08 - 10.08 = 0.00$$

$$\bar{D} = \frac{(-0.05) + (0.03) + 0.04 + 0}{4}$$

$$\bar{D} = 0.03$$

$$\text{variance} = \sigma^2 = 0.001684$$

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2}{(n-1)}}$$

$$= \sqrt{\frac{0.05^2 + 0.03^2 + 0.04^2 + 0}{(4-1)}}$$

$$= \sqrt{\frac{25 + 9 + 16}{3 \times 10^4}}$$

$$\sigma = 0.0408.$$

$$\text{mean probable error } (\gamma) = 0.6745 \sigma = 0.6745 \times 0.0408 = 0.0275$$

$$\text{stand. deviation of mean } (\sigma_m) = \frac{\sigma}{\sqrt{n}} = \frac{0.0408}{\sqrt{4}} = 0.0204$$

$$\text{st. de. of st. de. } (\sigma_{\sigma}) = \frac{1}{\sqrt{2}} \sigma_m = 0.0144$$

$$\text{mean probable error } (\gamma_m) = \frac{\gamma}{\sqrt{n-1}} = \frac{0.0275}{\sqrt{3}} = 0.015$$

$$\text{min. range of error} = \bar{x} - x_{\min} = 10.08 - 10.03 = 0.05$$

$$\text{max. " " } = x_{\max} - \bar{x} = 10.12 - 10.08 = 0.04$$

$$\text{Avg " " } = \frac{0.05 + 0.04}{2} = \pm 0.045.$$

case(i) If errors are given in the form of standard deviations ( $\sigma$ )

$$\text{Let } x = f(x_1, x_2, x_3, \dots, x_n) \\ = f(\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_n)$$

$$\sigma_{\text{resultant}} = \frac{\partial x}{\partial x} = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 \sigma_2^2 + \dots + \left(\frac{\partial x}{\partial x_n}\right)^2 \sigma_n^2}$$

case(ii) If errors are given in the form of variance ( $v = \sigma^2$ ).

$$\text{Let } x = f(x_1, x_2, x_3, \dots, x_n) \\ x = f(v_1, v_2, v_3, \dots, v_n)$$

$$v_{\text{resultant}} = \frac{\partial x}{\partial x} = \left(\frac{\partial x}{\partial x_1}\right)^2 v_1 + \left(\frac{\partial x}{\partial x_2}\right)^2 v_2 + \dots + \left(\frac{\partial x}{\partial x_n}\right)^2 v_n$$

case(iii) If errors are given in the form of probable error ( $r$ )  
 $\therefore (r \propto \sigma)$

$$x = f(x_1, x_2, \dots, x_n)$$

$$r_{\text{resultant}} = \frac{\partial x}{\partial x} = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 r_1^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 r_2^2 + \dots + \left(\frac{\partial x}{\partial x_n}\right)^2 r_n^2}$$

case(iv) If the errors are given in the form of uncertainty ( $\omega$ ).

(doubtfulness of the <sup>measurement</sup> error)

Uncertainty refers to the doubtfulness of the measurement and % limiting errors are not at all same.

$$x = f(\omega_1, \omega_2, \dots, \omega_n)$$

$$\omega_{\text{result}} = \frac{\partial x}{\partial x} = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 \omega_1^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 \omega_2^2 + \dots + \left(\frac{\partial x}{\partial x_n}\right)^2 \omega_n^2}$$

Q. 11

Two identical  $\Rightarrow R_1 = R_2 = 100\Omega \pm 1\%$  resistors

(10)

case (i) series connection.  $R_1 = R_2 = 100 \pm 1\Omega$   
 $\hookrightarrow \sigma_1 = \sigma_2 = 1.$ 

$$R_{eq} = R_1 + R_2$$

$$\frac{\partial R_{eq}}{\partial R_1} = 1 ; \quad \frac{\partial R_{eq}}{\partial R_2} = 1 ;$$

$$\frac{\partial R_{eq}}{R_{eq}} = \sqrt{\left(\frac{\partial R_{eq}}{\partial R_1}\right)^2 \sigma_1^2 + \left(\frac{\partial R_{eq}}{\partial R_2}\right)^2 \sigma_2^2} = \sqrt{1^2 \times 1^2 + 1^2 \times 1^2} = \sqrt{2}$$

$$\therefore R_{eq} = (100 + 100) \pm \sqrt{2} = 200 \pm 1.414 = 200 \pm \frac{1}{\sqrt{2}}\%$$
  
 $\hookrightarrow \sigma$   
 $\% = \frac{\sqrt{2}}{200} \times 100 = \frac{1}{\sqrt{2}}\%$

case (ii) parallel connection.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} ; R_{eq} = 50\Omega$$

$$\left(\frac{-1}{R_{eq}^2}\right) \frac{\partial R_{eq}}{\partial R_1} = \left(\frac{-1}{R_1^2}\right) ; \quad \left(\frac{-1}{R_{eq}^2}\right) \frac{\partial R_{eq}}{\partial R_2} = \frac{-1}{R_2^2}$$

$$\sigma_{result} = \frac{\partial R_{eq}}{R_{eq}} = \sqrt{\left(\frac{R_{eq}}{R_1}\right)^4 \sigma_1^2 + \left(\frac{R_{eq}}{R_2}\right)^4 \sigma_2^2} = \sqrt{2} \times \left(\frac{R_{eq}}{R_1 = R_2}\right)^2$$
$$= \sqrt{2} \times \left(\frac{50}{100}\right)^2 = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$$

$$\therefore R_{eq} = 50 \pm \frac{1}{2\sqrt{2}} = 50 \pm \frac{100}{50 \times 2\sqrt{2}} = 50 \pm \frac{1}{\sqrt{2}}\%$$



Note:- "n" no. of resistors are connected in series.

$$R_{eq} = R_1 + R_2 + \dots + R_n$$

$$\frac{\partial R_{eq}}{\partial R_1} = \frac{\partial R_{eq}}{\partial R_2} = \dots = \frac{\partial R_{eq}}{\partial R_n} = 1.$$

$$\therefore \frac{\partial R_{eq}}{R_{eq}} \Rightarrow \sigma_{\text{resultant}} = \sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^2}$$

Q:- (WB) chap 1 (conv 1)

$$R_1 = 20 \text{ k}\Omega \pm 5\% = 20 \text{ k} \pm 1 \text{ k}\Omega \rightarrow \sigma_1$$

$$R_2 = 30 \text{ k}\Omega \pm 10\% = 30 \text{ k} \pm 3 \text{ k}\Omega \rightarrow \sigma_2$$

case (i) :- parallel.  $R_{eq} = 12 \text{ k}\Omega$  i.e.  $= \frac{R_1 R_2}{R_1 + R_2} = 12 \text{ k}.$

$$\therefore \frac{\partial R_{eq}}{\partial R_1} = \left(\frac{R_{eq}}{R_1}\right)^2 = \left(\frac{12}{20}\right)^2; \quad \frac{\partial R_{eq}}{\partial R_2} = \left(\frac{R_{eq}}{R_2}\right)^2 = \left(\frac{12}{30}\right)^2$$

$$\therefore \sigma_{\text{result}} = \frac{\partial R_{eq}}{R_{eq}} = \sqrt{\left(\frac{\partial R_{eq}}{\partial R_1}\right)^2 \sigma_1^2 + \left(\frac{\partial R_{eq}}{\partial R_2}\right)^2 \sigma_2^2}$$

$$= \sqrt{1^2 \left(\frac{12}{20}\right)^4 + 3^2 \left(\frac{12}{30}\right)^4} = \frac{(12^2)}{(10)^2} \sqrt{\frac{1}{16} + \frac{9}{81}}$$

$$= 1.44 \sqrt{\frac{81 + 144}{16 \times 81}} = 1.44 \times \frac{15}{4 \times 9} = 9 \text{ k}\Omega.$$

case (ii) :-

## Characteristics of the instruments:-

1. Accuracy.
2. precision.
3. Linearity.
4. Sensitivity
5. Dead time
6. Dead zone.
7. Drift.
8. Threshold
9. Resolution
10. fidelity.

1. accuracy  $\Rightarrow$  degree of closeness.  $\Rightarrow$  GAE. (constant error).

Accuracy refers to the degree of closeness. It is an indication of the measured value. How much close to the true value. If the accuracy is mentioned by manufacturer is known as GAE i.e. guaranteed accuracy error.

CLASS ① instruments  $\Rightarrow$  if accuracy =  $\pm 1$  FSD i.e. 1% GAE.

2. precision (repeatable or consistency).

precision refers to repeatability or consistency, The most repeated value from the given set of recordings.

Ex:-  $I_T = 2A$ .

	<u>A</u>	<u>B</u>	
precised value (1.8A)	1.9A	1.8A	precised value. (1.5)A
	1.8A	1.5A	
	1.6A	1.5A	
	1.7A	1.7A	
	1.5A	1.5A	
	1.4A	1.6A	
	1.3A	1.5A	
	1.8A	1.5A	
	1.2A	1.4A	
	1.8A	1.5A	
1.1A	1.6A		

$\Rightarrow$  instrument 'B' is more precised but not accurate.

$\Rightarrow$  instrument 'A' is less precised but more accurate.

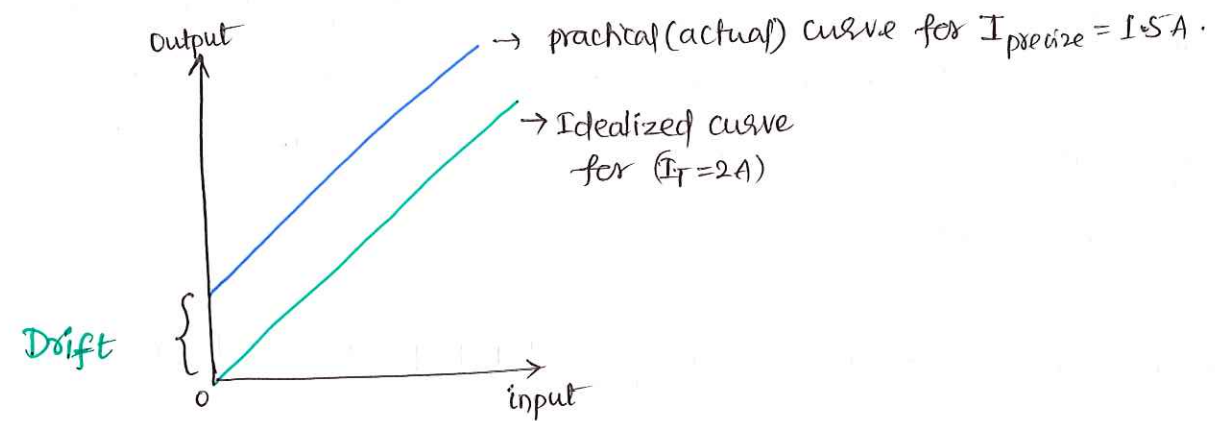
$\therefore$  precision ~~may~~ may not be the accurate indication.

Highly precise instrument doesn't mean that highly accurate.  
 Because the most precise instruments will give the wrong reading  
 so that precision never confirms accuracy.

→  $I_T = 2A$  ;      (1.5A) reading is      Reproducibility

out of 10 readings	⇒	repeated 5 times	⇒	50%
	⇒	" 8 "	⇒	8%
	⇒	" 10 "	⇒	100%

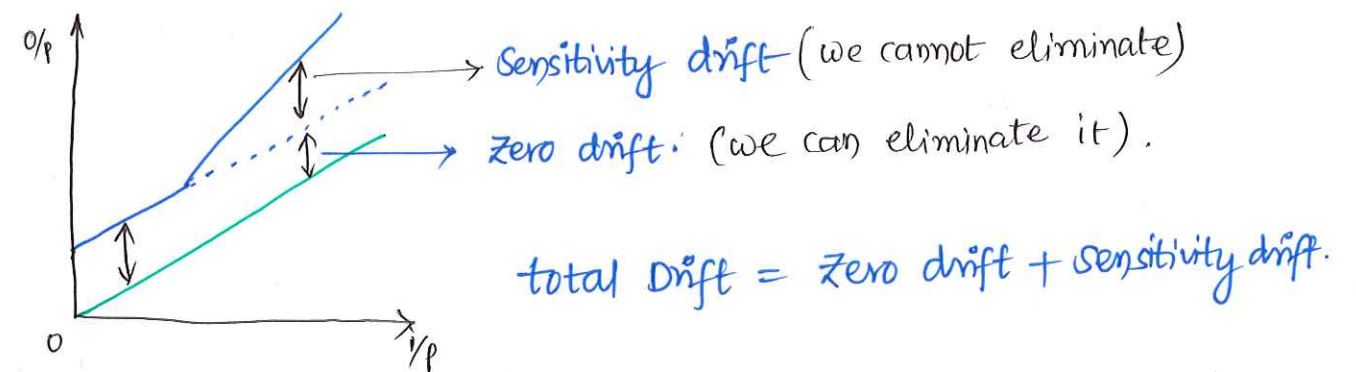
Reproducibility refers to the degree of repeatability.



\* Reproducibility :- Refers to the degree of repeatability.

A perfectly reproducible instrument is having zero drift.

Zero Drift can be eliminated by recalibrating the instrument



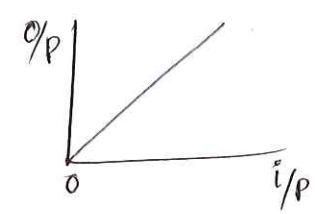


Linearity :- slope of curve (o/p vs i/p) = constant.

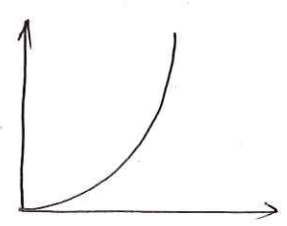
i.e. (Output)  $\propto$  (input).

proportional output ; slopes = constant.

i.e. Uniform scale readings.



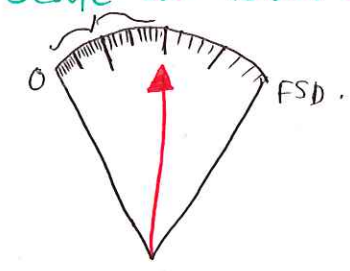
non-Linearity :-



slopes  $\neq$  constant  
(i.e. variable slope).

(Eg) (Output)  $\propto$  (input)<sup>2</sup>.

cramped scale at lower end.



Eg :-  $\theta \propto I^2$

$I = 1A \Rightarrow \theta = 1^2$

$I = 2A \Rightarrow \theta = 2^2$

$I = 5A \Rightarrow \theta = 5^2$

⋮

$I = 10A \Rightarrow \theta = 10^2$

$I = 20A \Rightarrow \theta = 20^2$

$I = 30A \Rightarrow \theta = 30^2$

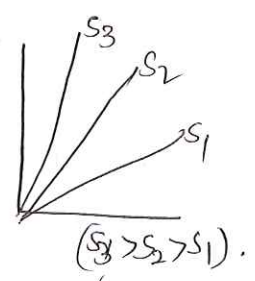
If the output follows the input with a proportional relationship then the instrument is said to be linear. otherwise, if the output follows the input with a square law relationship

then the instrument is said to be non-linear. for a non-linear instruments scale is cramped at lower end.

Sensitivity :- Slope of curve  $\equiv$  sensitivity =  $\frac{d(o/p)}{d(i/p)} = \frac{dy}{dx}$  ;

It is defined as the ratio of infinitesimal change in output to the change in input is known as sensitivity.

For linear instruments - constant sensitivity  
non-linear instrument - variable sensitivity.



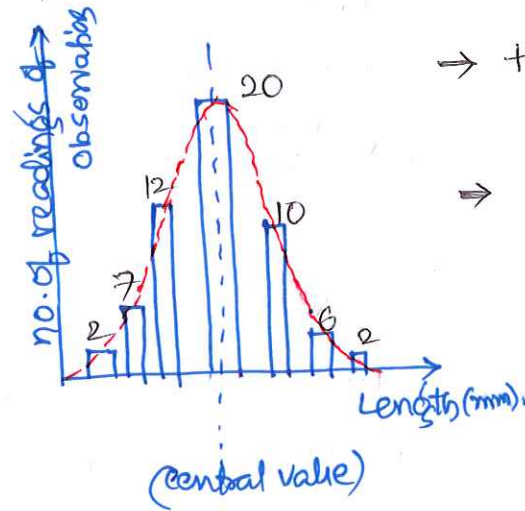
High sensitive (elements) instruments will respond for high small inputs.

units  $\Rightarrow$  mm/volt ; Degre/volt ; Radia/volt

(Ans) also.

Central value :- If we make a large no. of measurements and if the plus effects are equal to the minus effects, they would cancel each other and we would obtain the scatter round a central value. This condition is frequently met in practice.

Histogram :- When a no. of multisample observations are taken experimentally there is a scatter of data about some central value. One method of presenting test results in the form of a histogram.



→ Histogram is also called as a frequency distribution curve.

→ With more and more data taken at smaller and smaller increments the histogram would finally change into a smooth curve.

The most probable value of measured variable (variate) is the arithmetic mean of the no. of readings taken. Theoretically, an infinite no. of readings would give the best result, although in practice, only finite no. of measurements can be made.

Measure of Dispersion from the mean :-

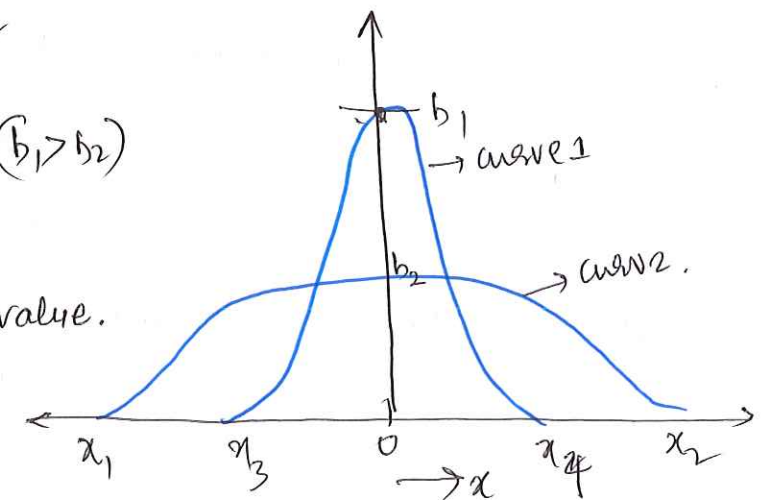
Dispersion :- The property which denotes the extent to which the values are dispersed about the central value called dispersion.

dispersion = spread = scatter

Curve 1 :- Greater precision ( $b_1 > b_2$ )

Curve 2 :- Lower precision

$x$  ⇒ deviation from the central value.





Dispersion is more for curve 2. That means . . . .

(13)

A large dispersion indicates that some factors involved in the measurement process are not under close control and  $\therefore$  it becomes more difficult to estimate the measured quantity with confidence and definiteness.

$$\text{Range} \Rightarrow (x_2 - x_1); (x_4 - x_3);$$

**Deviation :-** Deviation is the departure of the observed reading from the arithmetic mean of the group of readings.

$$d_1 = x_1 - \bar{x}$$

$$d_2 = x_2 - \bar{x}$$

$$\dots$$
$$d_n = x_n - \bar{x}$$

$$\sum d_i = d_1 + d_2 + \dots + d_n$$

$$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + \dots + (x_n - \bar{x})$$

$$= (x_1 + x_2 + x_3 + \dots + x_n) - n\bar{x}$$

$$= n\bar{x} - n\bar{x}$$

$$\therefore \bar{x} = \frac{\sum x_i}{n}$$

$$\sum d_i = 0.$$

$\therefore$  Algebraic sum of deviations is always zero.

Highly precise instruments yield a low value of average deviation between the readings. The average deviation is an indication of the precision of the instruments used in making the measurements.

$$\text{i.e. } \bar{D} = \frac{|d_1| + |d_2| + \dots + |d_n|}{n} = \text{avg. deviation.}$$

$$(\text{precision}) \propto \frac{1}{(\text{avg. deviation})}$$

Q:- There are 3 sets of data having average deviations of values 0.6, 0.3, and 0.5 for A, B, C set of data respectively. Then what is the correct order of precision of measurements of A, B, C set of data.

- (a)  $A > B > C$   
(b)  $A > C > B$   
(c)  $B > C > A$   
(d) we can't say until we get information about standard deviation.

Sol:- precision  $\propto \frac{1}{(\text{avg. deviation})}$   $\propto \frac{1}{(\text{dispersion} = \text{spread} = \text{scatter})}$

### Normal (or) Gaussian Distribution Curve of Errors :-

This is the basis for the major part of study of random errors. This type of distribution is most frequently met in normal practices.

The law of probability states the normal occurrence of deviations from average value of an infinite no. of measurements (or) observations can... be expressed as...

$$y = \frac{h}{\sqrt{\pi}} \exp(-h^2 x^2) ; y = \frac{1}{\sigma\sqrt{2\pi}} \exp(-x^2/2\sigma^2).$$

$x$  = magnitude of deviation from mean.

$y$  = no. of readings at any deviation  $x$ .

(The probability of occurrence of deviation  $x$ )

$h$  = a constant called precision index

$\sigma$  = standard deviation.

$\sigma$  = usually known as quantity of interest.



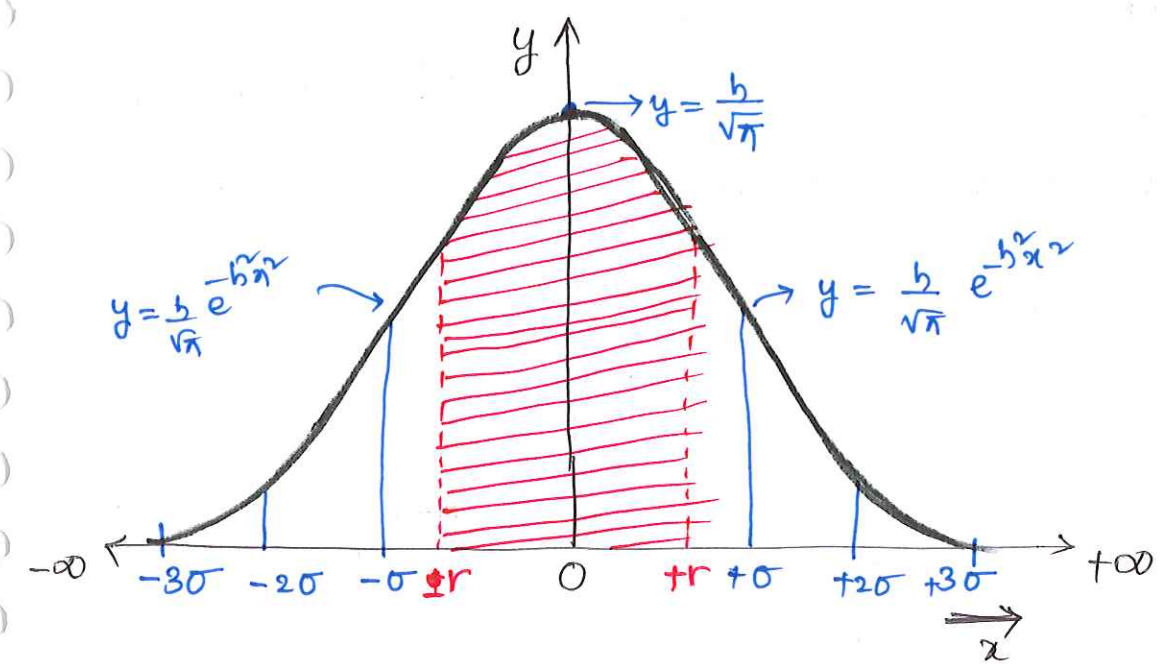


Fig:- Normal probability curve (or) Gaussian curve.

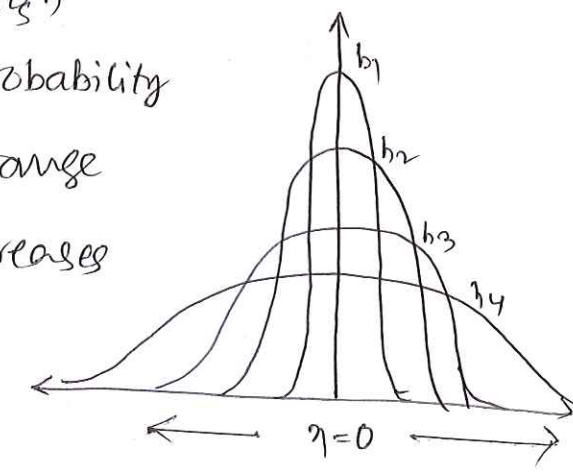
⇒ The value of  $b$  determines the sharpness of the curve since the curve drops sharply owing to the term  $(-b^2)$  being in the exponent. The sharp curve evidently indicates that the deviations are more closely grouped together around deviation  $x=0$ .

⇒ It is clear that the probability that a variate lies in a given range becomes less as the deviation of the range becomes greater.

If greater the value of  $b$ ... for a more probability less.

∴ Thus the name... **precision index** for  $b$  is reasonable.

→ A large value of  $b$  represents high precision of the data because the probability of occurrence of variates in a given range falls off rapidly as the deviation increases because the variates tends to cluster (becomes closer) into a narrow range.



## Probable Error:-

The confidence in the best value (most probable value) is connected with the sharpness of the distribution curve.

Total area of the Gaussian curve = 1.

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-3\sigma}^{3\sigma} \exp(-x^2/2\sigma^2) dx = 1.$$

A convenient measure of precision is the quantity ( $r$ ), called, probable error.

$$\frac{b}{\sqrt{\pi}} \int_{-r}^{r} \exp(-b^2 x^2) dx = \frac{1}{2}$$

$$\therefore r = 0.6745\sigma = \frac{0.4769}{b}$$

## average deviation for normal curve:

$$\bar{D} = \int_{-\infty}^{+\infty} |x| y dx$$

$$b = \frac{1}{\sqrt{\pi} \bar{D}} \Rightarrow \bar{D} = \frac{r}{0.4769\sqrt{\pi}} = \frac{r}{0.8453}$$

$$r = 0.8453 \bar{D}$$

$$PE = r = 0.6745\sigma = \frac{0.4769}{b} = 0.8453 \bar{D}.$$

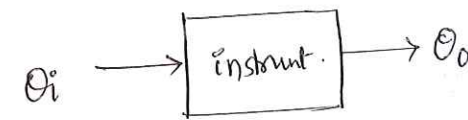
for finite readings ( $n$ ); - probable error ( $r_m$ ) =  $0.6745 \frac{\sigma}{\sqrt{n}}$

standard deviation of mean  $\sigma_m = \frac{\sigma}{\sqrt{n}}$

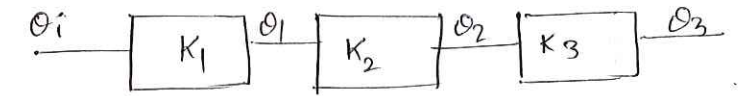
standard deviation of standard deviation  $\sigma_\sigma = \frac{\sigma}{\sqrt{2n}} = \frac{\sigma_m}{\sqrt{2}}$

$$S_V = \frac{R_m}{V_{FSD}} \left( \frac{\Omega}{\text{Volt}} \right) = \frac{1}{I_{FSD}} \quad 1/mA \quad R_m \rightarrow \text{meter resistance}; \quad (15)$$

$$V_{FSD} \rightarrow \text{Fullscale deflection};$$



$$S = K = \frac{\Delta O/P}{\Delta I/P} = \frac{\theta_o}{\theta_i}$$



$$S_{\text{th}} = \frac{\theta_3}{\theta_i} = \frac{\theta_o}{\theta_i} = \frac{\theta_o}{\theta_2} \cdot \frac{\theta_2}{\theta_1} \cdot \frac{\theta_1}{\theta_i} = K_1 K_2 K_3$$

$$(S_{\text{th}})_{\text{overall}} = (S_{V1}) \cdot (S_{V2}) \cdot (S_{V3})$$

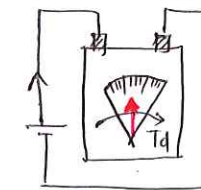
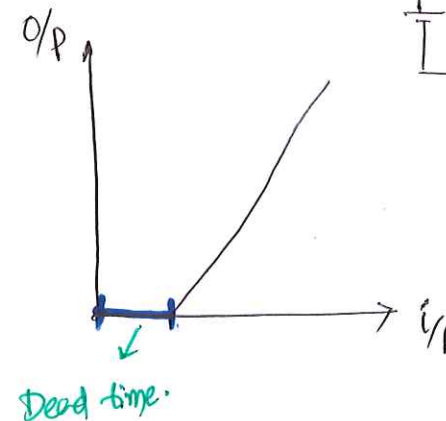
The overall sensitivity of a cascaded system is the multiplication of their individual sensitivities.

Q:- two meters having full scale currents of 50A, 100A respectively find their sensitivities in  $\Omega/\text{volts}$ , state which meter has greater sensitivity.

Sol:-  $S_{VA} = \frac{1}{50} \Omega/V = 0.02 \Omega/V$  ;  $S_{VB} = \frac{1}{100} \Omega/V$   
 $S_A = 20 \text{ m}\Omega/V$  ;  $S_B = 10 \text{ m}\Omega/V$

$$\therefore S_A > S_B$$

Dead Time :-



$\Rightarrow$  no-electrical inertia  
 $(\because \text{mass of electron is } 9.3 \times 10^{-31} \text{ kg})$   
 $\Rightarrow$  mechanical (ie. mass) inertia.

$$T_{\text{electrical}} = \frac{L \text{ (mH)}}{R \text{ (k}\Omega)} = \mu\text{sec}$$

$$T_{\text{elect.}} = RC = \text{msec}/\mu\text{sec}$$

$$T_{\text{mech}} = \frac{m}{B} \text{ seconds}$$

(m, B, K systems)  $m \rightarrow \text{mass}$   
 $B \rightarrow \text{friction}$



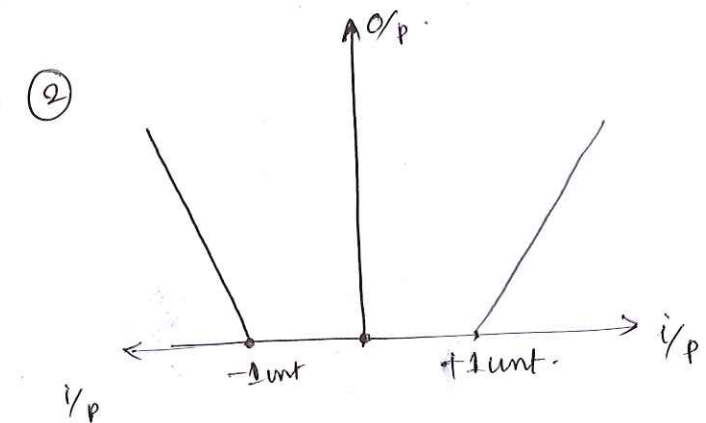
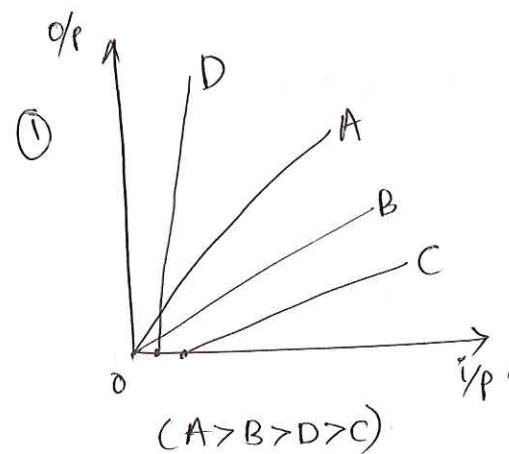
$$\therefore (\text{Time-const})_{\text{mech.}} > (\text{Time-constant})_{\text{electrical}}$$

The time taken by the instrument in order to give the response is known as dead-time.

There is no electrical ~~inertia~~ inertia b/c the mass of electron is very very small. Where as every mechanical body will offer some inertia so that it takes considerable amount of time in order to give the response. B/c mechanical time constants are always greater than that of electrical time constants.

**Dead zone** :- For the largest value of input. The response of the instrument is zero. Beyond this input value the instrument gives the response. The corresponding portion of input where the output is known as dead zone.

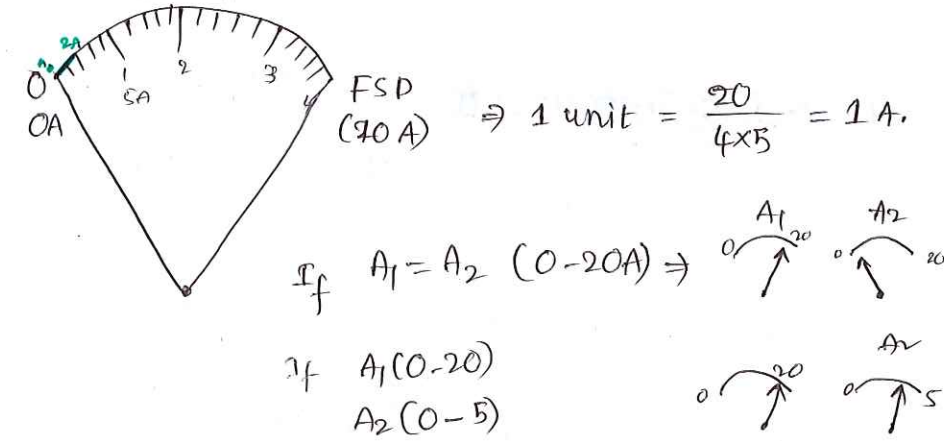
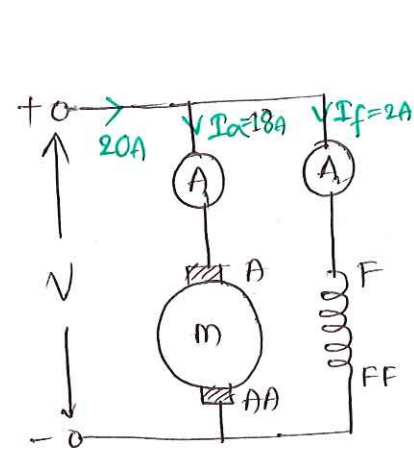
**Threshold** :- At what particular input value, the instrument will give the response is known as threshold (or) pick-up



① Identify the from the above plot which instrument is the best inst :- A

② Find the dead zone, threshold :-  $\pm 1$  unit  
 deadzone = 2 units

**Resolution** :- refers to clarity (or) certainty.



more no. of divisions  $\Rightarrow$  more clarity, more resolutions.

The smallest value of change in input that we can detect with more clarity (or) more certainty is known as resolution.

**Fidelity** :- The readings obtained how much faithfully represented in the record book is known as fidelity.

**Significant figures** :-

When we are representing the significant figures in a resultant variable always we will select the min. significant figure of the given variable.

**Types of standards.**

1. International stds  $\Rightarrow$  IEEE, ISO

Eg: std. voltage  $\Rightarrow$  Western company.

Western std. cell  $\Rightarrow$   $E = 1.0183 = 1.0183 \text{ volt}$

(Highest accuracy).  $\Rightarrow$  not available for common man (patents require).

2. National standards (primary standards). ISI.

(much more accurate), not available

3. secondary standards (Industrial standards).

Every industry has its own standards..., not available

4. Working standards  $\Rightarrow$  Least accurate.  
available to common-man.

**Secondary instruments**: In all these secondary instruments either current quantity (or) voltage quantity is converted into mechanical deflecting torque and by means of scale and pointer we can obtain the reading.

1. Indicating type instruments.
2. Recording type "
3. Integrating type "

1. These are the instruments which display the reading only at the time of measurement, It doesn't have any storage facility.

Eg: All analog instruments, PMMC, EMMC, MI, Ohmmeter, V/A, V/A, V/A, wattmeter, megger...

2. Eg: Substation recording type, magnetic tape recorder, XY-recorder, cockpit voice recorder, ECG (medical research patient monitoring).  
These are the instruments, which record the continuous variations of an electrical quantity. (Substation...)  
Eg:- seismograph, speedometer.

3. 1- $\phi$  Energy meters (domestic purpose) ; 3 $\phi$  energy (industrial).

These are the instruments, which will give the electrical energy supplied to a consumer over a specific period of time.



These are 3 essential components in all indicating instruments. (17)

1. Deflecting torque ( $T_d$ )

2. Controlling " ( $T_c$ )  $\Rightarrow$  (i)  $\propto \theta$  ; (ii) if  $\theta=0$ ; pointer should bring back to 0 pos

3. damping " ( $T_{damp}$ ).

The torque which is required to move the pointer from its initial position, due to continuous current is flowing through the instrument, continuous deflecting torque is exerted on the pointer so that always the pointer will be reaching full-scale reading, which is undesirable.

At steady-state both deflecting & controlling torques are equal in magnitude and acting opposite in direction so that the moving system produces oscillations, which is undesirable. To reduce the no. of oscillations (or) to damp out the no. of oscillations we are going to apply a torque is called damping torque.

If the damping torque is absent, nothing will happen, but it (pointer) will take more time to settle at final steady-state process value.

### Effects :-

1. Magnetic field effect.  $\Rightarrow$  PMMC, EMMC, (A), (V), (W)
2. Electro-static field effect  $\Rightarrow$  ESV-(V)
3. Electromagnetic field of attraction/repulsion.  $\Rightarrow$  M.I (A), (V)
4. Electromagnetic induction effect.  $\Rightarrow$  All induction type meters (A), (V), (W) Energy meter  $\checkmark$
5. Thermal effect/heating  $\Rightarrow$  Bolometer, RTD (resistance-temp-detector), Thermistor (temperature), Thermocouple, hot-wire
6. Chemical effect  $\Rightarrow$  DC amper-hour meter (VI) . Eg: vehicles, inverter.
7. Hall effect  $\Rightarrow$  (magnetic field measurement), flux/gauss meter, pointing vector type wattmeter.
8. piezoelectric effect  $\Rightarrow$  piezo-electric transducers.  $\downarrow$  rectifier also

pointing vector type instrument) It is used to measure the power loss density in a magnetic measurement.

PMMC

$$T_d \propto I$$

$$I = +ve \Rightarrow T_d \Rightarrow +ve$$

$$I = -ve \Rightarrow T_d \Rightarrow -ve$$

$$T_{dnet} = 0$$

\* cannot work for AC only DC.

Induction

(T/F)

only for AC

EMMC

$$T_d \propto I^2$$

$$I = +ve \Rightarrow T_d = +ve$$

$$I = -ve \Rightarrow T_d = +ve$$

$$T_{dnet} \neq 0$$

\* can work for AC & DC.

attraction/repulsion

$$T_d \propto I^2 \text{ (AC \& DC)}$$

ESV

$$T_d \propto V^2$$

(AC & DC)

Thermal

$$T_d \propto (\text{heat})$$

$$T_d \propto (I^2)(\text{time})$$

(AC & DC)

$$\theta \propto I^2$$

**Note :-** 1. Except PMMC for DC, induction type for AC remaining all other can be working for both AC as well as DC.

2. One and only one instrument is called PMMC always reads average value and remaining any other instrument always reads rms value including rectifier type instruments.

3. The rectifier type instruments are calibrated in such a manner by multiplying its form factor in order to read RMS value.



Reading of rectifier = (FF) x reading of PMMC

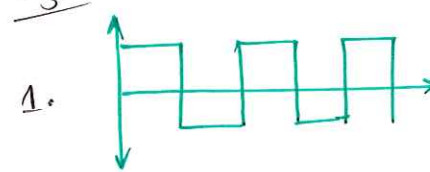
HWR  $\Rightarrow$  FF = 1.57  $\Rightarrow$  (2.22x)

FWR  $\Rightarrow$  FF = 1.11

\*\*\*

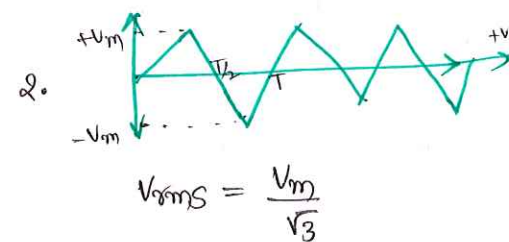
4. If the scale is multiplied by a factor of 1.11, it will give correct reading only for AC sinusoidal input full-wave rectifier. It gives wrong readings for AC sinusoidal input halfwave rectifier, squarewave input signal, triangular wave i/p signal and also for saw-tooth wave i/p signal.

Ex:-



$V_{RMS} = V_m$  ;  $V_{avg} = V_m$

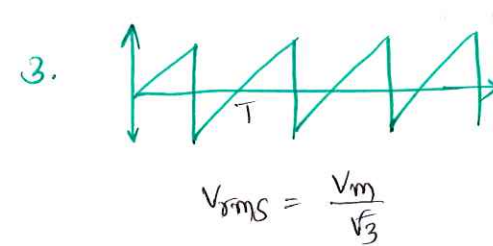
$K_f = \frac{V_{RMS}}{V_{avg}} = 1$



$V_{RMS} = \frac{V_m}{\sqrt{3}}$

$V_{avg} = \frac{V_m}{2}$

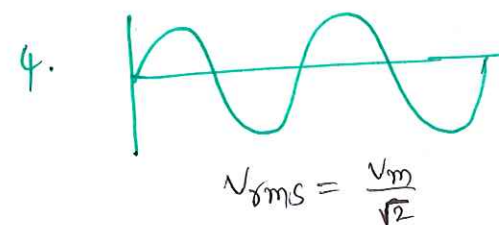
$K_f = \frac{V_{RMS}}{V_{avg}} = \frac{2}{\sqrt{3}} = 1.154$



$V_{RMS} = \frac{V_m}{\sqrt{3}}$

$V_{avg} = \frac{V_m}{2}$

$K_f = \frac{V_{RMS}}{V_{avg}} = \frac{2}{\sqrt{3}} = 1.154$

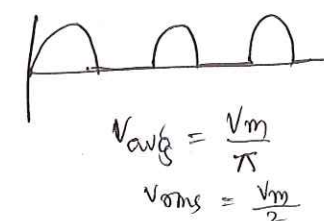


$V_{RMS} = \frac{V_m}{\sqrt{2}}$

$V_{avg} = \frac{2V_m}{\pi}$

$K_f = \frac{V_{RMS}}{V_{avg}} = \frac{\pi}{2\sqrt{2}} \Rightarrow (1.11)$

5.



$V_{avg} = \frac{V_m}{\pi}$

$V_{RMS} = \frac{V_m}{2}$

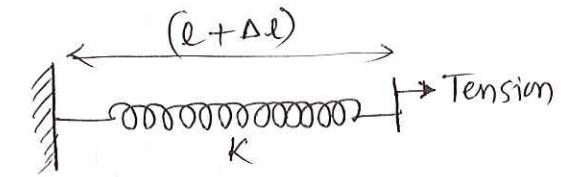
$K_f = \frac{V_{RMS}}{V_{avg}} = \frac{\pi}{2} = 1.57$



## Control (Spring) Torque :- (Tc).

1. Spring control technique.  $\left\{ \begin{array}{l} \rightarrow \text{Helical Spring} \Rightarrow \text{ESV of parallel plate type} \\ \rightarrow \text{Spiral Spring} \end{array} \right.$
2. Gravity Control

### Spring Control (i) Helical Spring



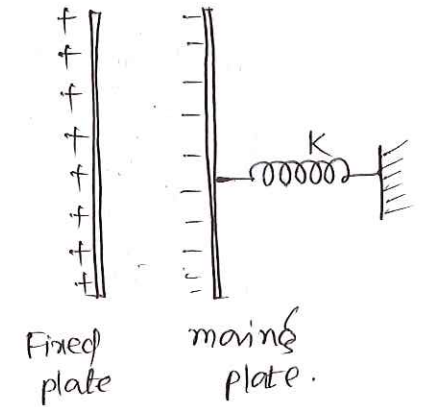
(Restoring force)  $\propto$  (displacement)

$$F_c \propto x$$

$$F_c = Kx$$

$$K = \frac{F_c}{x} \quad \frac{N}{m}$$

stiffness coefficient = Spring constant.



### (ii) Spiral Spring.

Torque  $\Rightarrow$  twisting moment

$$\text{torque} = \text{Force} \times \text{distance} \Rightarrow \tau = Fr \quad \text{Nm.}$$

$$T_c = K_c \theta \Rightarrow K_c = \frac{\tau_c}{\theta} \quad \frac{\text{Nm}}{\text{radians}} \Rightarrow \text{spring constant.}$$

Common material to prepare spiral spring.

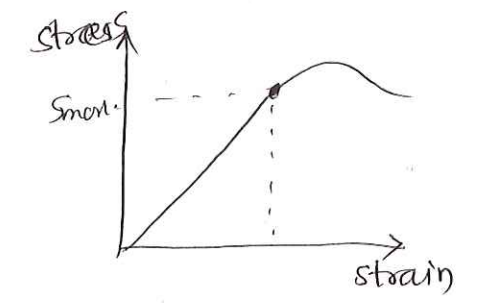
$\rightarrow$  phosphor bronze  $\Rightarrow K_c = 12 \times 10^9 \text{ kg/m}^2$

$\rightarrow$  Beryllium Copper (costly)

Some of the instruments, which has low resistance like mCoil mc milli-ammeter. In those instruments beryllium copper is preferred to prepare the spring but it is very costly.

### properties of Spring. (Requirements).

- (i) Non-magnetic material
- (ii)  $(\text{stress})_{\text{max}} < \text{Elastic limit}$
- (iii)  $\frac{L}{t} = \frac{\text{length}}{\text{thickness}} \cong 3000 ; \theta = 90^\circ$ .



$$T_c = \frac{E b t^3}{12L} \theta ;$$

$$T_c = K_c \theta .$$

- $E = \text{Young's modulus of elasticity (kg/m}^2\text{)}$
- $L = \text{Length of the spiral spring. (m)}$
- $t = \text{thickness of spring (m)}$
- $b = \text{width/depth of spring (m)}$
- $\theta = \text{deflection of pointer (radians)}$
- $K_c = \text{Spring constant}$

$$K_c = \frac{E b t^3}{12L} \left( \frac{\text{Nm}}{\text{radi}} \right)$$

$E = 12 \times 10^9 \text{ kg/m}^2$ . (for phosphor bronze).

$$\frac{L}{t} = \frac{E \theta}{2 S_{\text{max}}} \cong 3000 ; \theta = 90^\circ ;$$

Every control spring must satisfy the above requirements...  
 Some times spring is used as leads of the instruments that means it is a current carrying element. The area of cross-section of the spring must be sufficient in order carry the rated current otherwise the spring may suffer from internal heating problem, so that spring may be broken.

### Spring problems

1. Ageing effect.
  2.  $l_t = l_0 (1 + \alpha \Delta t)$ .
- $$K = \frac{F}{\Delta l} = \frac{F}{l + \Delta l} \Rightarrow l \uparrow \Rightarrow K \downarrow$$

Because of Error due to temperature the stiffness of the spring can be changed so that always the instrument gives the wrong readings. Due to ageing effect of the spring the stiffness const reduces so that the instrument always reads more. Spring control technique is costly compared to gravity control.

case(i):- If  $T_d \propto I \Rightarrow$  linear

We may obtain linear scale if the deflecting torque is proportional to the current flowing through the instrument.

$$T_d = K_d I \rightarrow (1)$$

$$T_c = K_c \theta \rightarrow (2)$$

at Steady-state  $\Rightarrow T_d = T_c$

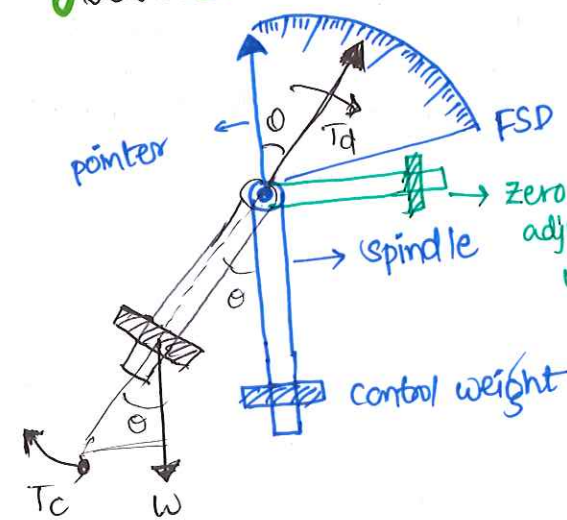
$$K_d I = K_c \theta$$

$$\theta = \left( \frac{K_d}{K_c} \right) I \Rightarrow \theta \propto I$$



case(ii):- If  $T_d \propto I^2 \Rightarrow$  non-uniform scale.  
 $\theta \propto I^2$

Gravity control :-



$$T_d = T_c = (m g \sin \theta)$$

adv

\* No ageing effect (b/c  $m g$  won't change).

$$\therefore T_c \propto (\sin \theta)$$

\* No current flow, no temperature effect.

\* Cost of the arrangement is low.

disadvantage:- (Level zero error) present.



In a gravity control technique a small weight is attached to the moving system. So that because of acceleration due to gravity there is a gravitational pull is developed so that it will bring the pointer towards reading value. (20)

### disadvantages

1. This technique is best suitable only for vertical placements, If these instruments are placed horizontally it may not give the proper reading.
2. The place where you are keeping the meter must be plane surface, If it is inclined surface there is a level zero error will occur, it can be adjusted by using zero adjusting weight.

application: This type of technique is used in, **Wall-mounted and panel board type instruments.**

$$T_c = K_g \sin \theta ; T_d = K_d I \quad (T_d \propto I)$$

$$K_d I = K_g \sin \theta$$

$$\boxed{I \propto (\sin \theta)} \Rightarrow \text{non-linear scaling.}$$

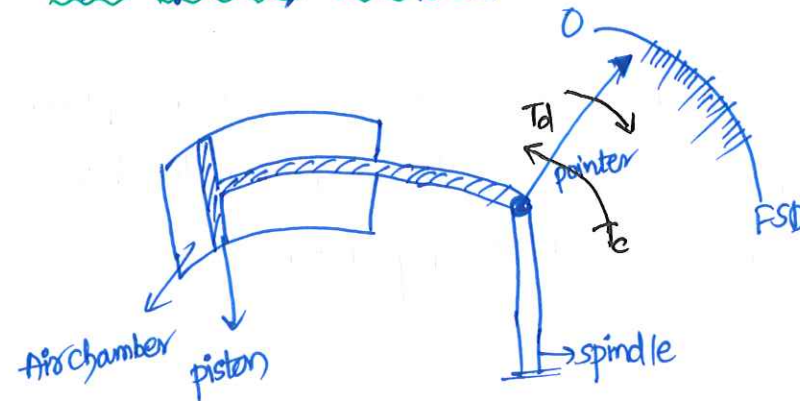
If  $(T_d \propto I^2)$

$$\boxed{I^2 \propto (\sin \theta)}$$

## Damping techniques :-

1. Air friction damping (MI, EMMC)
2. Fluid friction (Electrostatic voltmeter, wall-mounted, panel board type).
3. Eddy current damping (PMMC, Induction type)
4. Electromagnetic damping techniques. (Galvanometers)

## Air friction damping :-



Effectiveness order  $\Rightarrow$  (eddy current)  $>$  (fluid)  $>$  (air friction)

Priority order  $\Rightarrow$  (eddy current)  $>$  (air friction)  $>$  (fluid).

In this type of damping technique a light disc of aluminium vane (or) piston, which has a considerable area in order to develop the friction, which is placed inside the air-chamber, it is connected to the spindle so that the friction offered by air will oppose the motion of the pointer.

This arrangement requires less maintenance and cost is lesser.

## Fluid Friction Damping technique :-

It is the more effective form of damping technique in this type of damping technique an aluminium vane is placed inside the fluid chamber due to friction offered by the fluid the motion of the pointer is opposed.

The fluid must satisfy the following requirements.

(21)

- (i) Fluid shouldn't evaporate quickly.
- (ii) fluid viscosity should not change with temperature.
- (iii) The fluid should not have any corrosive action upon the metals of the instrument.
- (iv) The fluid must be very good insulator.

### Disadvantage

- (i) Due to leakage of the fluid it is difficult to keep the instrument clean. So that it requires more maintenance.
- (ii) cost is also more.

applications :- It is used in Electrost. V., panelboard, wall-mounted.

### Eddy Current Damping technique :-



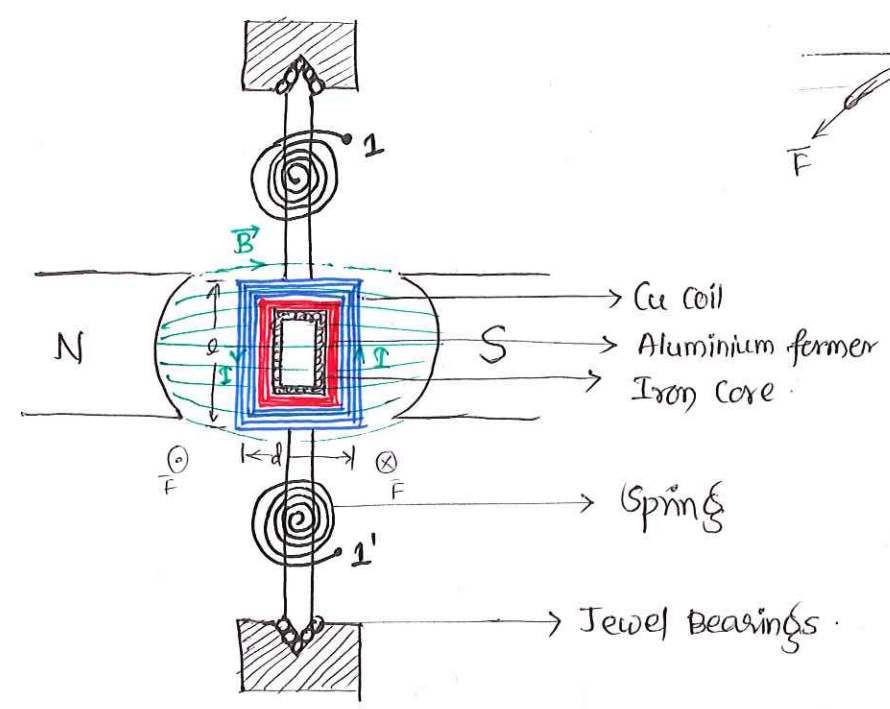


PMMC Instrument :-

\* U shaped - permanent Magnet

- ALNICO
- Hard Magnetic material
- Very high cohesive forces.
- Very strong internal magnetic field. ( $B = 0.1 \text{ Wb/m}^2$  to  $1 \text{ Wb/m}^2$ )

\* Stationary iron-core :- to make the magnetic field radial field.



$$\vec{F} = (\vec{i} \times \vec{B})$$

$$\vec{F} = B i l \sin \theta$$

Lorentz Force eqn  
 & Fleming's Left Hand Rule.  
 Here,  $\theta = 90^\circ$ ,  $\vec{B} \perp \vec{l}$   
 $\therefore F = B i l$  Net force

Torque =  $F \times l \times \text{dist}$   
 $T = F \times d$   
 $T = B i l d$   
 $T = B i A$   
 (per conductor)

For N turns  
 $\therefore T = (B i A) N$   
 $\therefore T_d = B i N A$  Nm

$T_d = (B N A) i$   
 $T_d = K_d i$   
 $\therefore \boxed{T_d \propto i}$

\* Jewel-bearings  $\Rightarrow$  To reduce friction  
made up of **SAPPHIRE**

\* Aluminium Former :-

- (i) provide the base for copper coil
- (ii) To provide eddy currents path.

\* Copper coil :-

- (i) To carry current
- (ii) To provide  $T_d$ .

\* Type of control (PMMC) - Spring control.

\* Spring (dual purpose)

(i) To provide controlling torque ( $T_c$ )

$$T_c \propto \theta \Rightarrow T_c = K_c \theta.$$

(ii) Used as Leads of instrument.  
(current carrying element).

at steady-state  $T_d = T_c$

$$K_d I = K_c \theta.$$

$$\theta = \frac{K_d I}{K_c} \Rightarrow \boxed{\theta \propto I.}$$

Uniform scale.

$$\text{Sensitivity} = \frac{d\theta}{dI} = \frac{K_d}{K_c} = \frac{BNA}{K_c} = \text{constant.}$$

$$\text{Sensitivity} = \frac{\text{mag. Flux density} (\text{No. of turns}) (\text{Area of Cross-section})}{(\text{stiffness of spring})}.$$

Due to aging effect of magnet

(i) magnetic field strength ( $B$ ) ↓ decreases.

$$\theta \propto B \Rightarrow \theta \downarrow \propto B \downarrow$$

(ii) ( $K_c$ ) spring constant/stiffness will decrease.

$$\uparrow \theta_c \propto \frac{1}{K_c \downarrow}$$

$$\therefore \boxed{\theta \propto \frac{B}{K_c}}$$

$$\frac{\delta \theta}{\theta} = \frac{\delta B}{B} - \frac{\delta K_c}{K_c}$$



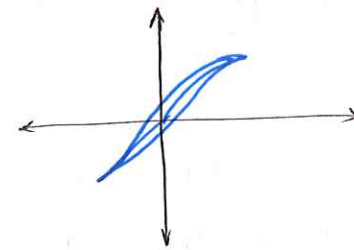
## advantages of PMMC

23

- \* Linear scale.
- \* produced torque is very high due strong magnetic field.
- \* Less weight of moving system  $\Rightarrow$  Less friction.
- \* High  $\left(\frac{T}{W}\right) = \frac{\text{torque}}{\text{weight}}$  ratio  $\Rightarrow$  high sensitivity.  
 $S = 20,000 \Omega/\text{volts}$  to  $30,000 \Omega/\text{volts}$ .

“In all analog instruments the torque to weight ratio will decide the **frictional errors**.”

- \* Frictional errors are reduced by jewel-bearings also.
- \* Aluminium former will have very thin hysteresis loop area.



Thin hysteresis curve for light weight aluminium former.

The word hysteresis means that the loading energy may not equal to the unloading energy, reduced hysteresis error in a PMMC instrument b/c of aluminium former, It has very thin hysteresis loop.

- \* Low power consumption ( $25 \mu\text{Watts} - 200 \mu\text{Watts}$ )  
So that No internal heating/temperature rise problem.  
reduced temperature errors, still any other temp. errors are possible it can be reduced by **swamping resistor**.
- \* A long open  $360^\circ$  circular scale is available.

\* Error due to external magnetic field is known as stray magnetic field error, In a PMMC instrument reduced stray magnetic field error b/c of very strong internal magnetic field.

\* More accuracy.

### disadvantages of PMMC :-

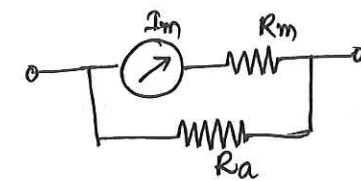
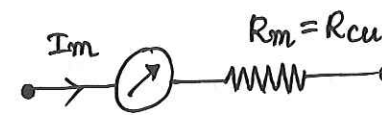
- \* Cannot work for ac
- \* Cost is more (pm, jewel-bearings) & delicate construction.

### Application :-

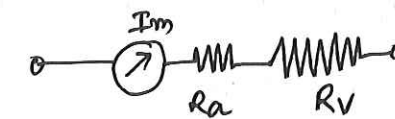
\* It is mainly used in air-craft systems, ~~air~~-space industry because of self-shielding property.  $\Rightarrow$  i.e. we shouldn't require any protecting cover b/c the operating field of meter is very strong.

**Note :-** All errors are lesser in PMMC instrument compared to any other instrument. So that this instrument has more accuracy & high sensibility.

### Electrical Equivalent ckt/diagram :-



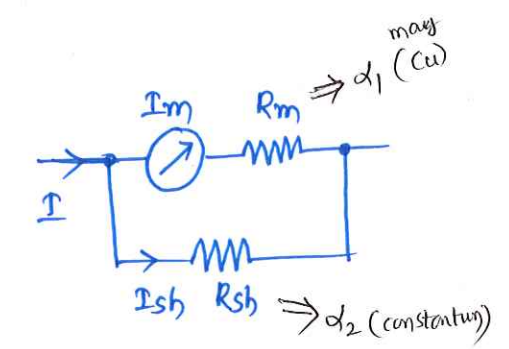
**Fig. PMMC instrument**  
also called ammeter  
i.e. of order (mA).



- \* Without any shunt multiplier  $\Rightarrow$  (0-5 mA)
- \* With internal shunt multiplier  $\Rightarrow$  (0-200 A)
- \* With external shunt multipliers  $\Rightarrow$  (0-5000 Amps).

- \* Without any series multiplier  $\Rightarrow$  (0-50mV)
- \* With series multiplier  $\Rightarrow$  (0-20,000V) (or) (0-30,000V).

Extension Range of PMMC Ammeter :-



$I = I_{extension} = I_m + I_{sh}$

$V_{sh} = V_m$

$\therefore I_{sh} R_{sh} = I_m R_m$

$R_{sh} = \frac{R_m I_m}{I_{sh}} = \frac{R_m (I - I_{sh})}{I_{sh}}$

$= \frac{R_m I_m}{(I - I_m)} = \frac{R_m}{(\frac{I}{I_m} - 1)}$

$R_{sh} = \frac{R_m}{(m - 1)}$

$m = 1 + \frac{R_m}{R_{sh}}$

multiplication factor (m)

$m = \frac{I}{I_m} = \frac{I_{extension}}{I_m} = \dots \times 10^6$

$\theta \propto I_m$

$\theta \propto (I_m \times m)$

$\theta \propto I_{extension}$

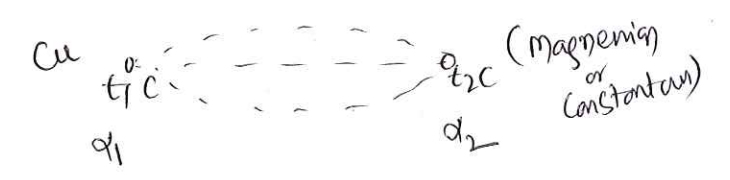
$\theta \propto I$

$R_{sh} \Rightarrow$  milli  $\Omega$ s (very very low).

Manganin (or) Constantan.

properties of shunt resistors :-

- (i) should carry large current without rise in temperature.
- (ii) resistance should not vary with time & temperature.
- (iii) shunt should have very low temperature coefficient.
  - $\therefore$  constant resistance is possible.
- (iv) Shunt should have very low thermal emf with copper.



DC instrument  $\Rightarrow$  Manganin  
 DC inston  $\Rightarrow$  Constantan



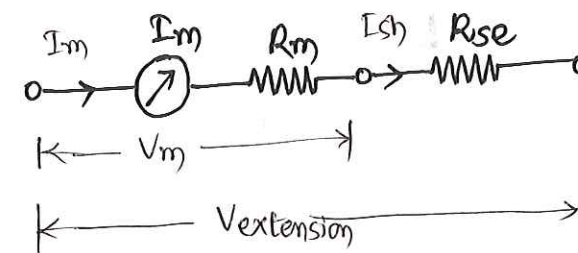
All these above properties can be satisfied by manganin.  
 Particularly for DC instruments manganin is preferred b/c it should have very low thermal emf with copper.  
 where as in the AC instruments always constantan is preferred because the produced thermal emf by Cu-constantan pair is always unidirectional. which will not have any affect on AC instrument readings.

DC instruments  $\Rightarrow$  Manganin.

AC instruments  $\Rightarrow$  Constantan

MANGANIN  $\Rightarrow$  (4% Ni + 84% Cu + 12% manganese)

Extension Range of PMMC voltmeter :-



$$I_m = I_{sh} \Rightarrow \frac{V_m}{R_m} = \frac{(V - V_m)}{R_{se}}$$

$$R_{se} = R_m (m - 1)$$

$$m = 1 + \frac{R_{se}}{R_m}$$

$m \rightarrow$  multiplication factor

$$m = \frac{V}{V_m} = 1 + \frac{R_{se}}{R_m}$$

$m \gg 1$  (always).

$$\theta \propto V_m$$

$$\theta \propto (V_m) m$$

$$\theta \propto V_m \times \frac{V}{V_m}$$

$$\theta \propto V$$

Series resistances are preferred (prepared by Eureka material).

R<sub>shunt</sub> → Manganin

R<sub>Series</sub> → Eureka

coil → Cuppor

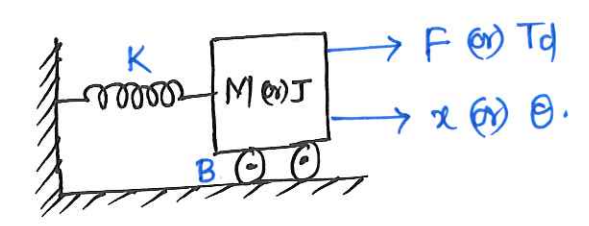
former → Aluminium

Spring → phosphor-bronze.

Jewel-bearing → Sapphire

PMMC ≡ 2<sup>nd</sup> order system of rotation :-

≡ All analog/measuring instruments.



$$F_d = m \frac{d^2x}{dt^2} + B \frac{dx}{dt} + Kx$$

(or)

$$T_d = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} + K_c\theta$$

$$F_d(s) = (ms^2 + Bs + K) \theta(s)$$

$$T/F = \frac{L(O/P)}{L(I/P)} = \frac{1}{ms^2 + Bs + K} \Rightarrow \text{CEqn } ms^2 + Bs + K = 0$$

$$s^2 + \left(\frac{B}{m}\right)s + \left(\frac{K}{m}\right) = 0 \quad \text{compared with 2<sup>nd</sup> order proto type system}$$

$$2\zeta\omega_n = \frac{B}{m} \quad \text{and} \quad s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = \frac{K}{m} \Rightarrow \omega_n = \sqrt{\frac{K}{m}} \Rightarrow \boxed{f_n = \frac{1}{2\pi} \sqrt{\frac{K}{m}} = \frac{1}{2\pi} \sqrt{\frac{K_c}{J}}}$$

$$\zeta = \frac{B}{2m\omega_n} \Rightarrow$$

$$\therefore \zeta = \frac{B}{2\sqrt{mK}}$$

If  $\zeta = 0 \Rightarrow$  undamped

$\zeta = 1 \Rightarrow$  critical damping

$\zeta < 1 \Rightarrow$  under damping (practical system).

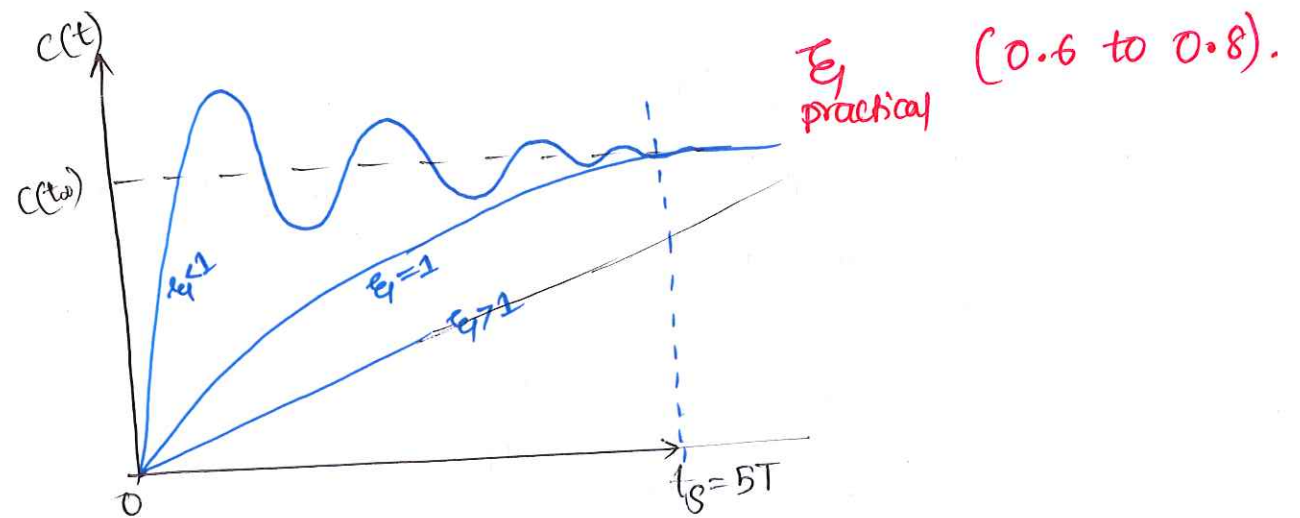
$\zeta > 1 \Rightarrow$  over damping

$t_{\text{settling}} = \frac{3}{\zeta\omega_n} \Rightarrow$  for 5% tolerance band

$t_s = \frac{4}{\zeta\omega_n} \Rightarrow$  for 2% tolerance band.

$\tau = \text{time period} = \frac{1}{\zeta\omega_n}$ ; if  $\zeta = 0$ ;  $\tau = \text{infinite}$ .

\*  $Q = \frac{1}{2\zeta}$ ; Underdamping  $\Rightarrow Q > 1$   
(Quality factor).



$\therefore$  All analog instruments use 2nd order systems, practically under-damped technique is preferred whose ' $\zeta$ ' value is in the order of 0.6 to 0.8, In a under damped system always the pointer is reaching full-scale value before coming to steady-state it makes few oscillations.



In critically damped system without making oscillation directly it moves to steady-state. (26)

Note :- (i) Potentiometer is zero order instrument.

$$T/F = \frac{V_o(s)}{X(s)} = K s^0 = \text{constant.}$$

(ii) Thermometer is 1st order instrument.

$$T/F = \frac{K}{(1+sT)}$$

$K \rightarrow$  static sensitivity ;  $T =$  time response.

(iii) All analog instruments are 2nd order instruments.

$$T/F = \frac{1/m}{(s^2 + \frac{B}{m}s + \frac{K}{m})}$$

$m =$  mass  
 $B =$  friction  
 $K =$  Spring constant.

{ PMMC }  $\Rightarrow$  Spring serves  
{ EMMC } dual purpose

( $T_c$  & current carrying element).

{ ESI }  $\Rightarrow$  Spring serves  
{ MI } only control torque

$\downarrow$   
If one of spring breaks-down the produced deflecting torque will be zero. Then the pointer always shows zero deflection.

$\downarrow$   
If spring breaks-down always the pointer will be swinging beyond the full-scale.

## Moving Iron Instruments :-

→ minimum reluctance principle

→ variable self-inductance concept.

$$\phi \propto I$$

$$N\phi = LI$$

$$N \frac{d\phi}{dt} = L \frac{dI}{dt} + I \frac{dL}{dt}$$

$$e dt = L dI + I dL$$

$$e I dt = LI dI + I^2 dL$$

$$\text{Energy supplied} = LI dI + I^2 dL \quad \text{--- ①} \quad \begin{array}{l} \text{change in energy stored} \\ = \frac{1}{2} (L+dL) (I+dI)^2 - \frac{1}{2} LI^2 \end{array}$$

$$\Delta E = \frac{1}{2} (L+dL) (I^2 + dI^2 + 2IdI) - \frac{1}{2} LI^2$$

$$= \frac{1}{2} L dI^2 + \frac{1}{2} \times 2IL dI + I^2 dL + dI^2 \cdot dL + IdI dL$$

$$= \left( \frac{1}{2} L dI^2 \right) + (IL) dI + \frac{1}{2} I^2 dL + (dL \cdot dI^2) + (IdI \cdot dL)$$

$$\cong (IL) dI + \frac{1}{2} I^2 dL \quad \text{--- ②} \quad \left( \frac{1}{2} L dI^2, dL \cdot dI^2, dI \cdot dL \text{ are negligible} \right)$$

Mechanical Workdone + change in energy stored  $\cong$  Energy supplied

$$\text{Mechanical workdone } dW = T_d \cdot d\theta$$

According to Law of conservation of energy...

$$dW + \frac{1}{2} I^2 dL + (IL) dI = LI dI + I^2 dL$$

$$dW = \frac{1}{2} I^2 dL$$

$$T_d \cdot d\theta = \frac{1}{2} I^2 dL$$

$$\therefore \boxed{T_d = \frac{I^2}{2} \frac{dL}{d\theta}}$$

If  $\frac{dL}{d\theta} = \text{constant}$  then  $(T_d \propto I^2)$

$\therefore$  AC & DC can be used.

Type of control :- Spring control (only control, not current carrying coil)

at steady-state  $T_d = T_c$

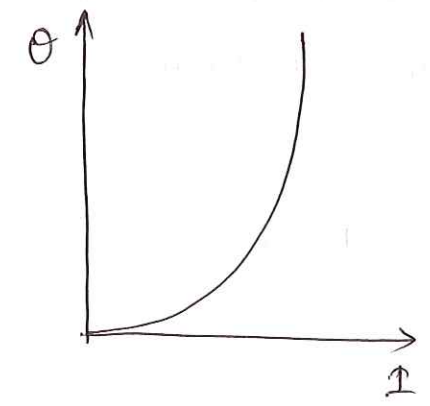
$$K_c \theta = \frac{I^2}{2} \frac{dL}{d\theta}$$

$$\therefore \boxed{\theta = \frac{I^2}{2K_c} \left( \frac{dL}{d\theta} \right)}$$

$\Rightarrow$  Attracted MI type ammeter.

$$\therefore \boxed{\theta \propto I^2}$$

non-linear (non-uniform) scale  
 $\therefore$  scale is cramped at lower end to obtain all readings.



$$I = \frac{V}{Z} = \frac{V}{|R+j\omega L|} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\boxed{\theta = \frac{V^2}{2K_c} \left( \frac{dL}{d\theta} \right)}$$

$\Rightarrow$  Attracted MI type voltmeter

$$\Rightarrow \boxed{\theta = \frac{V^2}{2K_c (R^2 + \omega^2 L^2)} \frac{dL}{d\theta}}$$

advantages

- \* works for both AC & DC.
- \* Reduced frictional errors because heavy part of the instrument is in stationary position that means coil.
- \* Robust construction
- \* Less cost.



### disadvantages :-

1. Non-linear scale  $\Rightarrow$  less accurate at lower ends of scale.

2. weak operating field b/c of electromagnet

$$B = 0.006 \text{ Wb/m}^2 - 0.0075 \text{ Wb/m}^2$$

$$B = (6\text{m} - 7.5\text{m Wb/m}^2).$$

$\therefore$   $T_d$  produced lesser,  $w \rightarrow$  moderate weight.

3.  $\therefore$  Low torque-weight ratio, Low sensitivity

$$S_v = (30 \Omega/V \text{ to } 40 \Omega/V).$$

4. Hysteresis error is more b/c of iron piece, it is a ferro magnetic material.

5. B/c of more hysteresis error the instrument may not follow the perfect square law. So that instrument accuracy is lesser.

6. More power consumption, more internal heating problem.  
more temperature error.

$$7. \theta \propto \frac{V^2}{Z^2} \propto \frac{V^2}{f^2} \propto \frac{(10^3)^2}{(10^5)^2} \propto 10^{-12}$$

This instrument severely suffers from **frequency error** particularly when we are measuring current & voltages in a low voltage & high frequency communication ckt.

Useful frequency range  $\Rightarrow (0 - 125) \text{ Hz}$ .

$\therefore$  MI is best suitable for measurement of V & I

at power frequency. At  $f = 50 \text{ Hz}$  the instrument

accuracy is also good.

Applications :-  $\therefore$  mainly used in Industrial application.

In this instrument the operating field is weak so that stray-magnetic field error is more. So we required some external protection i.e. provided by **wooden body/box**.

$$\theta = \frac{I^2}{2Kc} \left( \frac{dL}{d\theta} \right)$$

$\theta \propto I^2 \Rightarrow$  non-linear

$$\theta = \left( \frac{I}{2Kc} \right) \left( I \frac{dL}{d\theta} \right)$$

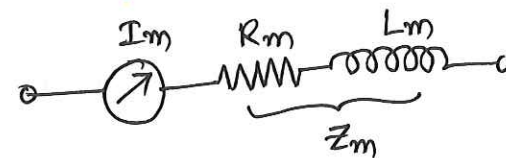
To have scale in linear form  $\therefore \theta \propto I \Rightarrow$  linear  
 $k', \theta = k'I$

$$I \frac{dL}{d\theta} = (\text{constant})_1 \quad (\text{or})$$

$$\theta \frac{dL}{d\theta} = (\text{constant})_2 \quad \text{Linear scale.}$$

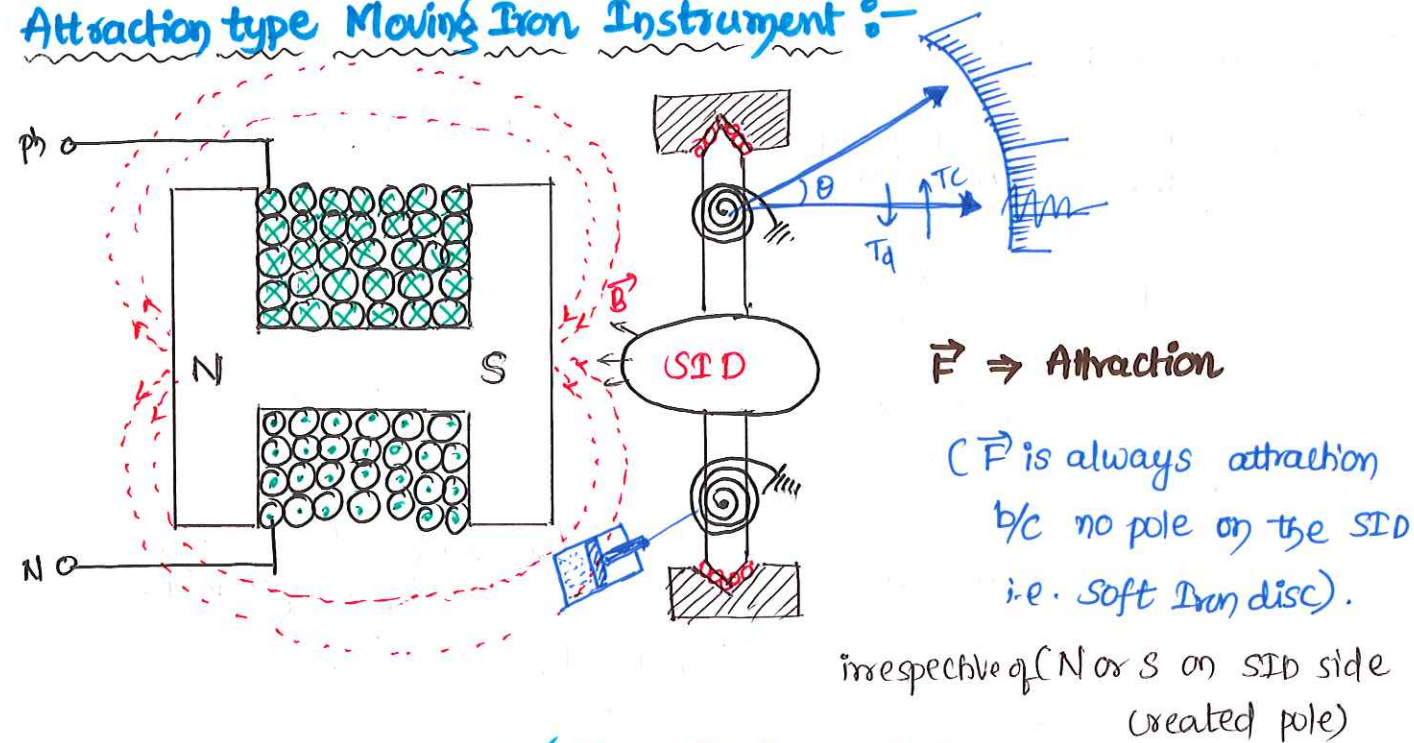
$\therefore$  linear scale  $\Rightarrow \theta \frac{dL}{d\theta} = \text{constant}$  (or)  $I \frac{dL}{d\theta} = \text{constant}$   
 & multiply non-linear scale with  $k'$  to get linear scale.

Electrical Equivalent ckt/diagram :-

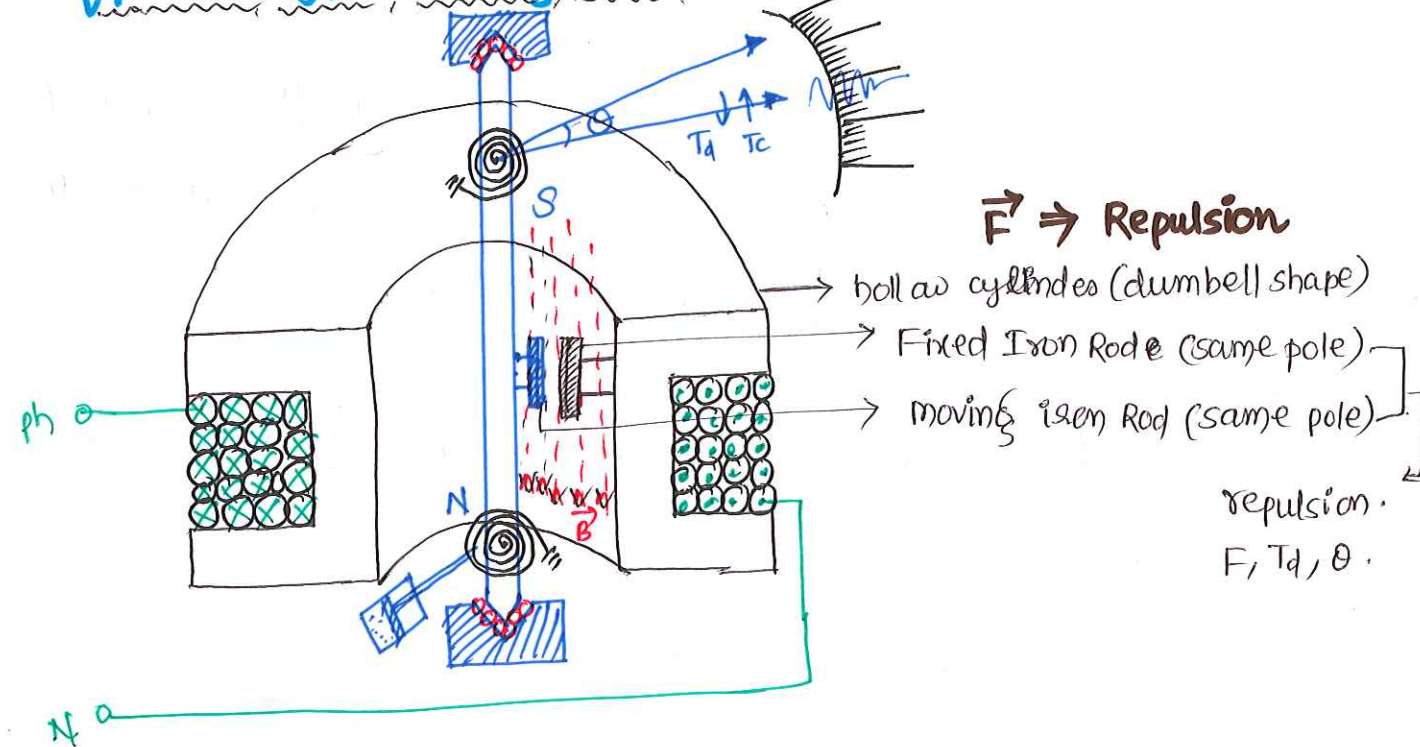


(Diagram)

### Attraction type Moving Iron Instrument :-



### Repulsion Type Moving Iron Instrument :-



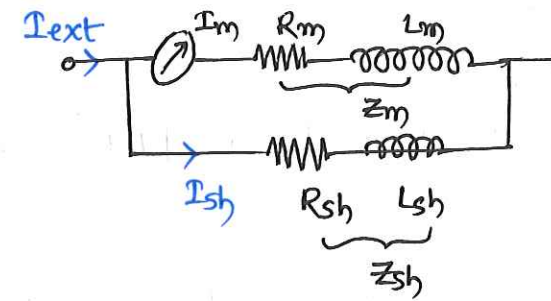
(Inside hollow dumbbell part .... Spindel is placed)

Flux is South to North, two rods get magnetise with same pole on each.



## Extension Range of MI ammeter :-

29



$$I_{\text{extension}} = I = I_m + I_{sh}$$

$$V_m = V_{sh}$$

$$I_m Z_m = I_{sh} Z_{sh}$$

$$\frac{I_m}{I_{sh}} = \frac{Z_{sh}}{Z_m}$$

$$\frac{I_m}{I_{sh}} = \frac{\sqrt{R_{sh}^2 + \omega^2 L_{sh}^2}}{\sqrt{R_m^2 + \omega^2 L_m^2}} \Rightarrow \frac{I_m}{I_{sh}} = \left(\frac{R_{sh}}{R_m}\right) \sqrt{\frac{1 + \omega^2 \left(\frac{L_{sh}}{R_{sh}}\right)^2}{1 + \omega^2 \left(\frac{L_m}{R_m}\right)^2}}$$

$$I_m = f(\text{frequency})$$

$$\frac{I_m}{I_{sh}} = \left(\frac{R_{sh}}{R_m}\right) \sqrt{\frac{1 + \omega^2 \tau_{sh}^2}{1 + \omega^2 \tau_m^2}}$$

$$\text{If } \tau_{sh} = \tau_m \Rightarrow \frac{L_{sh}}{R_{sh}} = \frac{L_m}{R_m}$$

$$\therefore \boxed{\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m}} \text{ i.e. independent of frequency.}$$

$$m = \frac{I_{\text{ext}}}{I_m} = 1 + \frac{R_m}{R_{sh}} \Rightarrow 1 + \frac{Z_{sh}}{Z_m} = m$$

$$R_{sh} = \frac{R_m}{(m-1)} ; \boxed{Z_{sh} = \frac{Z_m}{(m-1)}}$$

$$\therefore m = 1 + \frac{Z_m}{Z_{sh}} = 1 + \sqrt{\frac{R_m^2 + \omega^2 L_m^2}{R_{sh}^2 + \omega^2 L_{sh}^2}}$$

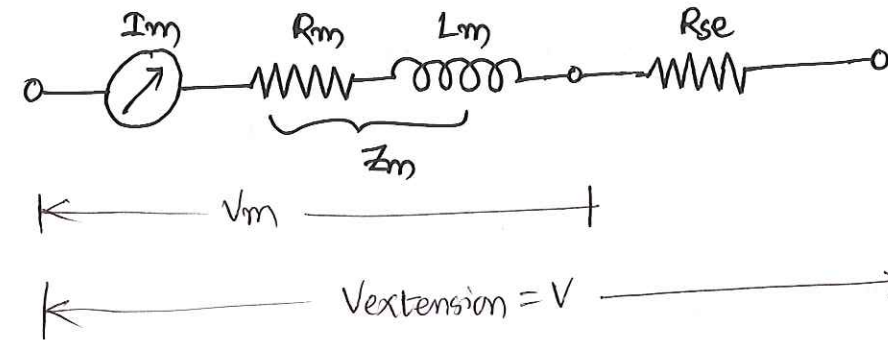
To make the MI ammeter independent of frequency the shunt time-constant should be equal to meter time-constant.

\* All errors are more in MI instrument as compared to PMMC except frictional error.

\*\*\*  $\Rightarrow$  which one of the following error is absent in PMMC instrument

- a) Hysteresis error
  - b) temperature error
  - c) stray-magnetic error
  - d) reactance error (or) frequency error.  $\rightarrow$  Zero error.
- } very very small magnitude

Extension range of MI voltmeter :-



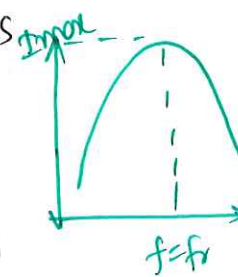
$$m = \frac{V_{ext}}{V_m} = \frac{V}{V_m} = \frac{I_m (Z_{total})}{I_m (Z_m)} = \frac{(R_m + R_{se}) + j\omega L_m}{R_m + j\omega L_m}$$

$$m = \sqrt{\frac{(R_m + R_{se})^2 + \omega^2 L_m^2}{R_m^2 + \omega^2 L_m^2}}$$

$\Rightarrow$  to nullify reactance of inductor can connect capacitor.

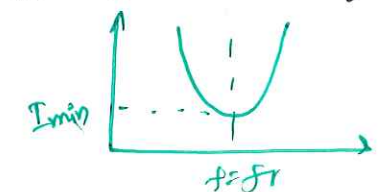
(i) If  $C_{se}$  is connected

RLC series resonance,  
 $Z_{min} = R$   
 $I = I_{min} = I_{coil}$   
 i.e. may saturate coil.

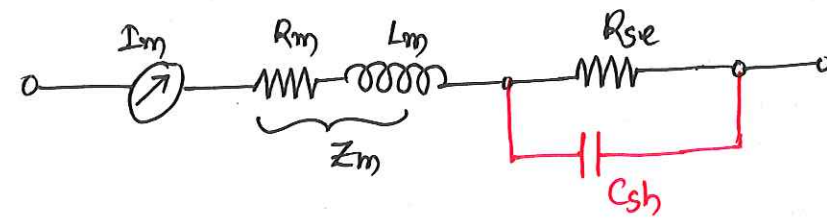


(ii) If  $L_{sh}$  is connected

RLC parallel, resonance  
 $X_L = X_C \Rightarrow Z_{max} = R$   
 $\therefore I = I_{min} = I_{coil}$   
 no... saturation of coil.



calculation of  $C_{sh}$  value :-



$$Z_{total} = (R_m + j\omega L_m) + \frac{R_{se} \frac{1}{j\omega C_{sh}}}{R_{se} + \frac{1}{j\omega C_{sh}}} \times 1$$

$$= (R_m + j\omega L_m) + \frac{R_{se}}{(1 + j\omega C_{sh} R_{se})} \times \frac{(1 - j\omega C_{sh} R_{se})}{(1 - j\omega C_{sh} R_{se})}$$

$$Z_{total} = R_m + j\omega L_m + \frac{R_{se}(1 - j\omega C_{sh} R_{se})}{1 + \omega^2 C_{sh}^2 R_{se}^2}$$

at resonance  $Z = \text{resistive}$ ,  $\text{Im}(Z_{total}) = 0$ .

$$\omega L_m - \frac{R_{se} \omega C_{sh}}{(1 + \omega^2 C_{sh}^2 R_{se}^2)} = 0$$

$$L_m = \frac{C_{sh} R_{se}^2}{(1 + \omega^2 C_{sh}^2 R_{se}^2)}$$

if  $1 \gg \omega^2 C_{sh}^2 R_{se}^2$

$$\therefore L_m = C_{sh} R_{se}^2 \Rightarrow \boxed{C_{sh} = \frac{L_m}{R_{se}^2}} \text{ Theoretical}$$

$$\boxed{C_{sh} = 0.41 \frac{L_m}{R_{se}^2}} \text{ practical (including "f" effect).}$$

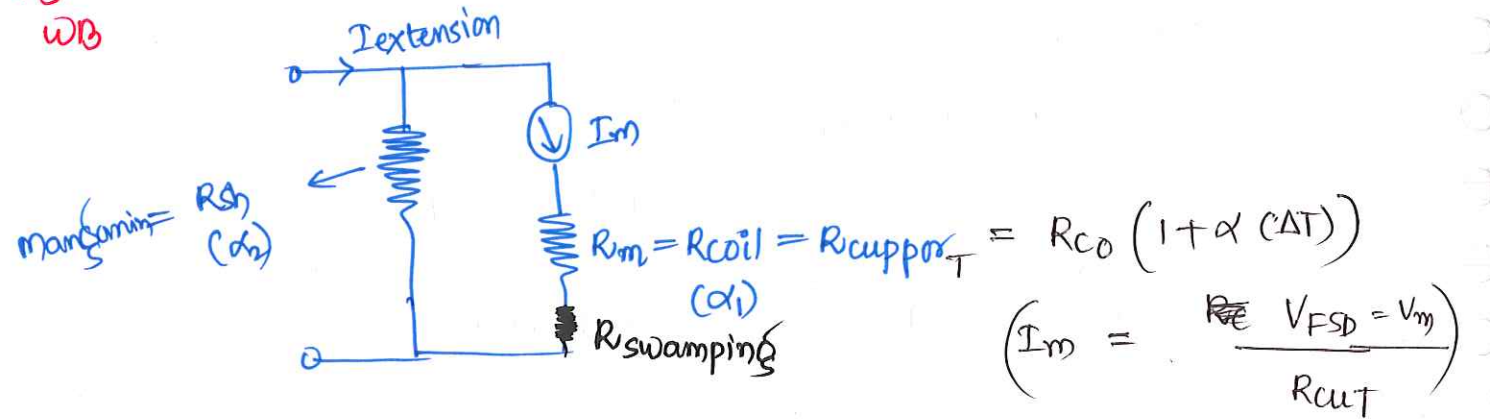


To make the MI voltmeter independent of frequency a capacitor must be connected in parallel with  $R_{se}$ , whose value of capacitance is  $0.41 \frac{L_m}{R_{se}}$ .

**Note:**— MI instrument works on the principle of change in self-inductance (or) minimum reluctance property.

PMMC  
 con. ③ ⇒  
 pg. 10  
 WB

**Swamping** resistance. in series with coil of meter.



$R_{sh} \neq f(\text{temperature})$  (or) very low  $(\alpha_T)$ .

$R_{swamping} \neq f(\text{temperature}) \Rightarrow \alpha_T = \text{constant}$ .  
 (manganin resistance).  $\therefore$  doesn't affected by ~~mangan~~ temperature rise.

practically  $R_{swamp} = (20 - 30) R_{coil}$

\*\*  $\therefore I_m = \frac{V_m}{(R_{cot_T} + R_{sw})}$

coil is prepared by Copper wire which has +ve temperature coefficient, shunt is prepared by manganin, which has constant temperature coefficient (or) negligible temperature coefficient. So that there is a temp. diff will exist b/w these.

To reduce the temperature difference both coil and shunt be made up of same material. For that a resistance (31) which is connected in series with the copper coil whose characteristics are similar to manganin (constant temperature coefficient). The value of resistance is around 20 to 30 times that of copper coil resistance is known as swamping resistance.

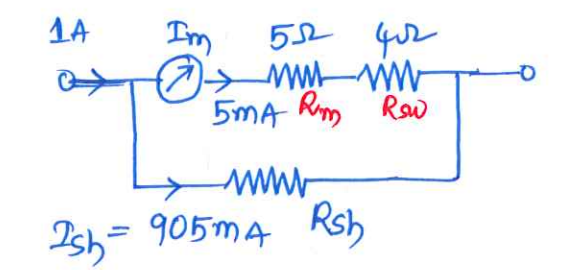
problem:-  $R_m = R_{coil} = 5 \Omega$ .

$$R_{sw} = 4 \Omega$$

$$I_m = 5 \text{ mA}$$

$$I = I_{ext} = 1 \text{ A}$$

$$R_{sh} = ?$$



$$\therefore R_{sh} = \frac{(R_m + R_{sw})}{\left(\frac{I}{I_m} - 1\right)} ; \quad m = \frac{I}{I_m}$$

$$R_{sh} = \frac{5 + 4}{\left(\frac{1}{5 \times 10^{-3}} - 1\right)} = \frac{9}{\left(\frac{10 \times 10^3}{5} - 1\right)} = \frac{9}{199} = 0.0452261 \Omega$$

$$R_{sh} = 45 \text{ m}\Omega$$

(P) which one of the following instrument works on the principle of change in mutual inductance

- Sol:-
- (a) PMMC
  - (b) MI
  - (c) EMMC
  - (d) All

# Electro magnetic moving coil (EMMC)

(or)

## Electro dynamo type instrument (EDM) :-

1. Fixed coil  $\Rightarrow$  electromagnet
2. Moving coil

$$T_d \propto i_1 i_2$$

$$T_d \propto \phi i_2$$

$$T_d \propto B i_2$$

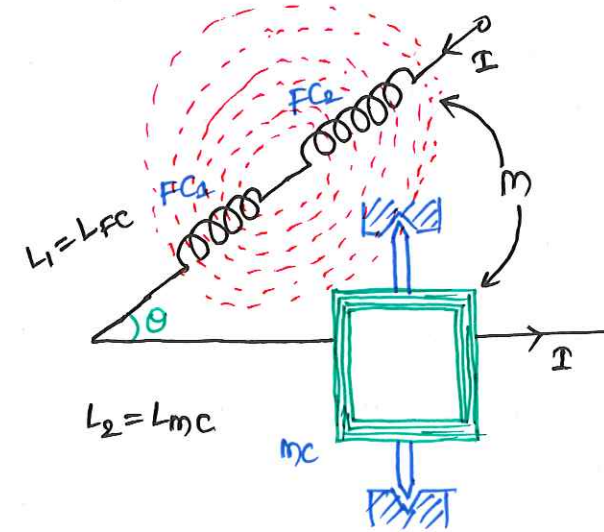
$$T_d = \frac{I^2}{2} \frac{dL}{d\theta} \Rightarrow mI$$

$$T_d = \frac{I^2}{2} \frac{d(m\phi)}{d\theta} \Rightarrow mI$$

$$L_{eq} = L_1 + L_2 + 2m$$

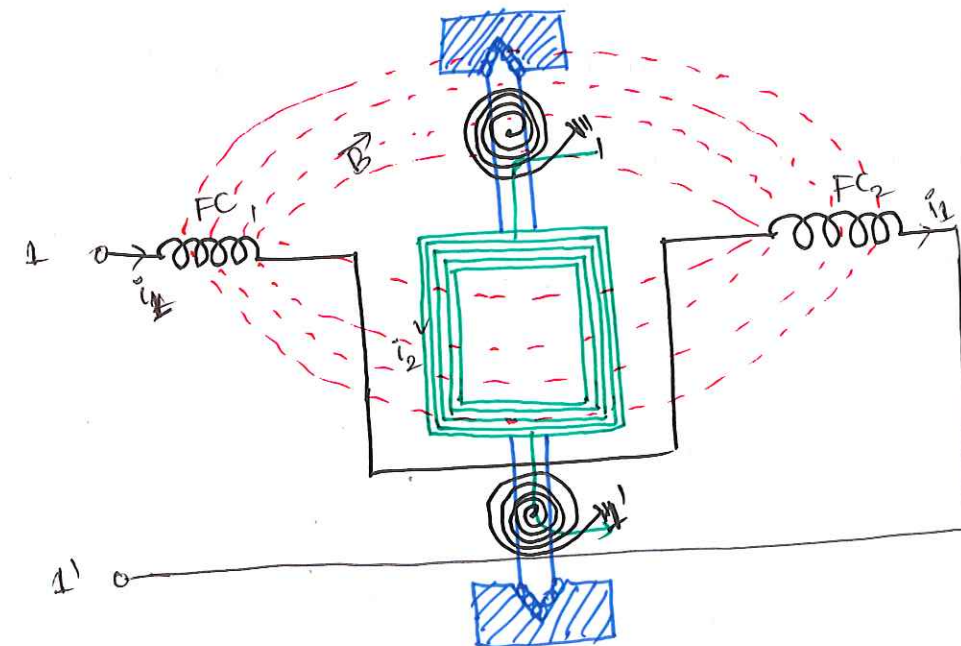
$$\therefore T_d = \frac{I^2}{2} \frac{d(L_1 + L_2 + 2m)}{d\theta}$$

$$\therefore T_d = I^2 \frac{dm}{d\theta} \quad \text{EMMC.}$$



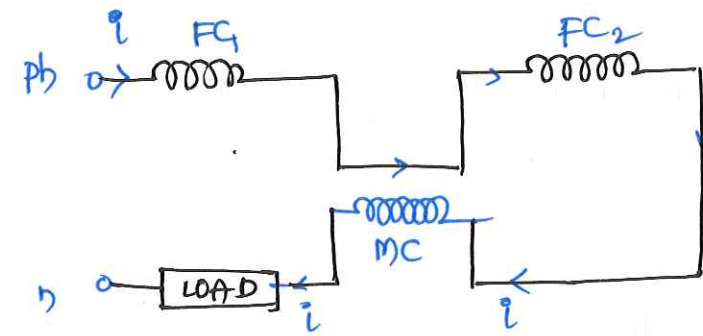
$L_1 = L_{fc} = \text{constant}$   
 $L_2 = L_{mc} = \text{constant}$   
 $m = \text{variable}$

$$\text{If } \frac{dm}{d\theta} = \text{constant} \Rightarrow T_d \propto I^2$$

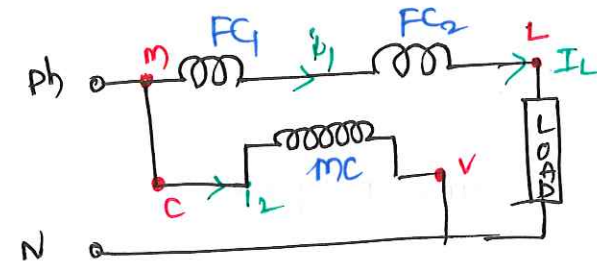




In case of (A), (V)  $\Rightarrow$  FC & M.C are in series  $\Rightarrow i_1 = i_2 = i$   
 $i_{FC} = i_{MC} = i$  (32)



In case of (W)  $\Rightarrow$  F.C & M.C are connected parallel  
 $i_1 \neq i_2$



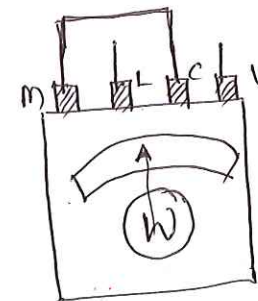
FC = current coil  
 connected in series with load.

Fixed coil  $\equiv$  current coil  $\equiv$  series coil  $\equiv$  field coil

$i_L = i_1 = \text{large} \therefore \Rightarrow$  thick-wire, weight of Fixed coil more

$i_2 = i_{PC} = \text{small} \therefore \Rightarrow$  thin-wire, weight of moving coil less

(pressure coil  $\equiv$  moving coil  $\equiv$  voltage coil).



$\Rightarrow$  air-core coil (no saturation & no hysteresis error).

The energy stored in the form of magnetic field....

$$E = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

$$\frac{dE}{d\theta} = 0 + 0 + I_1 I_2 \frac{dM}{d\theta}$$

Mechanical work done  $dw = T_d \cdot d\theta$

$$\therefore \frac{dw}{d\theta} = T_d$$

stored energy change  $\equiv$  mechanical work done on pointer.

$$\frac{dw}{d\theta} = \frac{dE}{d\theta}$$

$$T_d = I_1 I_2 \frac{dm}{d\theta}$$

$$\therefore T_d \propto I_1 I_2 \quad \text{if } \frac{dm}{d\theta} = \text{constant.}$$

$$T_d \propto \phi i_2$$

type of control :- Spring control (dual purpose)

(i) Controlling torque

(ii) Current carrying element.

$$T_c = K_c \theta.$$

at steady-state ;  $T_c = T_d.$

$$K_c \theta = I_1 I_2 \frac{dm}{d\theta}.$$

$$\therefore \theta = \frac{I_1 I_2}{K_c} \frac{dm}{d\theta} \quad \frac{dm}{d\theta} = \text{constant}$$

$$\theta \propto i_2 \rightarrow \text{EDM } \textcircled{W}.$$

In case of  $\textcircled{A}$  &  $\textcircled{V}$   $\Rightarrow i_1 = i_2 = I.$

$$\therefore \theta = \frac{I^2}{K_c} \frac{dm}{d\theta}$$

$$\theta \propto I^2 \rightarrow \text{EDW } \textcircled{A}, \textcircled{V}.$$

$\theta \propto I^2 \Rightarrow \therefore$  instrument works for both AC & DC  
 $\Rightarrow$  non-linear (or) non-uniform scale. (33)

$$i = \frac{V}{Z} \Rightarrow \boxed{\theta = \frac{I^2}{K_c} \frac{dm}{d\theta} = \frac{V^2}{Z^2 K_c} \frac{dm}{d\theta}}_{\text{EDM-(V)}}$$

$\therefore \boxed{\theta \propto \frac{V^2}{f^2}}$  frequency error  $\swarrow$  Low voltage  
 $\searrow$  High frequency

advantages :-

- \* works for AC & DC
- \* hysteresis errors are completely absent.  
b/c air-core coils
- \* EDMC type instruments are known as transfer instruments

The instruments which are working for both AC as well as DC, the instrument 1st it used for DC then without making any changes if the same instrument is used for AC supply. If it reads/gives the correct instruments, those instruments are (reading) said to be transfer instruments. These transfer instruments are used in the process of calibration of other instruments (AC)



### disadvantages :-

- \* Non-uniform scale.
- \* weak operating field b/c of electromagnets

$$B = 0.005 \text{ wb/m}^2 \text{ to } 0.006 \text{ wb/m}^2$$

$$\bar{B} = 5 \text{ mwb/m}^2 \text{ to } 6 \text{ mwb/m}^2$$

$$B_{\text{pmmc}} > B_{\text{MI}} > B_{\text{EMMC}}$$

$$\left(\frac{T_d}{W}\right)_{\text{pmmc}} > \left(\frac{T_d}{W}\right)_{\text{MI}} > \left(\frac{T_d}{W}\right)_{\text{EMMC}}$$

$$(S)_{\text{pmmc}} > (S)_{\text{MI}} > (S)_{\text{EMMC}} ; \text{ sensitivity.}$$

- \* Low torque to weight ratio.
- \* Low sensitivity.
- \* More power consumption.
- \* More internal heating problem.
- \* Temperature error is more
- \* Frictional error is more.
- \* stray magnetic field error is more.
- \*  $T_d \propto \frac{V^2}{f^2} \Rightarrow \theta \downarrow \propto \frac{V^2 \downarrow}{f^2 \uparrow}$

This instrument severely suffers from frequency error.

Useful frequency range : (0-125) Hz

\* In case low-grade instruments (accuracy is less around greater than  $\pm 5\%$  of full-scale deflection). (34)

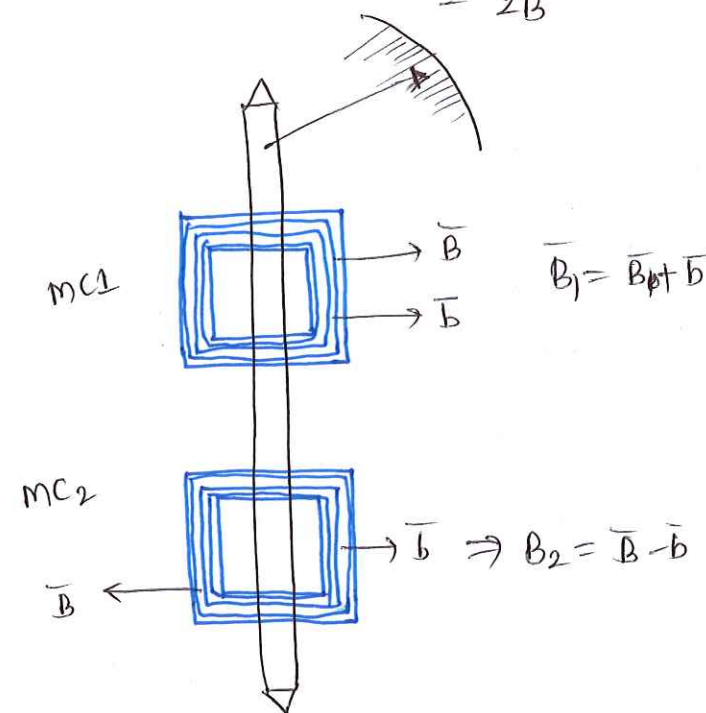
$$f = 10 \text{ Hz to } 1000 \text{ Hz}$$

\* Even we can use upto  $f = 10 \text{ kHz}$  by using.

### ASTATIC arrangement.

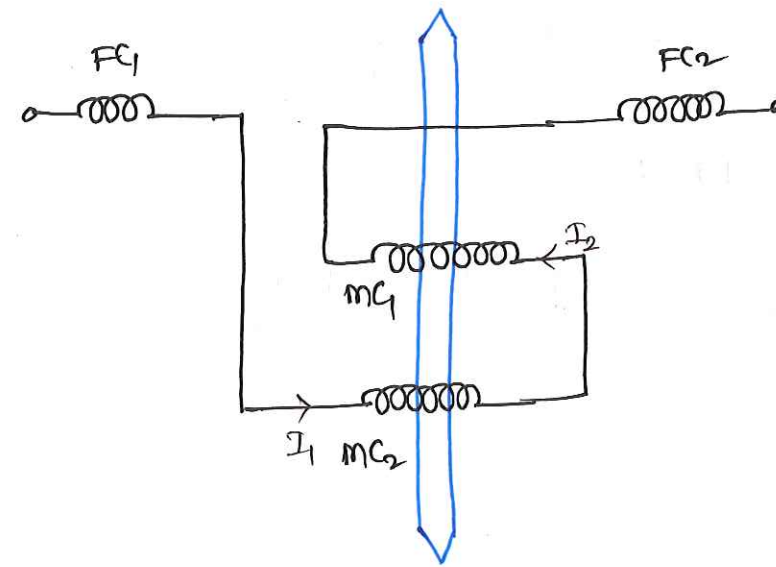
In this, there are two moving coils placed on same spindle, they are connected in such a manner. The flux produced by two coils equal in magnitude and acting opposite in direction. Let  $\vec{B}$  is the operating field produced by coil, Let  $\vec{b}$  is the stray-magnetic field.

$$\begin{aligned} \therefore \vec{B}_{\text{net}} &= \vec{B}_1 + \vec{B}_2 \\ &= \vec{B} + \vec{b} + \vec{B} - \vec{b} \\ &= 2\vec{B} \end{aligned}$$



### advantages of astatic arrangement

- \*  $Td \propto 2B$
- \* Accuracy increased
- \* Stray magnetic field error is absent.
- \* Sensitivity increases.
- \* frequency range of instrument is extended.



(P) The EMMC instrument follows the square law

(a) over a entire operating range of instrument.

(b)  $\theta = (0-90^\circ)$

(c)  $-90^\circ \leq \theta \leq 90^\circ$

(d)  $\theta = (-45^\circ \text{ to } 45^\circ)$

$$\theta = \frac{I_1 I_2}{k_c} \frac{dm}{d\theta} = \frac{I^2}{k_c} \frac{dm}{d\theta}$$

$$T_d = I^2 \frac{dm}{d\theta}$$

$$M = K \sqrt{L_1 L_2} \quad ; \quad K \rightarrow \text{coefficient of coupling.}$$

$K=1 \Rightarrow$  for perfectly couple.

$K=0 \Rightarrow$  No coupling (decoupled).

$$0 \leq K \leq 1.$$

$$0 \leq \cos\theta \leq 1.$$

$$M = \cos\theta \sqrt{L_1 L_2}$$

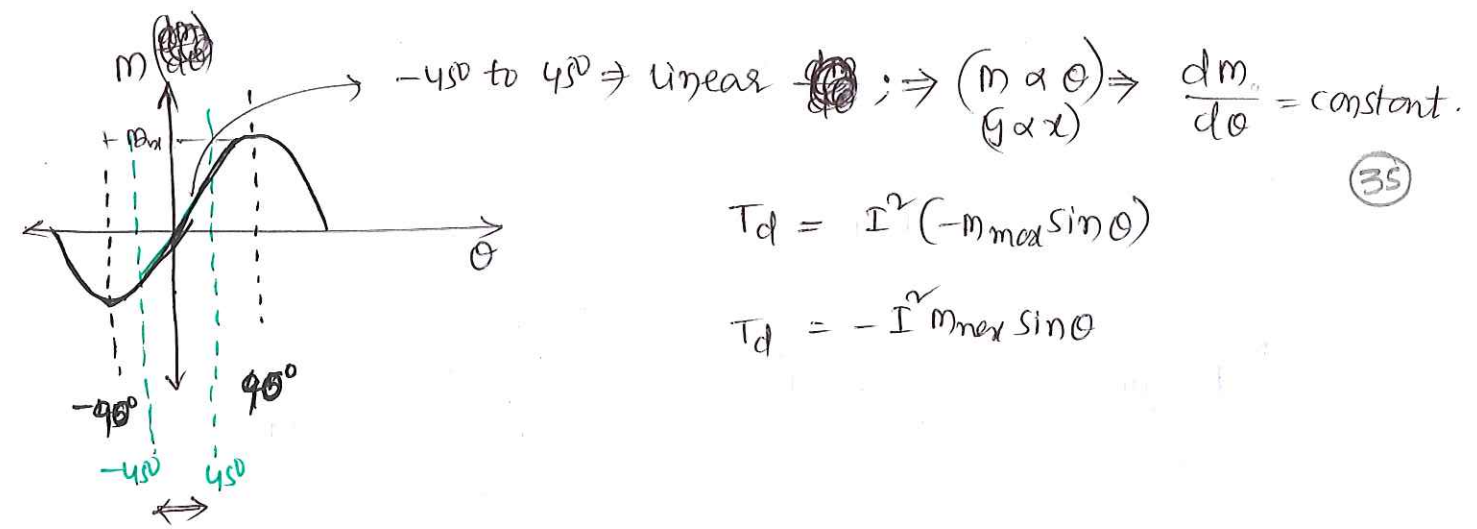
$$M_{\max} = \sqrt{L_1 L_2}$$

$$M_{\min} = 0.$$

$$\therefore M = M_{\max} \cos\theta.$$

$$\frac{dM}{d\theta} = -M_{\max} \sin\theta.$$

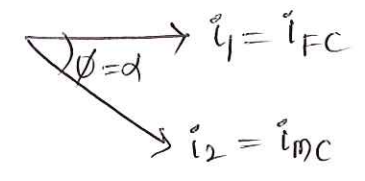




$$T_d = I^2 (-m_{max} \sin \theta)$$

$$T_d = -I^2 m_{max} \sin \theta$$

In case of AC Supply :- (EMMC)



Let  $i_1 = I_{m1} \sin \omega t$

$i_2 = I_{m2} \sin(\omega t - \phi)$

$\omega = 2\pi f = \frac{2\pi}{T}$

i.e.  $i_2$  lags  $i_1$  by  $\phi = \alpha$ .

$$\begin{aligned}
 P_{avg} &= \frac{1}{T} \int_0^T (i_1 i_2 \frac{dm}{d\theta}) dt \\
 &= \frac{1}{T} \int_0^T I_{m1} I_{m2} \sin \omega t \cdot \sin(\omega t - \phi) dt \\
 &= \frac{I_{m1} I_{m2}}{2T} \int_0^T 2 \sin \omega t \cdot \sin(\omega t - \phi) dt \\
 &= \frac{I_{m1} I_{m2}}{2T} \left[ \int_0^T [\cos \phi - \cos(2\omega t - \phi)] dt \right] \\
 &= \frac{I_{m1} I_{m2}}{2T} \left( \int_0^T \cos \phi dt - \int_0^T \cos(2\omega t - \phi) dt \right) \\
 &= \frac{I_{m1} I_{m2}}{2T} \left[ (\cos \phi) T - \left[ \frac{\sin(2\omega t - \phi)}{2\omega} \right]_0^T \right] \\
 &= \frac{I_{m1} I_{m2}}{2} \cos \phi - \frac{I_{m1} I_{m2}}{2} [0 - 0].
 \end{aligned}$$

$$\therefore T_{avg} = \frac{I_{m1} I_{m2} \cos\phi}{2}$$

$$T_{avg} = (I_{rms1}) (I_{rms2}) \cos\phi$$

$$(Torque)_{avg} = (I_{rms1}) (I_{rms2}) \cos(\phi = \alpha)$$

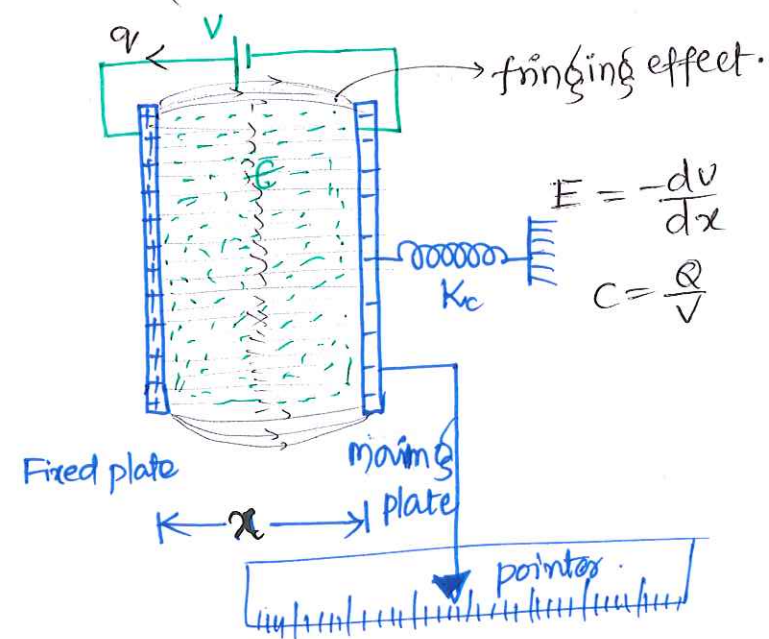
EDM type instrmt.

In case of AC supply, the average torque is calculated in terms of RMS value of fixed coil current and moving coil current.

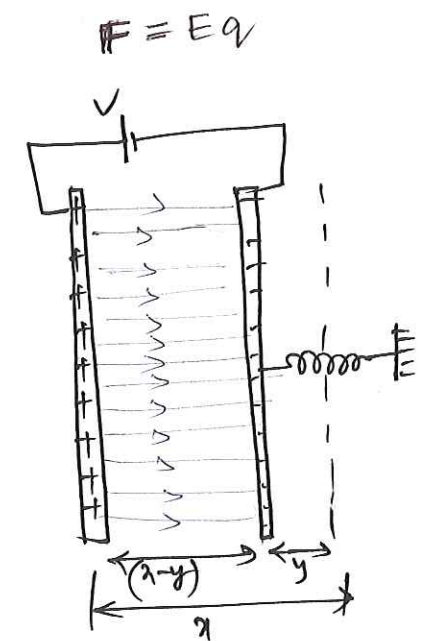
### Electrostatic Voltmeter :-

It works on the principle of change in capacitance, It reads only voltage b/c basically it is voltmeter.

(mobility of +ve charge) < (mobility of -ve charge)



Lines-moment of pointer on Non-linear scale.



$$C = \frac{AE}{x} \quad (\because C = \frac{AE}{d})$$

(36)

$$C' = \frac{AE}{(x-y)}$$

Pressure  $\uparrow$   $\Rightarrow$  Force  $\uparrow$   $\Rightarrow$  Capacitance  $\uparrow$   
(or)  
stress

$$C' = C + \Delta C$$

$$V' = V + \Delta V$$

$\therefore$  it is called as lo... as...

Capacitive transducer.

$$Q = CV$$

$$dQ = C dV + (dC) V$$

$$V dQ = (CV) dV + V^2 (dC)$$

$\Rightarrow$  No current flow through meter only displacement current not conventional current  $\therefore$  ESI ammeter won't exist.

$$\text{Energy supplied} = CV dV + V^2 dC$$

$$\text{Energy stored} = \frac{1}{2} CV^2$$

$$\text{Incremental energy stored} = \frac{1}{2} (C + dC) (V + dV)^2$$

$$\text{change in energy stored} = \frac{1}{2} (C + dC) (V + dV)^2 - \frac{1}{2} CV^2$$

$$= \frac{1}{2} (C + dC) (V^2 + dV^2 + 2V dV) - \frac{1}{2} CV^2$$

$$= \frac{1}{2} CV^2 + \frac{1}{2} C dV^2 + CV dV + \frac{1}{2} dC V^2 + \frac{1}{2} dC dV^2 + \frac{1}{2} dC (2V dV) - \frac{1}{2} CV^2$$

$$= \left( \frac{1}{2} C dV^2 + \frac{1}{2} dC dV^2 + V dC dV \right) + CV dV + \frac{1}{2} V^2 dC$$

$$\approx CV dV + \frac{1}{2} V^2 dC$$



Energy supplied = change in energy stored + mechanical energy stored in spring.  
(work done)

$$(v^2 dc + cv dv) = \left( \frac{1}{2} v^2 dc + cv dv \right) + (\text{Force}) \times (\text{displacement})$$

$$\frac{1}{2} v^2 dc = F_d dx = F_y$$

$$F_d = \frac{1}{2} v^2 \left( \frac{dc}{dx} \right)$$

$\therefore$   $F_d = \frac{v^2}{2} \left( \frac{dc}{dx} \right)$  Voltage responsive meter.  
both AC & DC. ✓

If  $\frac{dc}{dx} = \text{constant}$ ,  $\therefore F_d \propto v^2$

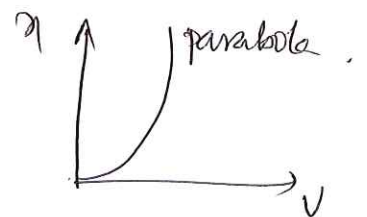
Type of control - Helical spring. 

$F_c = kx$   $k = \text{spring constant}$ .

At steady-state;  $F_d = F_c$

$$\frac{v^2}{2} \frac{dc}{dx} = kx$$

$$\therefore x = \frac{v^2}{2k} \frac{dc}{dx}$$



$\frac{dc}{dx} = \text{constant}$ ,  $x \propto v^2$   $\Rightarrow$  scale is non-linear

$\Rightarrow$  used to measure both AC, DC.

scale is non-linear, motion of the pointer is linear.

In a parallel plate electrostatic voltmeter, the motion of the pointer is linear but the scale is non-linear. (31)

### Advantages :-

- \*  $F_d \propto V^2$   
 $\therefore$  both ac & dc can measure.
- \* No magnetic field  $\Rightarrow$  No hysteresis error
- \* No stray magnetic field error.
- \* No current flow, only ~~has~~ static charge,  
 $\therefore$  So that No power loss. No internal heating problem.
- \* So that there is no temperature error.
- \* It draws practically zero current from the supply.
- \* The waveform & frequency errors are unimportant.

### Disadvantages :-

- \* Non uniform scale
- \*  $(F_d \propto V^2)$  only not for measurement of current.
- \* Only for high voltage measurements i.e. of the order (kV).
- \* When we are measuring low voltages, the produced deflecting force is very small, in the order of  $\mu$  Newtons.

$$F_d \propto (mV)^2 \Rightarrow \mu N \Rightarrow \text{no deflection}$$

$$F_d \propto (kV)^2 \Rightarrow MN$$

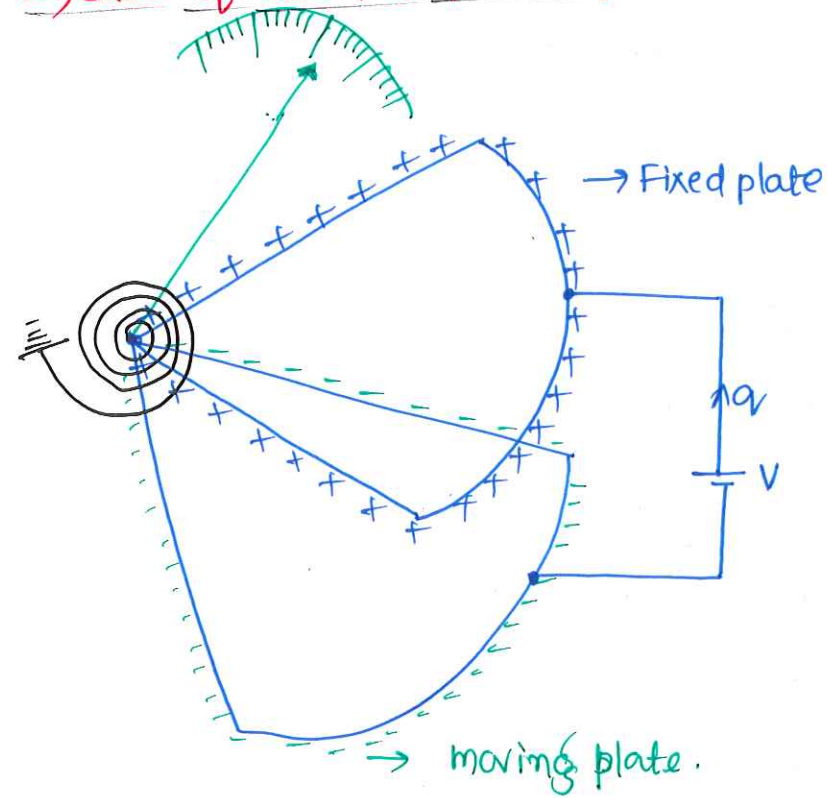
$\therefore$  This is not suitable for low voltage measurement but best & suitable for high voltages like in power systems

- \* Costly (expensive)

voltage range (500 volts - 250 kV).

In case of circular plates :-

(plates are rotating)



Mechanical work done  
 $= dw = T_d \cdot d\theta$

According to law of conservation of energy :-

Energy Supplied =  
 change in energy + Mech. Work done

$$cv dv + (dc) v^2 = cv dv + \frac{1}{2} v^2 dc + (T_d d\theta)$$

$$\frac{1}{2} v^2 dc = T_d d\theta$$

$$\therefore T_d = \frac{v^2}{2} \left( \frac{dc}{d\theta} \right)$$

A = circular plate area

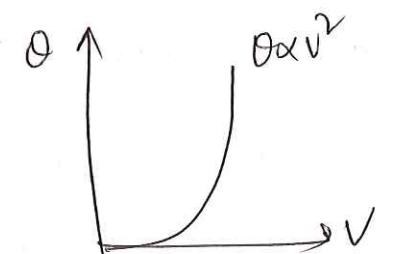
if  $\frac{dc}{d\theta} = \text{constant} \Rightarrow T_d \propto v^2 \Rightarrow AC \& DC$

Type of control :- Spring control (spiral spring)  $T_c = K_c \theta$

at steady state  $T_c = T_d$

$$K_c \theta = \frac{v^2}{2} \frac{dc}{d\theta}$$

$$\theta = \frac{v^2}{2 K_c} \frac{dc}{d\theta}$$



if  $\frac{dc}{d\theta} = \text{constant}$  ;  $\theta \propto v^2 \Rightarrow$  non-linear, rotational (or) circular scale.



### Types of ESV :-

(38)

1. Repulsion type  $\Rightarrow$  ( $< 10\text{KV}$ )  $\Rightarrow$  (Gold Leaf ESV)
2. Attractive type  $\Rightarrow$  ( $> 20\text{KV}$ )  $\Rightarrow$  (Kelvin-absolute electrometer)
3. Attractive & repulsion type  $\Rightarrow$  (Symmetrical type ESV)  
( $10\text{KV} - 20\text{KV}$ )

$$F_d = \frac{V^2}{2} \left( \frac{dC}{dx} \right) = \frac{V^2}{2} \left( \frac{e}{x} \right) \quad ; \quad C = \frac{A\epsilon}{x} = \frac{A\epsilon_0\epsilon_r}{x}$$

$$= \frac{V^2}{2} \frac{A\epsilon}{x}$$

$$F_d = \frac{V^2}{2} \frac{A\epsilon}{x^2} \Rightarrow F_d = \frac{V^2}{2x^2} (A\epsilon)$$

$$F_d = \left( \frac{A\epsilon}{2} \right) \left( \frac{V^2}{x^2} \right)$$

$$V = \sqrt{\frac{2 F_d x^2}{A\epsilon}}$$

$$V = x \sqrt{\frac{2 F_d}{A\epsilon}}$$

Kelvin-absolute electrometer.

If any instrument gives the reading in terms of the physical parameters (or) constants of instrument... then it is called Absolute instrument.

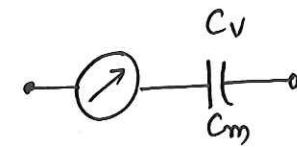
Absolute meters.

- relay current meter balance meter - current
- tangent galvanometer
- Lorentz force meter - resistance
- Kelvin-absolute electrometer - voltage

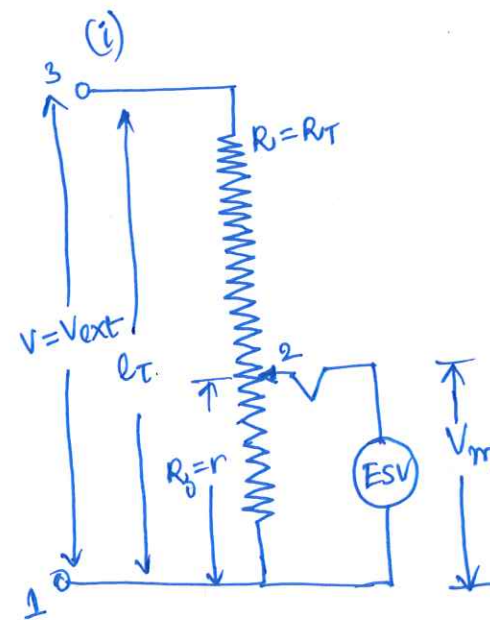
Absolute instruments are used in the process of calibration of other instruments.

- $F_d = \frac{V^2 dC}{2 dm}$
- (i) Kelvin-absolute electrometer.  $V = x \sqrt{\frac{Fd(2)}{AE}}$
  - (ii) Rayleigh current balance meter  $F_d = I^2 \frac{dm}{dx}$
  - (iii) Loretz force meter
  - (iv) Tangent galvanometer.

Extension Range of ESV :-



- (i) It can be obtained by using potentiometer
- (ii) By using external capacitor (a) series capacitor
- (iii) Multiple series capacitors



$$\left\{ \begin{array}{l} \text{Length} \\ \text{Ratio} \end{array} \right\} = \left\{ \begin{array}{l} \text{Voltage} \\ \text{Ratio} \end{array} \right\} = \left\{ \begin{array}{l} \text{Resistance} \\ \text{Ratio} \end{array} \right\}$$

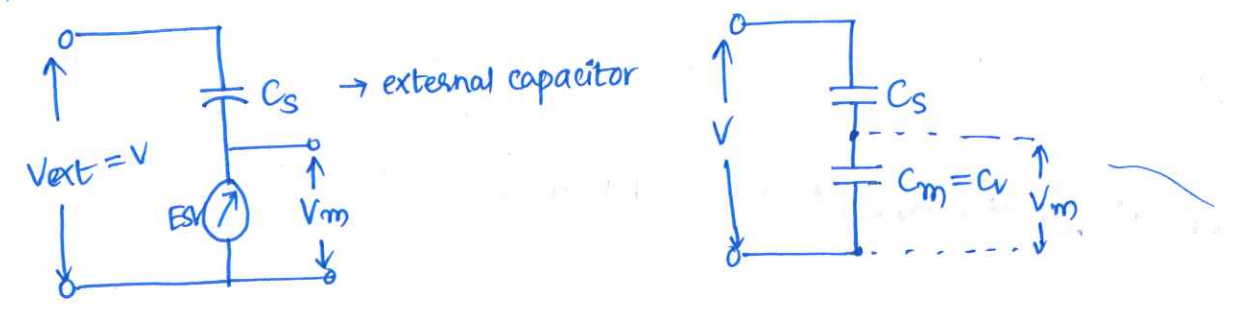
$$\frac{x}{l_t} = \frac{R_0}{R_t} = \frac{V_m}{V_{ext}}$$

$$\frac{x}{l_t} = \frac{r}{R} = \frac{V_m}{V} = \frac{1}{m}$$

$$m = \text{multiplication factor} = \frac{V_{ext}}{V_m} = \frac{V}{V_m}$$

$$\therefore m = \frac{R}{r} \quad \therefore \Rightarrow V_{ext} = V = \left( \frac{R}{r} \right) V_m$$

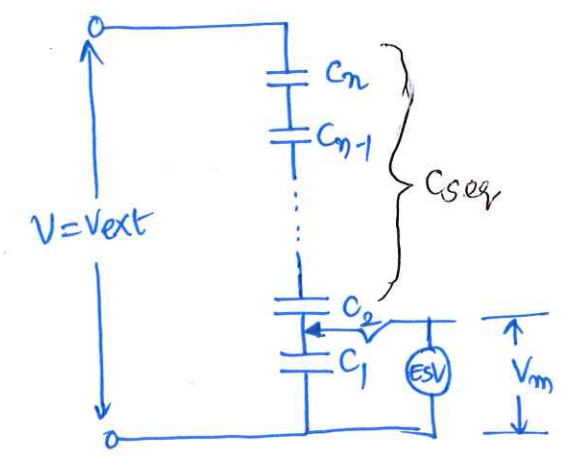
(ii)



$$V_m = \left( \frac{C_s}{C_s + C_m} \right) V_{ext} \Rightarrow m = \frac{V_{ext}}{V_m} = 1 + \frac{C_m}{C_s} = \text{multiplication factor.}$$

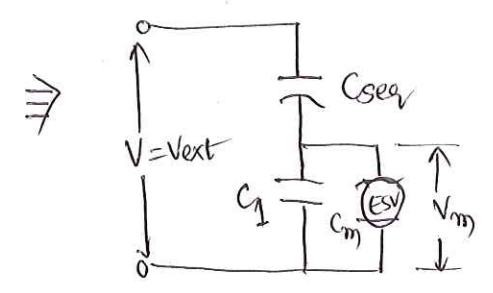
$$\therefore C_s = \frac{C_m}{(m-1)}$$

(iii) Multiple series capacitors



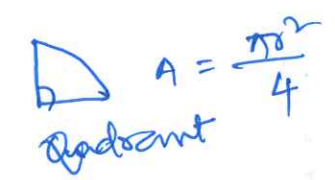
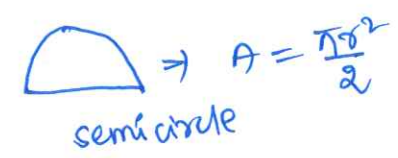
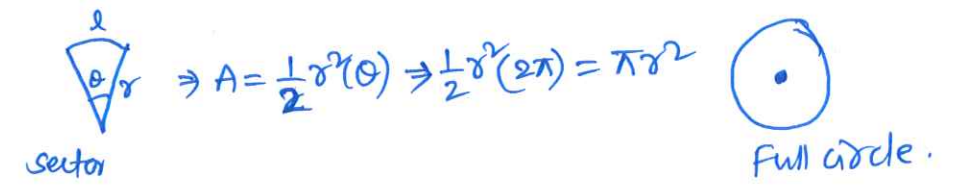
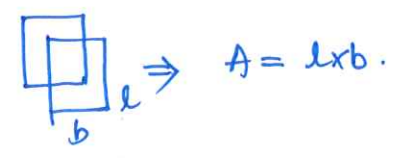
Let  $C_{seq}$  is the equivalent capacitance of series capacitor except the capacitance across ESV meter.

$$\frac{1}{C_{seq}} = \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$



$$\therefore V_m = \left( \frac{C_{seq}}{C_{seq} + C_1 + C_m} \right) V_{ext}$$

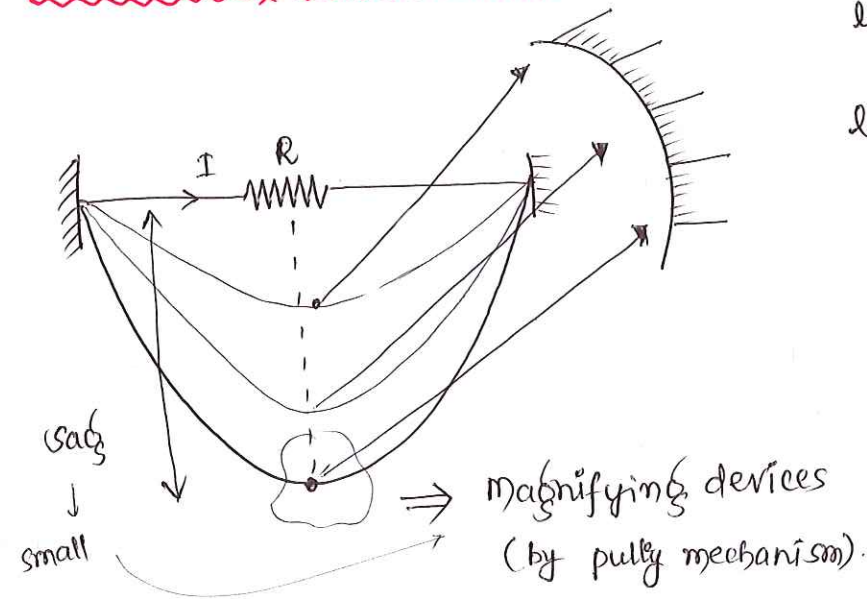
$$\therefore m = \frac{V}{V_m} = 1 + \left( \frac{C_m + C_1}{C_{seq}} \right)$$





which one of the following instrument works on the principle of heating effect ..... Thermal instrument.

### Thermal Instrument :- (RF instruments); Radio frequency



$$l_t = l_0 (1 + \alpha \Delta t)$$

$$l_t - l_0 = l_0 \alpha \Delta t$$

$$\Delta l \uparrow = l \alpha \Delta t \uparrow$$

$$\theta \propto (\text{sag}) \propto (\Delta l) \propto (\Delta t) \propto (\text{heat produced}) \propto (I^2 R).$$

Let  $\Rightarrow$   $l \rightarrow$  length of wire in 'm'

$R \rightarrow$  resistance of wire in  $\Omega$

$I \rightarrow$  current flow.

$H \rightarrow$  Rate of heat generation in "watt"

$S_a \rightarrow$  Heat dissipating surface (area) -  $m^2$

$\Delta t \rightarrow$  temperature rise in  $^{\circ}C$

$U \rightarrow$  coefficient of heat dissipation in  $\text{watt}/m^2^{\circ}C$ .

$\alpha \rightarrow$  temperature coefficient

$$\therefore U = \frac{H}{S_a(\Delta t)} \Rightarrow \Delta t = \frac{H}{U S_a} = \frac{\Delta l}{l \alpha}$$

$$\therefore \Delta l = \frac{l \alpha H}{U S_a}$$

$$\therefore \Delta l = \frac{l \alpha}{U S_a} (I^2 R) \quad \text{Hot wire ammeter}$$

$\Theta \propto (\Delta l (s a \xi)) \propto I^2$   $\Rightarrow$  working for both AC & DC  
 it reads always RMS value.  
 $\Rightarrow$  scale is non-uniform.

(40)

$$\Delta l = \frac{l \alpha}{U_{Sa}} (I^2 R) \Rightarrow \Delta l = \frac{l \alpha}{U_{Sa}} \left( \frac{V^2}{R} \right)$$

Hotwire voltmeter.

### Advantages :-

- \* Works for both AC & DC
  - \* No magnetic field  $\therefore$  there is no stray-magnetic field error.
  - \* No hysteresis error.
  - \* No reactance term in the principle, **No frequency error.**
- $\therefore$  Thermal instruments can be used in a frequency range of several mega Hz. i.e. upto radio frequency range.  
 So that thermal instruments popularly known as RF instruments.

### Disadvantages :-

- \* Non-uniform scale.
- \* Difficult construction.
- \* The **ambient temperature** will affect the actual readings of the instruments. So that the instrument **accuracy is lesser.**
- \* Thermal instruments are **slow (sluggish) in response** because the time taken by the wire to get heat up.
- \* **Skin effect** is more pronounced when we are used at high frequency.

$$\text{Skin effect} \propto \left( \frac{d^2 f \eta r}{\rho} \right)$$

$$\therefore (\text{Skin effect}) \propto f$$

- \* To reduce the skin effect in this instruments tubular or hollow conductors are preferred  $I > 3 \text{ Amp}$ .
- \* Thermal instruments has limited over-loading capability around 150%. otherwise wire gets damaged (breaks).

In produced sag is given by

$$sag = s = \sqrt{\frac{1}{2} l \Delta l}$$

### Magnification of sag :-

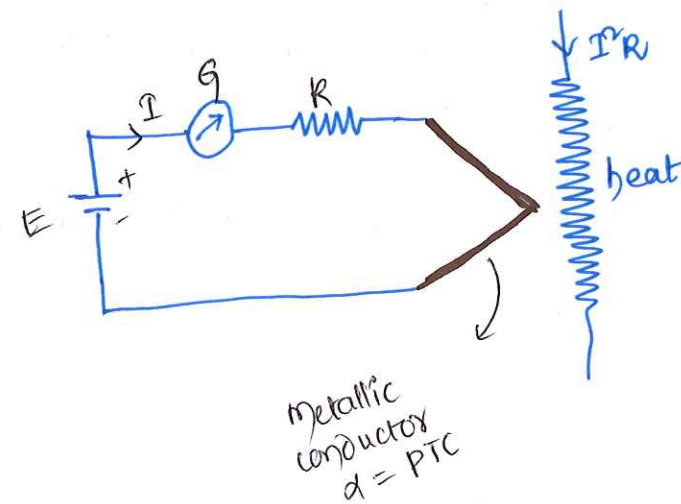
It is defined as the ratio of produced sag to the change in length.

$$m = \frac{sag}{\Delta l} = \frac{\sqrt{\frac{1}{2} l \Delta l}}{\Delta l} = \sqrt{\frac{l}{2(\Delta l)}}$$

$$m = \sqrt{\frac{l}{2(\Delta l)}}$$

### Resistance Temperature Detector :-

RTD is a temperature measuring device, which works on the principle of change in resistance of a metallic conductor due to heat produced by heater element, which has positive temperature coefficient. (PTC)



- by
- (i) conduction — contact
  - (ii) convection
  - (iii) Radiation. } non-contact type.



$$I_1 = \frac{E}{R}$$

$$I_2 = \frac{E}{R_1} = \frac{E}{(R + \Delta R)}$$

(41)

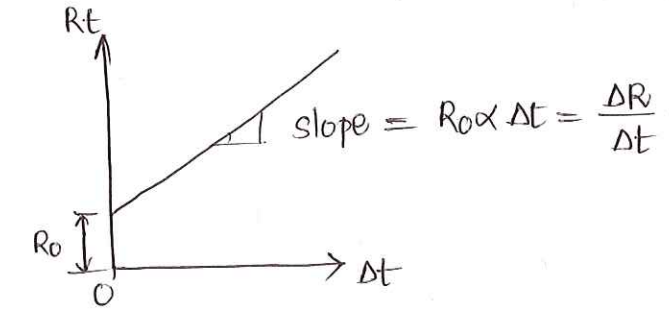
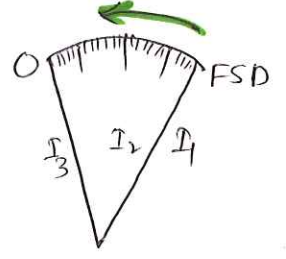
$$R_t = R_0(1 + \alpha \Delta t)$$

$$\therefore I_2 < I_1$$

$$\Delta R = (R_t - R_0) = R_0 \alpha \Delta t$$

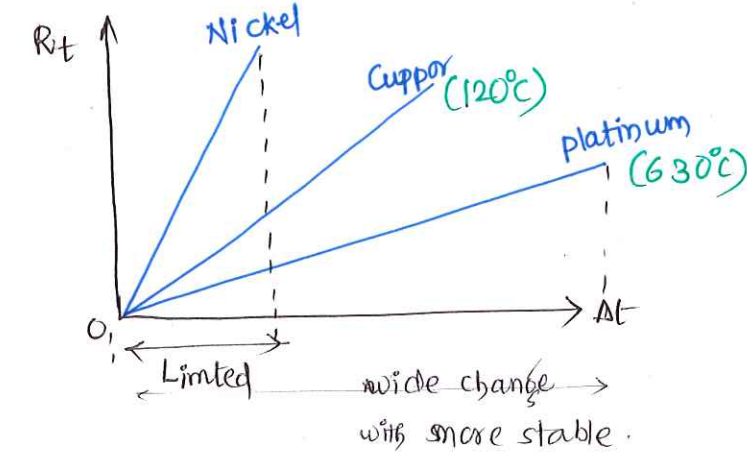
$$I_3 < I_2 < I_1$$

$\therefore$  Galvanometer should have reverse scale



(RTD) most commonly used material for RTD  $\Rightarrow$  platinum.

platinum is preferable b/c it is highly stable at higher temperature. It can be used upto 630°C.

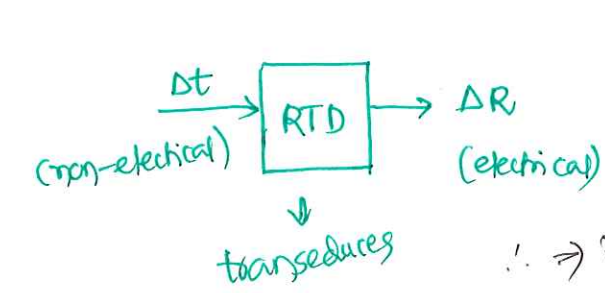


Copper is used for moderate temperature range only. Around (120°)

Silver & Gold are commonly used materials b/c of low resistivity. (Ag & Au)

In general, the temperature range of RTD we can use upto 183°C.

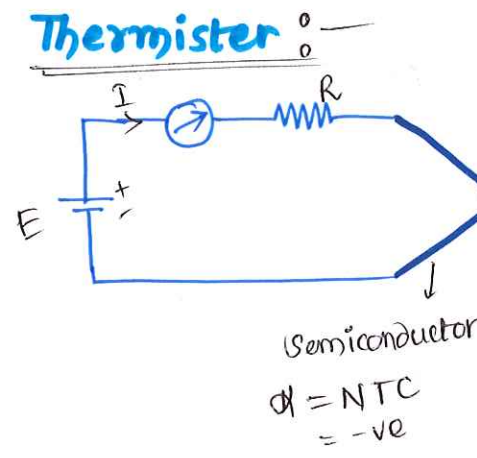
Sensitivity  $\Rightarrow$  (S<sub>Ni</sub> > S<sub>Cu</sub> > S<sub>platinum</sub>).



$$\Rightarrow S_{RTD} = \frac{\Delta R}{\Delta t}$$

Sensitivity  $\propto \frac{1}{\text{temp. range}}$

$\therefore$  it is also called passive transducer



$$R_t = R_0 (1 - \alpha \Delta t)$$

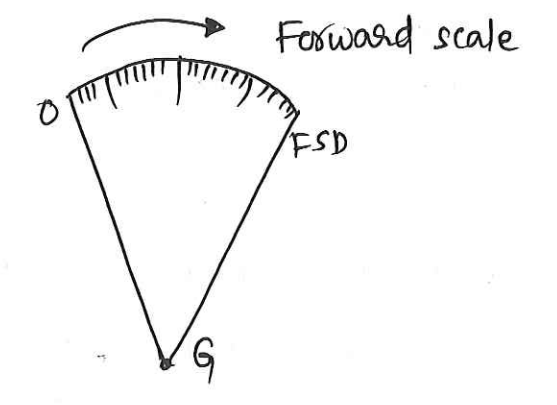
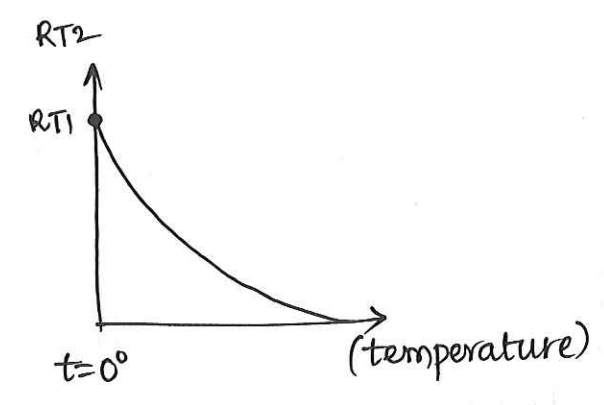
$$\therefore \Delta R_t = R_t - R_0$$

(temp)  $\uparrow \Rightarrow$  (Resistance)  $\downarrow$

It decreases exponentially not linear

(-ve temp. coeff is replaced in place of PTC to overcome reverse scale in RTD... but it is now called as Thermister)

$$R_{T2} = R_{T1} \cdot e^{-\beta \left( \frac{1}{T_1} - \frac{1}{T_2} \right)}$$



$t = \infty^\circ\text{C}$  (All semiconductors will behave like conductors (or) Supercond)

$\beta \rightarrow$  coefficient of temperature. in  $^\circ\text{Kelvin}$ .

$T_1, T_2 \rightarrow$  kelvin temperature.

$$\beta = (3000^\circ\text{K to } 4000^\circ\text{K})$$

Thermister has more sensitivity than RTD.

Temp. range  $\Rightarrow (-55^\circ\text{C to } 15^\circ\text{C})$   
 (or)  
 $(-65^\circ\text{C to } 15^\circ\text{C})$

Semiconductor material  $\Rightarrow$  Sintered mixture of metallic oxides.  
(Fe, Co, Ni + oxides) (42)

platinum, indium  $\Rightarrow$  zero Temp. coeff.

{ d-block, semiconductor }  $\Rightarrow$  -ve temp. coefficient.  
{ Electrolytic solutions }

All metals  $\dots \dots \Rightarrow$  positive temp. coeff.

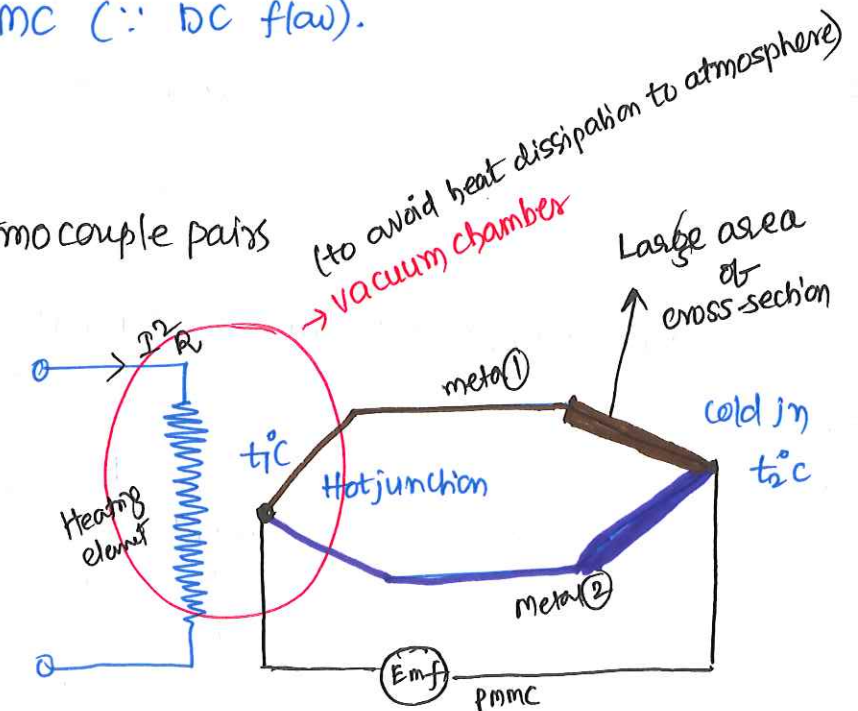
**working principle**  $\Rightarrow$  Thermistor is a temperature measuring device which works on the principle of change in resistance of a ~~me~~ semiconducting material due to heat produced by heater element which has negative temp. coeff.

### Thermo-couple :-

It works on the principle of **Seebeck effect**. There are two dissimilar metals are maintained at two different temperatures namely hot junction and cold junction, there is some electron flow is observed from hot junction to cold junction is known as Seebeck effect. The produced **electron flow is always unidirectional**. It can be measured by **PMMC** ( $\because$  DC flow).

Cu-Co **Copper - Constantan**  
Fe-Co **Iron - Constantan**  
**Chromel - Alumel**

Thermocouple pairs





$$\therefore \text{produced emf} = e = a \cdot (\Delta t) + b (\Delta t)^2$$

where,  $\Delta t \rightarrow$  rise in temperature (or)  
temperature difference b/w junctions.

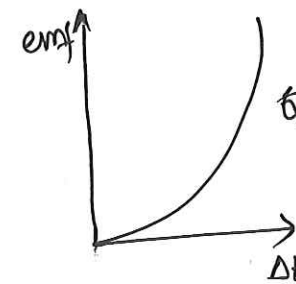
$a \rightarrow$  constant (or) V per  $^{\circ}\text{C}$

$$a = \frac{\text{emf}}{(\Delta t)} = \text{sensitivity of thermocouple.}$$

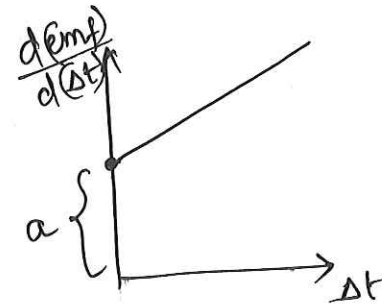
$$a \Rightarrow 40 \mu\text{V}/^{\circ}\text{C} \text{ to } 50 \mu\text{V}/^{\circ}\text{C}$$

$b \rightarrow$  constant (or) V per  $^{\circ}\text{C}^2$

$$b \Rightarrow (1 \mu\text{V}/^{\circ}\text{C}^2 \text{ to } 2 \mu\text{V}/^{\circ}\text{C}^2)$$



Quadratic non-linearity  $\Rightarrow$  Thermocouple  
exponential  $\Rightarrow$  Thermistor.

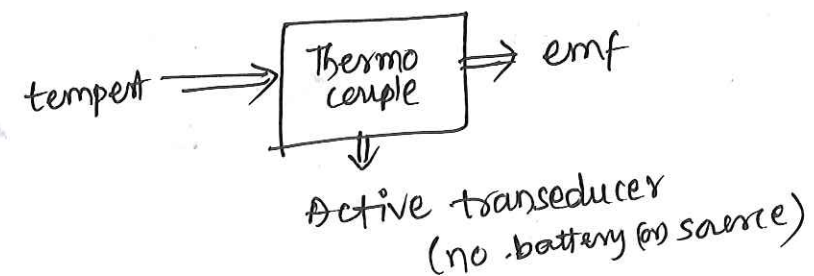


$$\frac{d(\text{emf})}{d(\Delta t)} = a + 2b(\Delta t)$$

rate of production of electron flow @  
emf change due to temperature change

$$\text{slope} = 2b.$$

$$\text{emf} \propto (\Delta t) \propto (\text{heat}) \propto (I^2 R)$$



RTD, Thermister  $\Rightarrow$  passive transducer (source)

(43)

Thermocouple  $\Rightarrow$  Active transducer (no source)

### Note

$\rightarrow$  The produced emf in thermocouple is in DC nature, it is in the order of several milli-volts, The produced emf is detected by **pmmc** but the scale is calibrated to read **RMS value of AC current** flowing through the heater element.

$\rightarrow$  RTD, thermisters are passive transducers, whereas thermocouple is an active transducer b/c it doesn't require any external source.

Temperature range of Thermocouple upto  $1100^{\circ}\text{C}$ . Even we can use upto  $2000^{\circ}$ .

Q: Arrange the above instruments in the order of decreasing of sensibility

1. RTD
2. Thermister
3. Thermocouple.

$$S_{\text{Thermister}} > S_{\text{RTD}} > S_{\text{Thermocouple}}$$

$\downarrow$                        $\downarrow$                        $\downarrow$

(-55 $^{\circ}$  to 150)      1803 $^{\circ}$                       1100 $^{\circ}\text{C}$

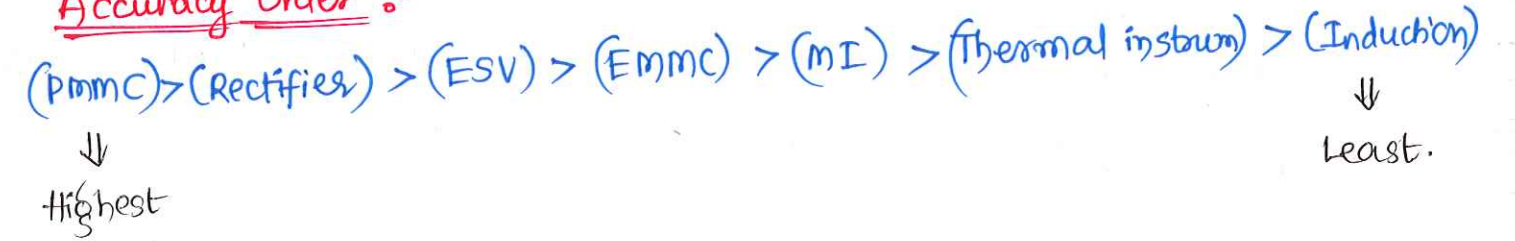
Non-linearity  $\Rightarrow$  Thermister  $>$  Thermocouple  $>$  RTD



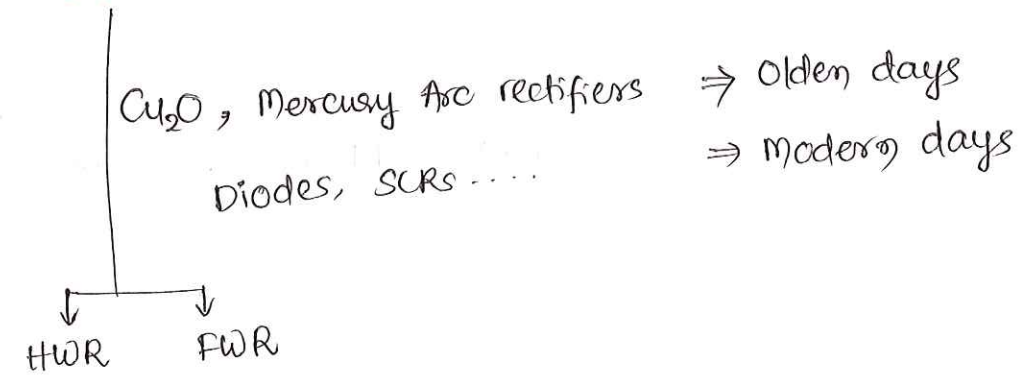
Note

In spite of low accuracy and difficult construction, thermal instruments are popularly used for the measurement of current & voltage at low voltage & high frequency communicationckt and in wire-less networks

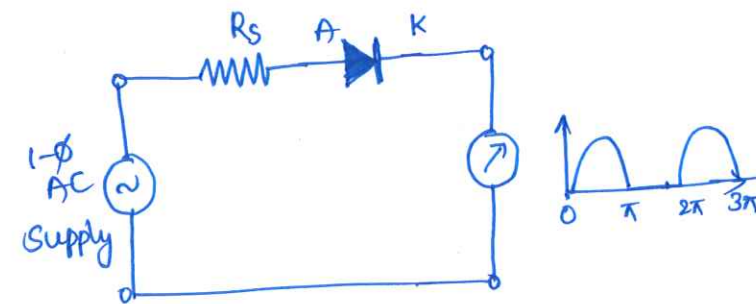
Accuracy Order :-



Rectifier Type of Instruments :-



1. Half Wave Rectifier :-



$$V_{AC} = V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$V_{DC} = V_{avg} = \frac{V_m}{\pi} = \frac{\sqrt{2} V_{RMS}}{\pi}$$

$$\frac{V_{DC}}{V_{AC}} = \frac{V_{avg}}{V_{RMS}} = \frac{\sqrt{2}}{\pi} = 0.45$$

$$I_{AC} = \frac{V_{AC}}{(R_s + R_m)}$$

$$I_{DC} = \frac{V_{DC}}{(R_s + R_m)}$$

$V_{DC} = 0.45 V_{AC}$

$I_{DC} = 0.45 I_{AC}$



$$\frac{1}{(I_{AC FSD})} = 0.45 \times \frac{1}{(I_{DC FSD})}$$

$$S_{AC(HWR)} = 0.45 (S_{DC(HWR)})$$

$$(R_S + R_m) = 0.45 \times V_{AC} \times \frac{1}{I_{DC}} = 0.45 V_{AC} \cdot S_{DC}$$

$$R_S = (0.45 S_{DC}) V_{AC} - R_m$$

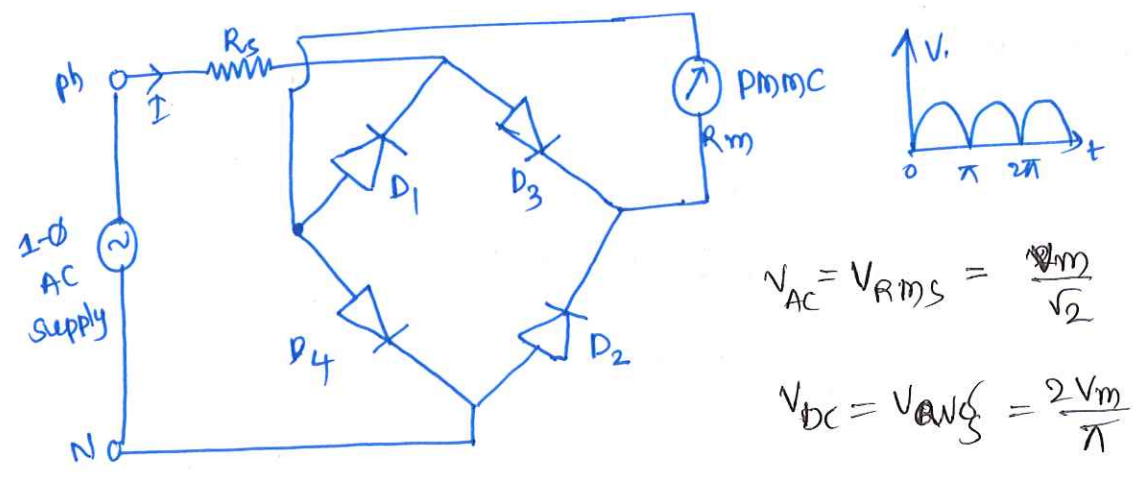
$$R_S = S_{AC} \cdot V_{AC} - R_m \Rightarrow \text{if } R_D = 0$$

if  $R_D \neq 0 \therefore R_S = S_{AC} V_{AC} - R_m - R_D$

$$FF = K_f = \frac{V_m/\sqrt{2}}{V_m/\pi} = \frac{\pi}{\sqrt{2}} = 2.22$$

(Reading of HWR type instrument) = (2.22) (PMMC meter reading)

Full wave Rectifier :-



$$V_{AC} = V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$V_{DC} = V_{avg} = \frac{2V_m}{\pi}$$

$$V_{DC} = \frac{2}{\pi} (\sqrt{2} V_{RMS}) = 0.9 V_{DC}$$

$$I_{AC} = \frac{V_{AC}}{R_S + R_m} ; I_{DC} = 0.9 I_{AC} \therefore S_{AC} = 0.9 S_{DC}$$

$$(R_s + R_m) = (0.9) V_{AC} S_{DC} = S_{AC} V_{AC}$$

$$R_s = S_{AC} V_{AC} - R_m \Rightarrow \text{ideal diode } R_D = 0$$

$$R_s = S_{DC} V_{AC} - R_m - 2R_D \Rightarrow \text{practical identical diodes } R_D \neq 0.$$

$$FF = \frac{V_{RMS}}{V_{DORG}} = \frac{V_{AC}}{V_{DC}} = \frac{2.89}{2} = 1.11$$

$$\therefore (\text{Reading of FWR reading}) = (1.11) (\text{PMMC reading})$$

$$\text{Sensitivity of FWR} = 2 (\text{Sensitivity of HWR})$$

DC (or) AC

$$S_{AC \text{ FWR}} = 2 \cdot S_{DC \text{ HWR}}$$

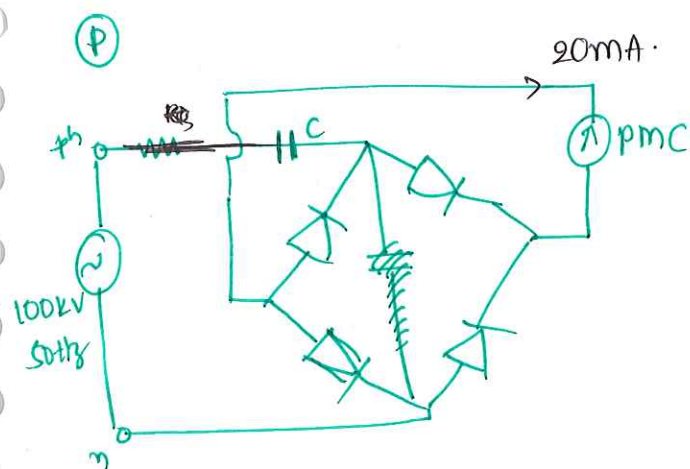
### Advantages:

- \* Uniform scale
- \* Working for both AC & DC
- \* Low power consumption
- \* High sensitivity in the order of several thousands  $\Omega/V$   
(1000  $\Omega/V$  to 2000  $\Omega/V$ )
- \* No frequency error, It can be used over a wide frequency range in the order of several kHz (or) MHz range.

### Disadvantages:

- \* It needs separate calibration for each input waveform.
- \* It has different sensitivity like AC as well as DC sensitivity
- \* It consists of a diode which is a semi-conductor device, the diode resistance is changed due to change in temperature

\* The diode will offer some capacitance known as stray capacitance of the diode. Due to this capacitance the instrument readings are affected. (45)



$$C = ?$$

$$I_{DC} = (0.9) I_{AC}$$

$$= 0.9 \times \frac{V_{AC}}{(R_s + R_m)}$$

$$R_s = \frac{1}{j\omega C} = X_C$$

$$I_{DC} = 0.9 \times \frac{V_{AC}}{(X_C + R_m)}$$

$$\therefore I_{DC} = 0.9 \times \frac{V_{AC}}{X_C}$$

$$20 \text{ m} = \left( j\omega C \times 0.9 \times 100 \times 10^3 \right)$$

$$V_{AC} = V_{rms} = 100 \text{ kV}$$

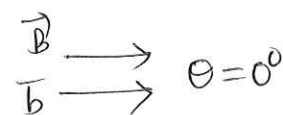
not mention,  $R_m = 0$

$$\therefore C = \frac{20 \times 10^{-3}}{100 \times 10^3 \times 0.9 \times 100} = \frac{2}{9\pi} \times 10^{-8} = 7.07 \times 10^{-10}$$

$$C = 707 \text{ pF}$$

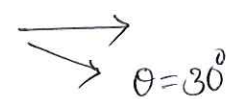
Let  $B$  = operating field.  
 $b$  = stray field.  $\theta(\vec{b}, \vec{B})$

$$B_{\text{resultant}} = \sqrt{B_1^2 + B_2^2 + 2B_1B_2 \cos \theta}$$

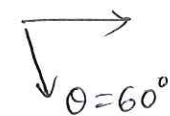


$$B_R = \sqrt{B_1^2 + B_2^2 + 2B_1B_2}$$

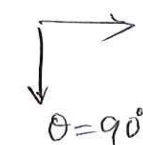
(parallel aiding)



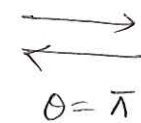
$$B_R = \sqrt{B_1^2 + B_2^2 + \sqrt{3}B_1B_2}$$



$$B_R = \sqrt{B_1^2 + B_2^2 + B_1B_2}$$



$$B_R = \sqrt{B_1^2 + B_2^2}$$



$$B_R = \sqrt{B_1^2 + B_2^2 - 2B_1B_2}$$

(parallel opposite)



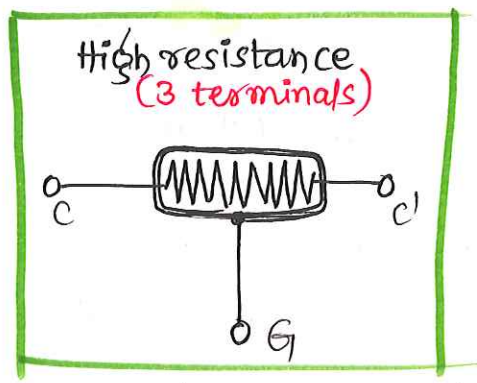
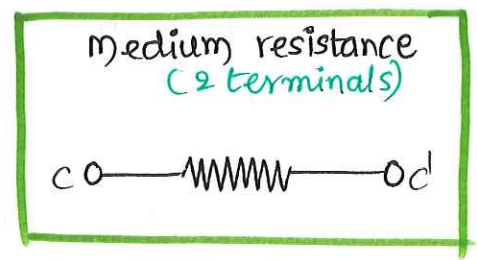
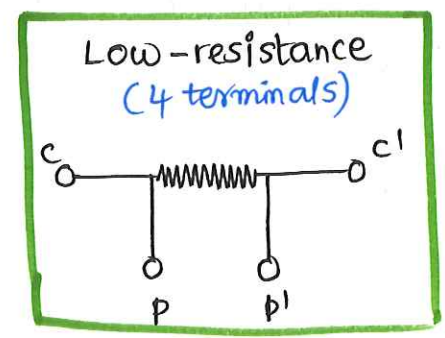
Summary

.....

# Measurement of Resistance

The resistance is classified only for the purpose of measurements

- (i) Low resistance ( $< 1 \Omega$ ), armature wdg resi, series field wdg, <sup>Compensat</sup> wdg resi
- (ii) Medium resistance ( $1 - 100 k\Omega$ ) (or) ( $1 - 0.1 M\Omega$ )
- (iii) High resistance ( $> 100 k\Omega$ )
  - ammeter shunt resistance, diode forward bias resistance, link resistance (Lead n), slide wire resistance.
  - shunt field wdg resistance, resistance of heating elements (heaters) All electrical appliance equipment resistors in domestic purposes. (daily life).
  - insulation resistance of underground cable, insulators (power system). insulation resistance of dielectrics, power diode - reverse bias resistance. Series multiplier resistance of voltmeter, input resistance of CRO. I/p resistance Op-Amp, FETs ... etc.



$C, C'$  - current terminals  
 $P, P'$  - voltage/potential "

$$R = \frac{V_{PP'}}{I_{CC'}}$$

4 probe Method.

### Errors :- (Difficulties)

1. Due to contacts
2. Due to leads
3. Due temperature change
4. Due to thermal emf

### Difficulties faced in measurement of highresi

#### (or) Errors :-

1. Error due to leakage currents (these can be avoided by using guard wire)
2. Error due to Specimen capacitance.
3. i.e. Error due to electro-static field effect

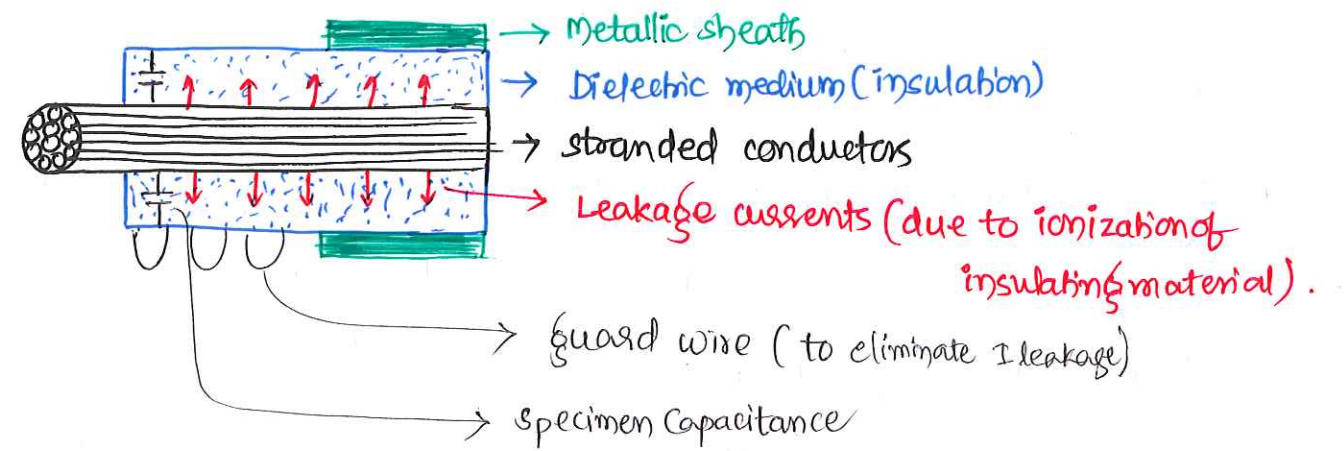


Fig: Underground power cable cross-sectional view pertaining to difficulties of high resistance measurement.

Low resistance can be measured by

- (i)  $A-V$  method
- (ii) potentiometer
- (iii) Kelvin double bridge  $\rightarrow$  most accurate ( $R \Rightarrow 0.1 \mu\Omega$  to  $1 \Omega$ ).
- (iv) All above..

Measurements can be measured by (medium)

- (i)  $V-A$  method
- (ii) substitution method
- (iii) Ohmmeter  $\begin{cases} \rightarrow$  series type. \\  $\rightarrow$  shunt type. \end{cases}

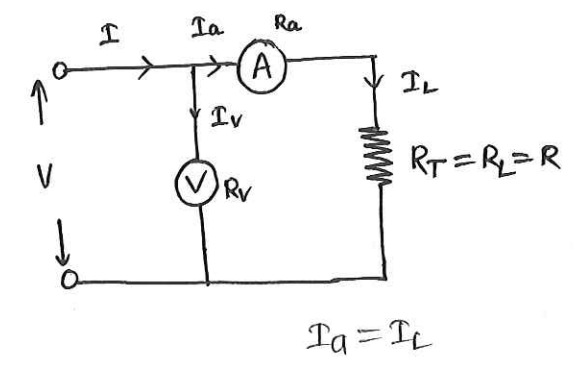
(iv) wheatstone bridge.  $\rightarrow$  accurate more

High-resistance measured by

- (i) loss of charge
- (ii) Direct deflection method
- (iii) Megger device
- (iv) mega-ohm bridge method.  $\rightarrow$  most accurate.



Voltmeter-Ammeter Method :-



$$R_{measured} = R_m = \frac{V}{I_a} = \frac{V_{supply}}{I_a}$$

$$= \frac{I_a R_a + I_L R_L}{I_a}$$

$$R_m = (R_a + R_L)$$

$$R_m = R_T + R_a$$

Error =  $(A_m - A_T) = R_m - R_T = R_a$

$\therefore$  Error = +ve ;  $\Rightarrow$  (measured) > (true value)  $\therefore R_m > R_T$ .

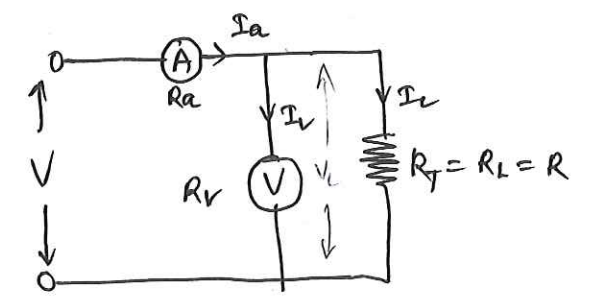
% Error =  $\frac{R_m - R_T}{R_T} \times 100 = \frac{R_a}{R_T} \times 100$

Ideally  $\Rightarrow$  if we try  $(R_a \rightarrow 0 \Omega)$

**Note :-** voltmeter-ammeter is best suitable for measurement of large value of resistance in a medium resistance range.

The produced error is positive b/c of ammeter connected on load side. Ideally ammeter resistance is zero but practically it is never possible, it is only known as loading effect of ammeter.

Ammeter-voltmeter Method :-



$$R_m = \frac{V}{I_a} = \frac{I_L R_L}{I_a} = \frac{V_L}{I_a}$$

$$= \frac{I_L R_L}{I_L + I_v} = \frac{V_L}{\frac{V_L}{R_L} + \frac{V_L}{R_v}}$$

$$R_m = \frac{R_L R_v}{R_L + R_v}$$

$$\frac{1}{R_m} = \frac{1}{R_L} + \frac{1}{R_v}$$

$$R_m = \frac{R_L R_V}{R_V \left(1 + \frac{R_L}{R_V}\right)} \Rightarrow R_m = \frac{R_L}{\left(1 + \frac{R_L}{R_V}\right)}$$

$$R_m = R_L \left(1 + \frac{R_L}{R_V}\right)^{-1}$$

$$R_m = R_L \left(1 - \frac{R_L}{R_V} + \left(\frac{R_L}{R_V}\right)^2 - \frac{R_L^3}{R_V^3} + \dots\right)$$

$$R_m \approx R_L \left(1 - \frac{R_L}{R_V}\right)$$

$$\therefore R_m = R_L - \frac{R_L^2}{R_V}$$

$$\text{Error} = R_m - R_T = -\frac{R_L^2}{R_V}$$

$$\text{Error} = -ve \Rightarrow (\text{measured}) < (\text{true value})$$

$$\therefore \% \text{ Error} = \frac{R_m - R_T}{R_T} \times 100 = -\frac{R_T}{R_V} \times 100 \quad (\because R_L = R_T = R)$$

ideally  $R_V \Rightarrow \infty \Omega$  (error  $\rightarrow 0$ ). never possible.

Note: ① Ammeter-voltmeter method is best suitable for measurement of low resistances, the produced error is negative, error is because of voltmeter connected on load side. Ideally voltmeter resistance is  $\infty$ , practically it is not possible it is only known as Loading Effect of voltmeter.

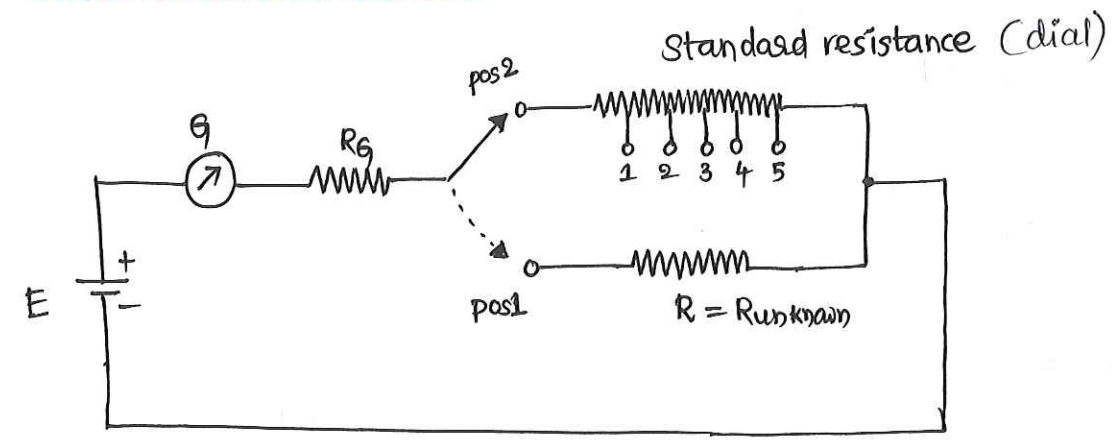
② To get same error in both the methods

$$\% \text{ error}_{(A-V)} = \% \text{ error}_{(V-A)}$$

$$\left| -\frac{R_L}{R_V} \right| = \left| +\frac{R_A}{R_L} \right|$$

$$\therefore R_L^2 = R_A R_V \Rightarrow R_L = \sqrt{R_A R_V}$$

## Substitution Method :-



case (i) :- Switch is moved to position '1'.  $\Rightarrow I_G = I_1$

$$\therefore I_1 = \frac{E}{(R_G + R)} \quad \text{--- (1)}$$

case (ii) :- Switch is moved to position '2'  $\Rightarrow I_G = I_2$

adjust the dial on standard resistance box...

ensure ...  $I_2 = I_G = I_1$

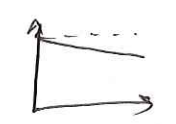
$$\therefore I_2 = \frac{E}{(R_G + R_{\text{switch}})} = I_1 = \frac{E}{(R_G + R)} \quad \text{--- (2)}$$

$\therefore$  Unknown resistance (R) =  $R_{\text{switch}}$  (reading on dial resistance)

advantage :- One of the best method to measure resistance (practical)

disadvantages :- \* Choice of availability of standard resistance box

\* For long time measurement in laboratories, standard battery may be changed.



$\rightarrow$  Let  $\theta_1 \rightarrow$  deflection of (G) with unknown resistance

$$\theta_1 \propto \frac{1}{(R_G + R)} \quad \rightarrow (3)$$

Let  $\theta_2 \rightarrow$  deflection of (G) with standard resistance.

$$\theta_2 \propto \frac{1}{(R_G + S)}$$



$$\frac{\theta_1}{\theta_2} = \frac{R_G + S}{R_G + R}$$

$$\therefore R = \frac{\theta_2}{\theta_1} (R_G + S) - R_G$$

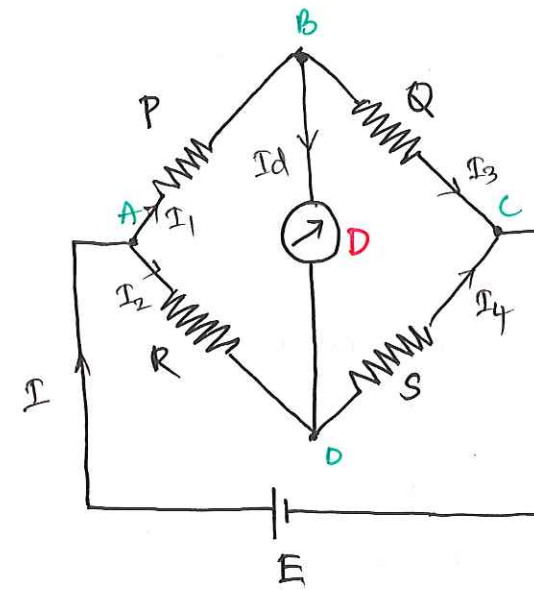
$E = \text{constant (assumed)}$ .

### Wheatstone Bridge :- (Comparison method).

$P, Q, S \Rightarrow$  known resistances  
 $R \Rightarrow$  unknown resistance  
 $E \Rightarrow$  DC battery

$P, Q, R, S \Rightarrow$  ratio arms resistance.

$\rightarrow \Rightarrow$  Diagonal galvanometer detector.



$$e = V_{BD} = V_{BA} + V_{AD}$$

$$= V_{AD} - V_{AB}$$

$$e = I_2 R - I_1 P$$

$$\Rightarrow I_1 P = I_2 R$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{R}{P} \quad \text{--- (1)}$$

$$\text{(or)} \quad e = V_{BD} = V_{BC} + V_{CD}$$

$$= V_{DC} - V_{BC}$$

$$e = S I_4 - I_3 Q \Rightarrow \frac{I_4}{I_3} = \frac{Q}{S} \quad \text{--- (2)}$$

Under bridge balance condition;  $e = 0$ ,  $I_d = 0$ .

$$\therefore I_1 = I_3 \quad \& \quad I_2 = I_4$$

$$\therefore \frac{I_1}{I_2} = \frac{I_3}{I_4} \Rightarrow \therefore \text{from (1) \& (2)}$$

$$\frac{P}{R} = \frac{S}{Q} \Rightarrow \boxed{\frac{P}{Q} = \frac{R}{S}}$$

$$\text{Detector sensitivity} = \text{voltage sensitivity} = \frac{\text{change in deflection}}{\text{change in voltage across detector}} \quad (19)$$

$$\therefore S_V = S_D = \frac{\Delta \theta}{\Delta V} = \frac{\Delta \theta}{e} \Rightarrow \Delta \theta = e S_V$$

$$\text{Bridge sensitivity} = \frac{(\text{change in deflection})}{(\text{unit change in resistance})} = \frac{\Delta \theta}{\left(\frac{\Delta R}{R}\right)} = S_B$$

$$\therefore \Delta \theta = S_B \left(\frac{\Delta R}{R}\right)$$

$$\therefore e S_V = S_B \left(\frac{\Delta R}{R}\right) \Rightarrow \boxed{S_B = \frac{R e S_V}{\Delta R}}^{**}$$

at unbalanced point ( $R' = R + \Delta R$ )

$$e = V_{TB} = V_{oc} = V_{AD} - V_{AB} = V_B - V_D$$

$$\boxed{e = E \left( \frac{R'}{R+S} - \frac{P}{P+Q} \right)}^{**} \quad (20)$$

$$e = E \left( \frac{R'}{R+S} - \frac{P}{P+Q} \right)$$

$$R' = R + \Delta R$$

$$e = E \left( \frac{(R+\Delta R)}{(R+S+\Delta R)} - \frac{P}{P+Q} \right)$$

$$e = E \left( \frac{R+\Delta R}{(R+S)+\Delta R} - \frac{R}{R+S} \right)$$

$$= E \left[ \frac{(R+\Delta R)(R+S) - R[(R+S)+\Delta R]}{(R+S)^2 + \Delta R(R+S)} \right]$$

$$= E \left[ \frac{\cancel{R^2} + \cancel{RS} + \cancel{\Delta R R} + \cancel{\Delta R S} - \cancel{SR} - \cancel{R^2} - \cancel{R \Delta R}}{(R+S)[(R+S)+\Delta R]} \right] = \frac{E(\Delta R \cdot S)}{(R+S)^2 + \Delta R(R+S)}$$

at balance point

$$\frac{P}{Q} = \frac{R}{S}$$

$$\frac{Q}{P} = \frac{R}{R+S}$$

$$\frac{Q}{P} + 1 = \frac{R}{R} + 1$$

$$\frac{P+Q}{P} = \frac{R+S}{R}$$

$$e = \frac{S \cdot \Delta R \cdot E}{(R+S)^2 + \Delta R(R+S)}$$

$$(R+S)^2 \gg \Delta R(R+S)$$

$$\therefore e = \frac{S \cdot \Delta R \cdot E}{R^2 + S^2 + 2SR} = \frac{(\Delta R) E}{\frac{R}{RS} (R^2 + S^2 + 2RS)}$$

$$e = \frac{R \Delta R \cdot E}{R \left( \frac{R^2}{S} + \frac{S^2}{R} + 2 \right)}$$

$$\Delta V = e = \frac{\left( \frac{\Delta R}{R} \right) E}{\frac{R}{S} + 2 + \frac{S}{R}} \quad (\text{or}) \quad e = \frac{\left( \frac{\Delta R}{R} \right) E}{\frac{P}{Q} + 2 + \frac{Q}{P}}$$

$$S_V = \frac{\Delta \theta}{\Delta V} ; \Rightarrow S_V = \frac{\Delta \theta}{e} ; S_B = \frac{\Delta \theta}{\left( \frac{\Delta R}{R} \right)}$$

$$\therefore e S_V = \left( \frac{\Delta R}{R} \right) S_B$$

$$\frac{\left( \frac{\Delta R}{R} \right) E \cdot S_V}{\frac{P}{Q} + 2 + \frac{Q}{R}} = \left( \frac{\Delta R}{R} \right) S_B$$

$$\therefore S_B = \frac{e S_V}{\left( \frac{\Delta R}{R} \right)} = \frac{E \cdot S_V}{\frac{P}{Q} + 2 + \frac{Q}{R}}$$

$S_{B \max}$  ; if  $\frac{P}{Q} = \frac{Q}{P} = 1$  i.e.  $R = S = P = Q$



$$S_{B_{max}} = \frac{S_V E}{4}$$

condition for obtaining max. bridge sensitivity, it is possible if the ratio of ratio arm resistances of ratio of arm (or) equal to unity. i.e.  $P=Q=R=S$ , the value is  $\frac{E}{4}$  times that of detector sensitivity.

$$e = V_{th} = V_{oc} = \frac{E \left( \frac{\Delta R}{R} \right)}{\frac{P}{Q} + 2 + \frac{Q}{P}} = \frac{E \left( \frac{\Delta R}{R} \right)}{\frac{R}{S} + 2 + \frac{S}{R}} \quad (1)$$

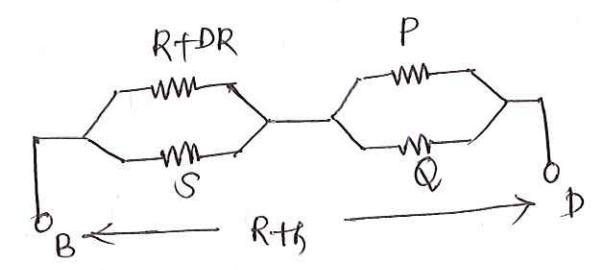
$$\theta = E \left( \frac{R + \Delta R}{R + \Delta R + S} - \frac{P}{P + Q} \right)$$

$$R_{th} = S \parallel (R + \Delta R) + P \parallel Q$$

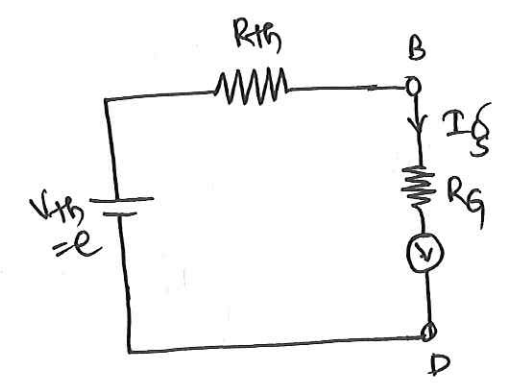
$$R_{th} = \frac{PQ}{P+Q} + \frac{S(R + \Delta R)}{R + S + \Delta R}$$

if  $\Delta R = 0$

$$R_{th} = \frac{PQ}{P+Q} + \frac{RS}{R+S}$$



Thevenins equivalent ckt of Wheatstone Bridge



$$R_{th} = \frac{PQ}{P+Q} + \frac{RS}{R+S}$$

$$V_{th} = e = \frac{\left( \frac{\Delta R}{R} \right) E}{\frac{R}{S} + 2 + \frac{S}{R}}$$

$$I_{th} = I_G = \frac{V_{th}}{R_{th} + R_G}$$

For ideal (G) ;  $R_G = 0$  ;  $I_{th} = I_G = \frac{V_{th}}{R_{th}}$

→ If all arms are wheatstone Bridges of equal resistances then

$$R = P = Q = S.$$

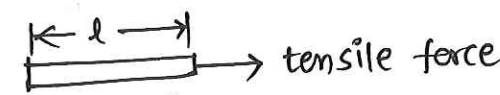
$$R_{th} = P \parallel Q + R \parallel S = R \parallel R + R \parallel R = \frac{R}{2} + \frac{R}{2} = R.$$

$$\therefore R_{th} = R, \quad V_{th} = V_{oc} = 0.$$

$$S_{B_{max}} = \frac{S_v E}{4}$$

→  $P = Q = S = R$ ; but one of the arm is under stress (strain).

Case (i) :-



length ↑; area ↓  
but volume = constant.



(compressive force)



$$R_{th} = \frac{(R + \Delta R)S}{(R + S + \Delta R)} + \frac{PQ}{P + Q}$$

$$= \frac{(R + \Delta R)R}{(R + R + \Delta R)} + \frac{R \cdot R}{R + R}$$

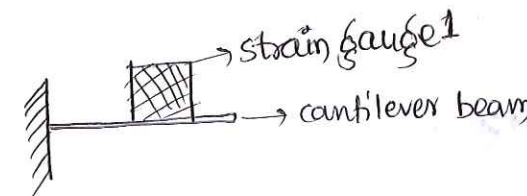
$$R_{th} = \frac{(R + \Delta R)R}{(2R + \Delta R)} + \frac{R}{2}$$

$$V_{th} = E \left( \frac{(R + \Delta R)R}{(R + R + \Delta R)} - \frac{R}{R + R} \right)$$

$$= E \left( \frac{R^2 + R \Delta R}{2R + \Delta R} - \frac{1}{2} \right)$$

$$= E \left[ \frac{2R^2 + 2R \Delta R - 2R - \Delta R}{2(2R + \Delta R)} \right]$$

$$= E \frac{\Delta R}{2(2R + \Delta R)}$$



$$V_{th} = E \left( \frac{\Delta R}{4R + 2\Delta R} \right)$$

$$4R \gg 2\Delta R.$$

$$\therefore V_{th} = \frac{E \Delta R}{4R}$$

(Quarter bridge) equation

$R = P = Q = S$ ; one arm  $(R + \Delta R)$ .

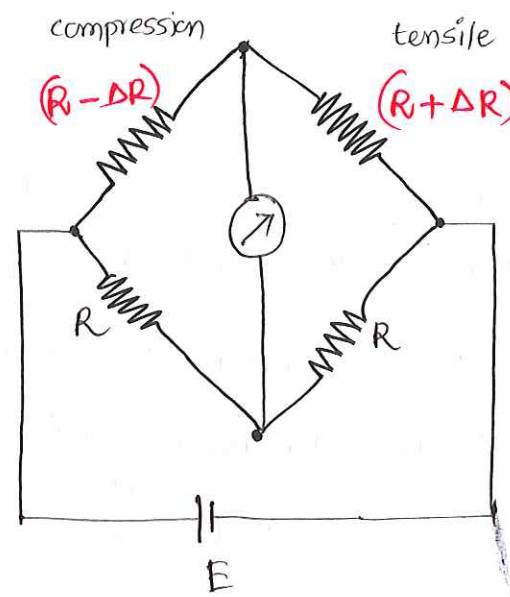
$$V_{th} = \frac{E}{4} \left( \frac{\Delta R}{R} \right)$$

$$S_Q = \frac{\Delta V}{\left( \frac{\Delta R}{R} \right)} = \frac{V_{th}}{\frac{\Delta R}{R}} = \frac{E}{4}$$

The change of resistance is occurred in one of the arm of the bridge due to applied tensile force and remaining arms of the bridge are same is known as **Quarter Bridge**. (51)

→ **Case(ii)**

The change of resistance is occurred in adjacent arms with opposite strength is known as half bridge.



(adjacent arm) more sensitivity than (opposite arms) change.

$$V_{\frac{1}{2}} = \frac{E}{2} \left( \frac{\Delta R}{R} \right)$$

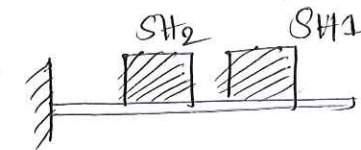
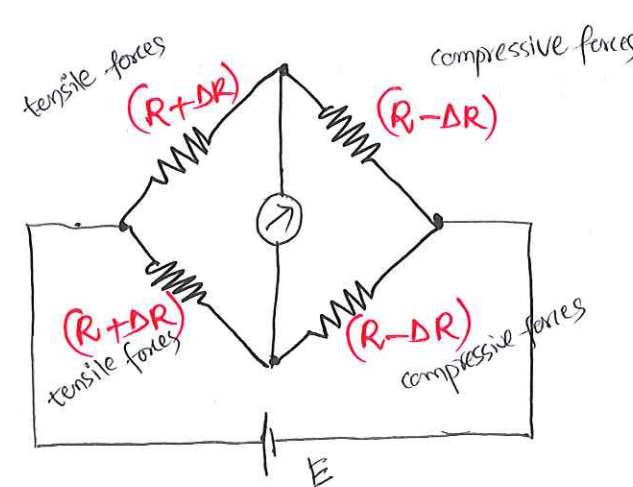
$$S_{\frac{1}{2}} = \frac{V_{\frac{1}{2}}}{\left( \frac{\Delta R}{R} \right)} = \frac{E}{2}$$

$$S_{HB} = 2 S_{QB}$$

H → Half Bridge  
Q → Quarter Bridge

→ **Case(iii)**

If the change of resistance occurs in all arms the wheat-stone bridge opposite strain in the adjacent arm (or) opposite strain in opposite arms is known as full bridge.



$$V_{\frac{1}{2}} = \frac{E}{1} \left( \frac{\Delta R}{R} \right)$$

$$S_{FB} = \frac{V_{\frac{1}{2}}}{\left( \frac{\Delta R}{R} \right)} = E = 4 S_{QB} = 2 S_{HB}$$

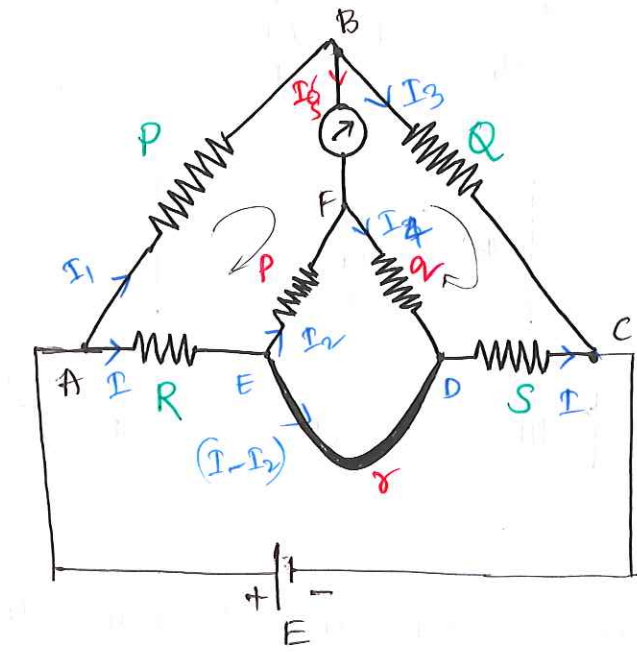


$$S_{QB} : S_{HB} : S_{FB} = \frac{1}{4} : \frac{1}{2} : 1 = 1 : 2 : 4$$

S → Sensitivities ; Q → Quarter ; H → Half ; F → Full Bridge .

### Kelvin's Double Bridge (0.1 μΩ - 1 Ω)

It is one of the most accurate bridge to measure the low resistance in the order of 0.1 μΩ to 1 Ω. In this bridge a link resistance is connected b/w the known standard resistance and test resistance



outer arm resistances ⇒ P, Q  
inner arm resistances ⇒ p, q

- S ⇒ Known resistance
- R ⇒ Unknown resistance (or) Test resistance .
- γ ⇒ link resistance .

At Bridge balance, ⇒  $I_d = 0$

$$\therefore I_1 = I_3 ; I_2 = I_4 ;$$

To analyze the bridge apply KVL in ABFEA loop.

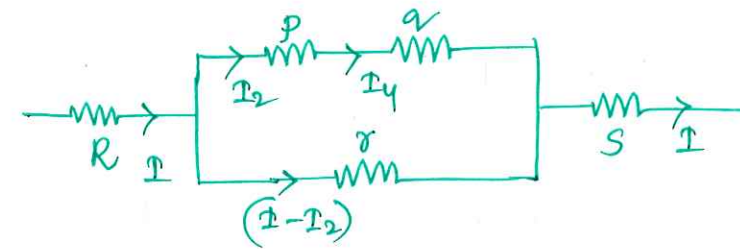
$$I_1 P + i(I_1 - I_2) - IR = 0$$

$$I_1 P = I_2 p + IR \quad \text{--- ①}$$

Apply KVL in "BCDFB" loop.

(52)

$$I_3 Q = I_4 q + S I \quad \text{--- (2)}$$



$$I_2 = I_4 = \frac{I \times r}{(P+q+r)}$$

$$\frac{I_1 P}{I_3 Q} = \frac{I_2 P + I R}{I_4 q + S I} = \frac{\frac{I P r}{P+q+r} + R I}{\frac{I q r}{P+q+r} + S I}$$

$$\frac{P}{Q} = \frac{\left[ \frac{P r}{P+q+r} + R \right]}{\left[ \frac{q r}{P+q+r} + S \right]}$$

$$\frac{P}{Q} \left[ \frac{q r}{P+q+r} + S \right] = \left( \frac{P r}{P+q+r} \right) + R$$

$$R = \frac{P}{Q} S + \frac{P}{Q} \frac{q r}{P+q+r} - \frac{P r}{q+r+P} \times \frac{q}{q}$$

$$R = \frac{P}{Q} S + \left( \frac{P}{Q} - \frac{P}{q} \right) \left[ \frac{q r}{P+q+r} \right]$$

case(i) If  $\frac{P}{Q} = \frac{P}{q} \Rightarrow \frac{P}{Q} = \frac{R}{S}$  (not possible) (very difficult)

case(ii) if  $r=0 \Rightarrow R = \frac{P}{Q} S \Rightarrow \frac{P}{Q} = \frac{R}{S}$  (practically possible!)  
i.e. lead resistance = 0.

(i) By maintaining the ratio of outer arm resistance is equal to the ratio of inner arm resistance ( $\frac{P}{Q} = \frac{P'}{Q'}$ ) we can convert kelvin double bridge into wheatstone bridge to measure medium resistance. This is not possible practically b/c the resistances are temperature dependent.

(ii) By shorting the ( $r=0$ ) (link resistance) we can measure medium resistance using kelvin double bridge.

---

Q:- which one of the following is an indicating type meter

- (i) Series type ohmmeter.
  - (ii) Shunt type ohmmeter.
  - (iii) megger, multimeter
  - (iv) All are correct.
- 

Q:- which one of the following meter is having  $\infty$  to  $0.2$  scale.

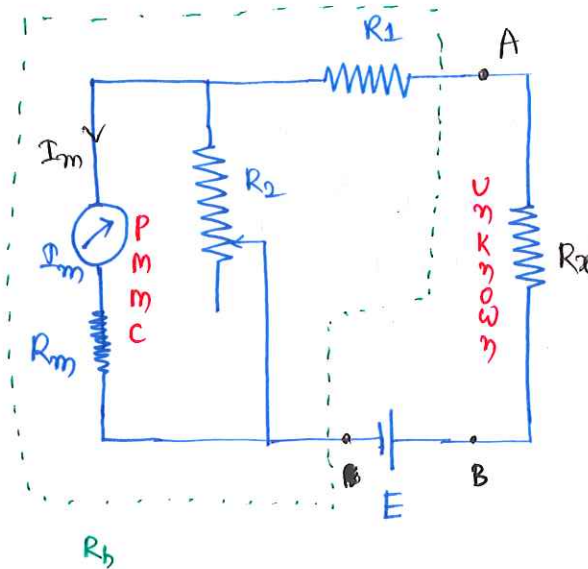
- (i) series type ohmmeter
  - (ii) shunt " "
  - (iii) both
  - (iv) None of the above
-



Ohmmeter is an indicating type instruments b/c PMMC meter is involved in their ckt. There are two types of ohmmeters (53)

- (i) Series type
- (ii) Shunt type

### Series Type Ohmmeter :-



- $R_1 \rightarrow$  series resistor (or) current limiting resistor
- $R_2 \rightarrow$  shunt resistor (or) zero adjusting resistor
- $R_x \rightarrow$  Unknown resistor
- $R_b \rightarrow$  equivalent ckt resistance

$$R_b = R_2 \parallel R_m + R_1$$

$$R_b = \frac{R_2 R_m}{R_2 + R_m} + R_1$$

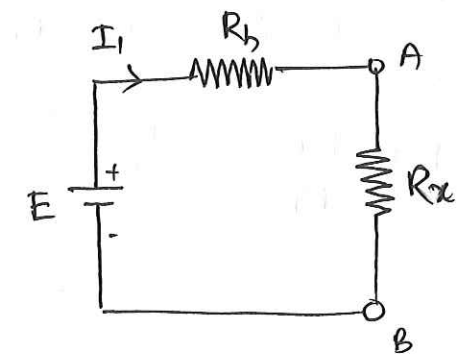
$$I_m = I_1 \left( \frac{R_2}{R_2 + R_m} \right)$$

$$\therefore I_m = \frac{E}{(R_b + R_x)} \left[ \frac{R_2}{R_2 + R_m} \right]$$

case (ii) :- Short ckt

$$R_x = 0 \Rightarrow I_1 = I_{sc} = I_{max}$$

$$I_1 = \frac{E}{R_b} ; I_m = \max = I_{mfSD}$$

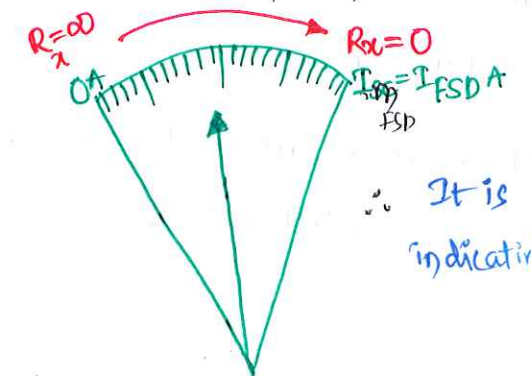


$$I_1 = \frac{E}{R_b + R_x}$$

case (i) :- Open - ckt.

$$\text{i.e. } R_x = \infty \Rightarrow I_1 = 0$$

$$\Rightarrow I_m = 0$$



$\therefore$  It is indicating type.

$$\propto \frac{1}{R_x}$$

$$\theta \propto \frac{1}{R_x} \Rightarrow (\text{deflection}) \propto \frac{1}{(\text{unknown resistance})}$$

$$I_{mFSD} = \frac{E}{R_b} \left( \frac{R_2}{R_2 + R_m} \right)$$

$$\frac{I_m}{I_{mFSD}} = \frac{\left( \frac{E}{R_b + R_x} \right) \left( \frac{R_2}{R_2 + R_m} \right)}{\left( \frac{E}{R_b} \right) \left( \frac{R_2}{R_2 + R_m} \right)}$$

$$\boxed{\frac{I_m}{I_{mFSD}} = \frac{R_b}{R_b + R_x}} \quad \text{where; } R_b = \left( \frac{R_m R_2}{R_m + R_2} \right) + R_1$$

Let  $x$  be the fraction of full-scale current

$$x = \frac{I_m}{I_{mFSD}} = \frac{R_b}{R_b + R_x}$$

$$1 + \frac{R_x}{R_b} = \frac{1}{x} \Rightarrow \boxed{R_x = R_b \left( \frac{1}{x} - 1 \right)}$$

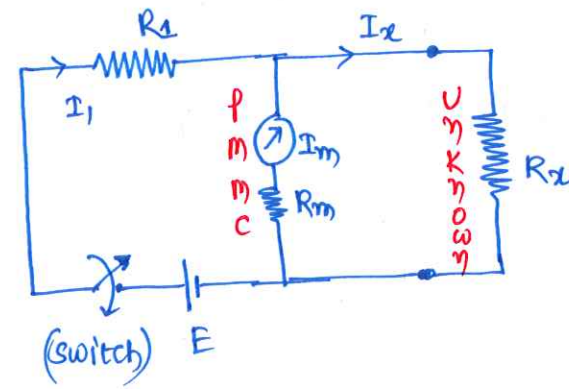
### Applications :-

- \* It is most commonly used meter in industries.
- \* It is used to measure the resistance of heating elements
- \* Used to measure the resistance of field coils of an electrical machine
- \* To check the *semiconductors diodes... healthy position.*
- \* Sorting of resistors in the laboratory & giving labels to them.
- \* It is also used to check the continuity of windings of electrical machines.

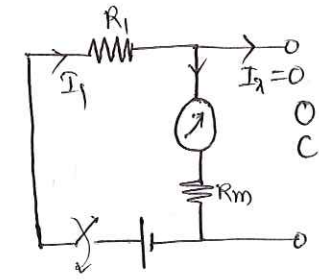
# Shunt type Ohmmeter

(switch is needed)

(54)



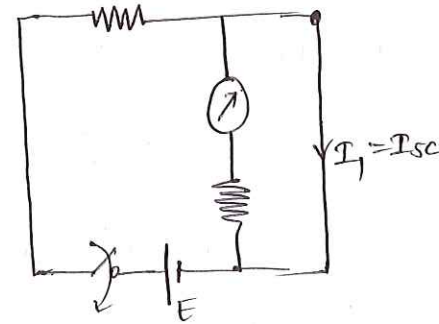
Case (i) :- If  $R_x = \infty \Rightarrow$  O.C



$$I_m = I_{mFSD}$$

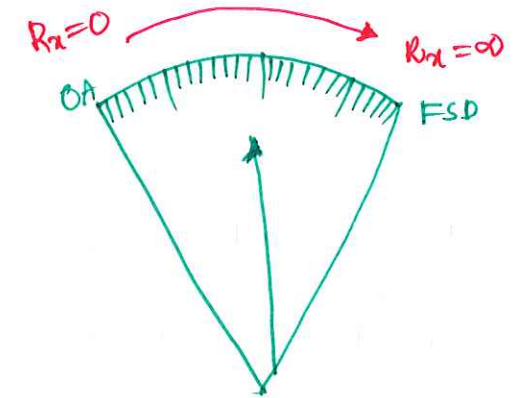
$$\therefore I_1 = I_m = I_{mFSD} = \frac{E}{(R_1 + R_m)}$$

Case (ii) :-  $R_x = 0$ ; S.C.



$$I_m = 0; I_1 = \frac{E}{R_1}$$

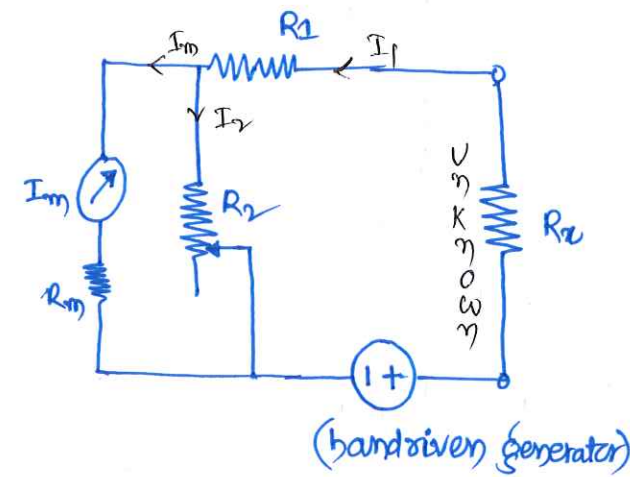
$$\theta \propto R_x \Rightarrow$$



$\therefore$  deflection  $\propto$  (unknown resistance)



## Megger :-



$$E_{min} = V_{min} = 40V$$

to get sufficient deflection (40-200V).

- \* The minimum voltage required to produce the deflection is around 40V.
- \* Also called as Direct reading/indicating type instrument.
- \* It is mostly used in industries to measure the high resistance.
- \* Megger is also called as **moving coil type instrument**.
- \* Used to know the continuity of the cables in power systems applications.

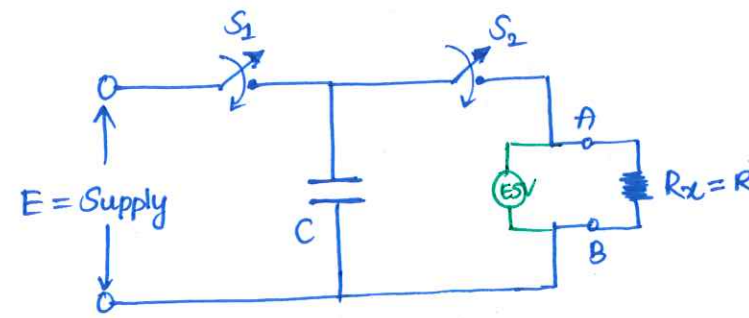
- \* Ratio type Ohmmeter
- \* Indicating type meter
- \* Similar to Series type Ohmmeter.

$$\frac{I_m}{I_{mFSD}} = \frac{R_b}{R_b + R_x}$$

- \* It (hand driven generator) used to generate sufficient voltage in order to obtain deflection.

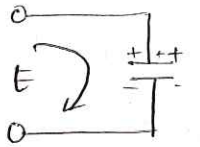
## Loss of charge method :-

CPRI ⇒ Central Power Research  
Institute... (55)  
certification...



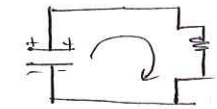
case (i) :-  $S_1$  - closed ;  $S_2$  - Open

capacitor will be fully charged ...



case (ii) :-  $S_2$  - closed ;  $S_1$  - Open.

capacitor discharges through unknown resistance.



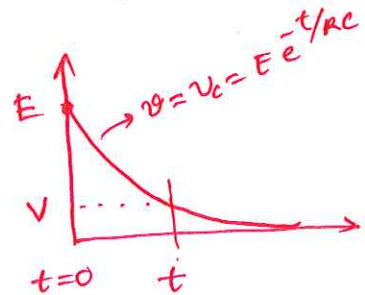
∴ ckt behaves like a discharging RC ckt.

Voltage across capacitor

$$V_c = E e^{-t/RC} \quad (\text{i.e. exponential decay}).$$

here  $R = R_x$ .

$$\therefore V = E e^{-t/RC} \Rightarrow \frac{E}{V} = e^{t/RC}$$



$$t = RC \ln\left(\frac{E}{V}\right) \Rightarrow R = \frac{t}{C \ln\left(\frac{E}{V}\right)}$$

$t$  = time taken to read  $V$  voltage from (Switch 2 closed).  
(discharging time in seconds)

$$\log_e x = \ln x = \frac{2.303 \log_{10} x}{10}$$

$$\therefore R = \frac{t}{2.303 C \log_{10}\left(\frac{E}{V}\right)} = \frac{0.4343 t}{C \log_{10}\left(\frac{E}{V}\right)}$$

### advantages :-

- \* It is one of the best method to measure the insulation resistance of an underground cable because the cable itself offers some capacitance so that auxiliary capacitors are sufficient.

### Disadvantages :-

- \* To measure the high voltage across the insulation resistance an electrostatic voltmeter is used, it **ESV-meter Capacitance** works on the principle of change in capacitance so the meter capacitance will **affect** the actual capacitance of the ckt.

---

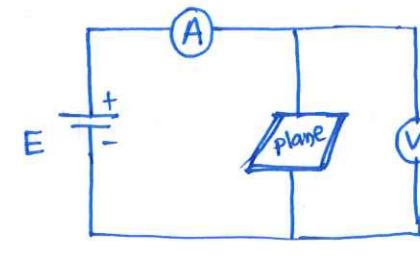
Q:- which one of the following method is used to measure the surface resistivity of an insulating material.

- Direct deflection method
-



## Direct Deflection Method :-

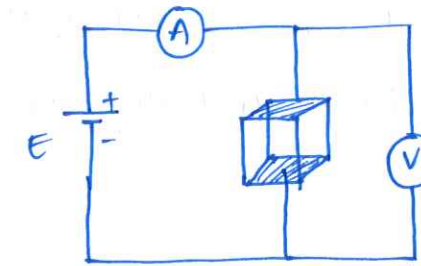
$$R_m = R_{\text{plane}} = R_{\text{surface}} = \rho_s \left( \frac{l}{A} \right) = \frac{V}{A}$$



Surface resistivity

$$R_m = \rho_s \left( \frac{1 \text{ unit}}{1 \times 1 \text{ cm}} \right) = \rho_s$$

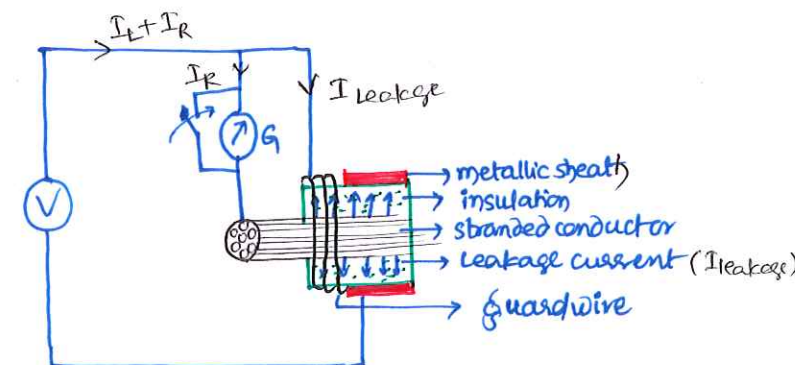
$$R_m = \rho_s$$



volume resistivity.

$$R_m = R_{\text{cube}} = \frac{V}{A} = \rho_v \frac{l}{A} = \rho_v \frac{1 \text{ unit}}{1 \text{ cm} \times 1 \text{ cm}}$$

$$R_m = \rho_v$$



By using Direct deflection method an insulation resistance of an underground cable is measured. The galvanometer (G) measures the current b/w the conductor and metallic sheath. The voltage is measured b/w the conductor and metallic sheath.

### Difficulties faced in the measurement of high resistance :-

- (i) The insulation resistance of an insulating material may be comparable with the actual value of resistance so that due to electrostatic field effect there is an ionization of insulating medium so that leakage currents produced.

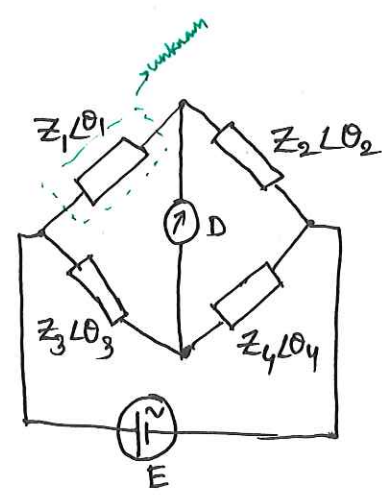
The leakage all comparable to that of actual currents. So that we must eliminate these leakage currents from the measure

These can be carried-out by using **guard-wire**.

(ii) Due to electro-static field effect **stray-charges** can appear in the measuring ckt, It will cause the error.

(iii) In the measurement of insulation resistance the **specimen** will offer some **capacitance**, on application of direct voltage a **large amount of charging current** flows initially and it decays after a short interval.

DC Bridges	AC Bridges
* Used to measure resistance only	* Used to measure inductance(L), capacitance(C), mutual inductance(M).
* Type of detector D'Arsonval Galvanometer	* Type of detectors 1. Vibrational Galvanometer 5 Hz to 100 Hz $\Rightarrow$ power frequency 2. Head phones (250 - 3000) Hz $\Rightarrow$ audio frequency 3. Tunable amplifiers (10 Hz - 100 K) Hz Radio frequency range.
* Type of Supply (i) DC Supply (ii) DC Battery	* Type of supply $\Rightarrow$ AC supply (i) Fixed-free oscillator upto $f = 1000$ Hz power o/p upto 1 Watt (ii) Cristal oscillator upto $f = 125$ KHz power o/p upto 7 watts. (iii) Interruptors (electronic oscillator) upto $f = 50$ Hz - 100 Hz required power o/p $< 1$ W. (iv) Micro phone hummers $f = 500 - 3000$ Hz very small power o/p. (v) Alternator, $f = 120$ KHz large power o/p



- $\angle \theta_1 = 0^\circ \Rightarrow$  pure resistor
- $\angle \theta_1 = 90^\circ \Rightarrow$  pure inductor
- $\angle \theta_1 = (0^\circ - 90^\circ) \Rightarrow$  combination of R & L (not series or parallel)
- $\angle \theta_1 = -90^\circ \Rightarrow$  pure capacitor
- $\angle \theta_1 = (90^\circ \text{ to } 0^\circ) \Rightarrow$  combination of R & C

continued



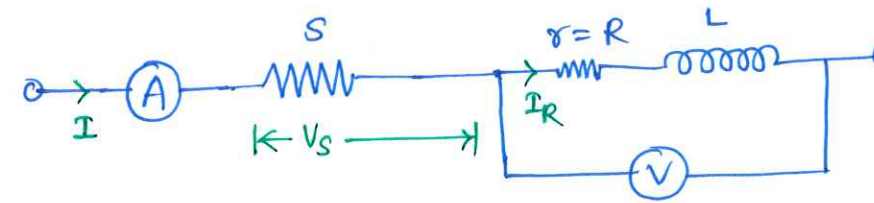
continuation

DC Bridges	AC Bridges
<p>* Only magnitude condition need to satisfy.</p> $\frac{P}{Q} = \frac{R}{S} \Rightarrow R_1 R_4 = R_2 R_3$ $Z_1 Z_4 = Z_2 Z_3$	<p>* Both magnitude &amp; phase angle should satisfy.</p> $ Z_1 Z_4  =  Z_2 Z_3 $ $L\theta_1 + L\theta_4 = L\theta_2 + L\theta_3$

- (A-V) method
  - (V-V) method
  - (V-A) method
- } Basic elementary methods  
(Loading effect of meters)
- Maxwell's inductance → no capacitor, not meant for Q factor measure.
  - Maxwell's inductance-capacitance → Low Q ( $Q < 1$ )
  - Hay's bridge → Accurate-slow balance - High Q bridge. ( $Q > 10$ )
  - Owen's bridge → medium Q, two capacitor
  - Schering bridge → Loss angle, capacitance,  $\tan \delta$ , dissipation factor
  - Anderson bridge → 5 point,  $1 \times Q$ , faster balance, fixed capacitor
  - Desautels bridge
- \* Bridge doesn't have capacitance
  - \* Bridge doesn't measure Q-factor of coil
- } Maxwell inductance
- \* Bridge used to measure Q of low Q coils → Maxwell indu-capac-bridges
  - \* Bridge measure high Q → Hay's bridge
  - \* Bridge - slow balance bridge → Hay's bridge
  - \* " - Accurate bridge → Hay's bridge
  - \* Bridge consists of two capacitors. - Owen's bridge
  - \* " 5 point bridge - Anderson.

## (A-V) Method :-

58



$R, L \Rightarrow$  unknown

$$I = I_S = I_R \quad \text{Ⓢ} = I_R (R + j\omega L)$$

$$V_S = I_S S \quad \text{Ⓢ} = I_R R + j\omega L I_R$$

$$V_S = \text{Ⓢ} S \quad \text{Ⓢ} = I R + j\omega L I$$

it is simple series ckt

Ideal Ⓢ & Ⓢ are

considered  $\Rightarrow$  i.e.

Loading effects are neglected.

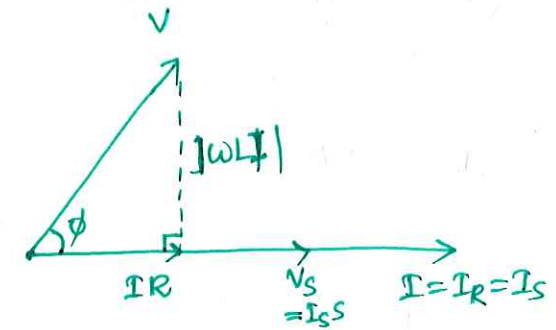
$S \Rightarrow$  standard resistance.

$I = I_S$  is in phase with  $V_S$

$V_S$  is in phase with  $I_S = I$ .

$I = I_R$  lags  $V$  by ' $\phi$ '.

$V$  leads  $I_R = I$  by  $\phi$ .



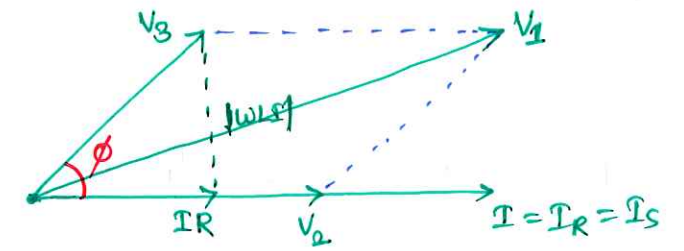
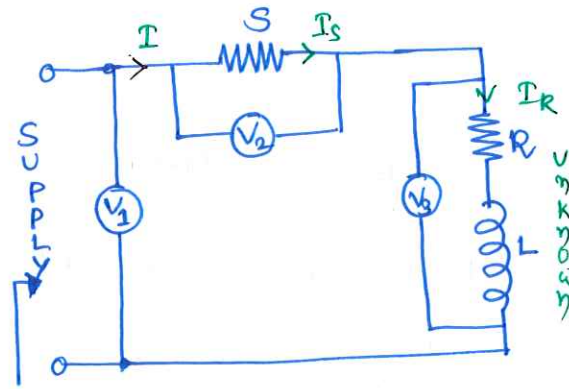
$$\cos \phi = \frac{I R}{V} \Rightarrow R = \frac{V}{I} \cos \phi$$

$$R = \frac{\text{Ⓢ}}{\text{Ⓢ}} \cos \phi$$

$$\sin \phi = \frac{\omega L I}{V}$$

$$L = \frac{V}{\omega I} \sin \phi \Rightarrow L = \frac{\text{Ⓢ}}{\omega \text{Ⓢ}} \sin \phi$$

### 3-V method :-



$$V_1 = V_2 = I_S \cdot S ; I = I_S = I_R$$

$$I_S = \frac{V_2}{S}$$

$$I = I_S \text{ is in phase with } V_2 \quad \vec{V}_1 = \vec{V}_2 + \vec{V}_3$$

$$V_2 = I_R R + j\omega L I_R$$

$$V_2 \text{ is " " } I_S = I$$

$$V_3 = I_R (R + j\omega L)$$

$$I = I_R \text{ lags } V_3 \text{ by } \phi$$

$$V_1 = I R + j\omega L I$$

$$V_3 \text{ leads } I_R = I \text{ by } \phi$$

$$\vec{V}_1 = \vec{V}_2 + \vec{V}_3$$

$$V_1^2 = \sqrt{V_2^2 + V_3^2 + 2V_2V_3 \cos\phi}$$

$$\cos\phi = \frac{V_1^2 - V_2^2 - V_3^2}{2V_2V_3}$$

$$\cos\phi = \frac{IR}{V_3}$$

$$\Rightarrow R = \cos\phi \frac{V_3}{I}$$

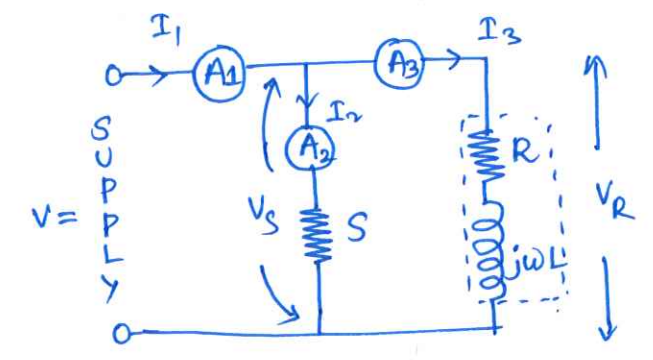
$$R = \frac{V_3}{V_2} S \cos\phi$$

$$L = \frac{V_3}{V_2} \frac{S}{\omega} \sin\phi$$

$$\sin\phi = \frac{\omega L I}{V_3}$$



3-A method :-



$$V = V_S = V_R$$

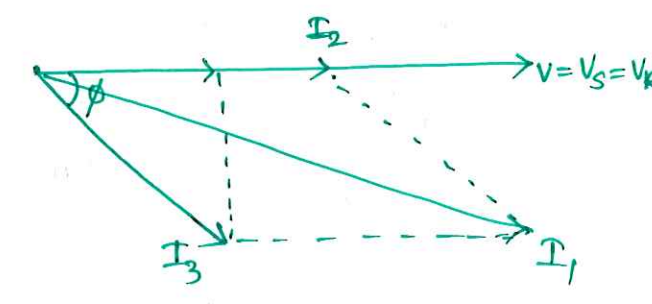
$$\bar{I}_1 = \bar{I}_2 + \bar{I}_3$$

$$I_1^2 = I_2^2 + I_3^2 + 2I_2I_3 \cos \phi$$

$$\cos \phi = \frac{I_1^2 - I_2^2 - I_3^2}{2I_2I_3}$$

$I_3$  lags  $V_R$  by  $\phi$ .

$I_2R$  is in phase with  $V_S = V = V_R$ .



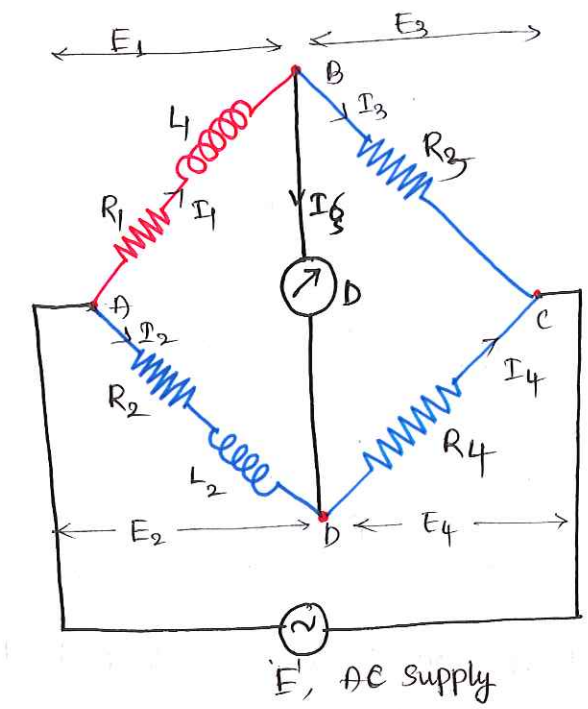
mirror image of 3-V method

$$\cos \phi = \frac{I_2 R}{V_R}$$

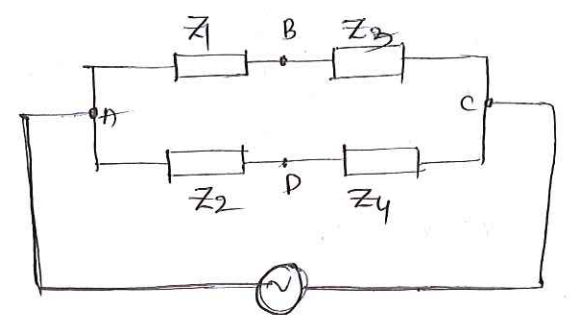
$$\therefore R = \frac{V}{\cos \phi} \left( \frac{I_2}{I_3} \right)$$

# Maxwell's Inductance Bridge

always  $\neq$  as  $\Rightarrow$  Unknown



Bridge is under balance  
 $I_G = 0$ ;  $I_1 = I_3$ ;  $I_2 = I_4$



$\vec{E}_1 = \vec{E}_2$ ;  $\vec{E}_3 = \vec{E}_4$  ( $\because E_{BD} = 0$ )  
 $\vec{E} = \vec{E}_2 + \vec{E}_4 = \vec{E}_1 + \vec{E}_3$

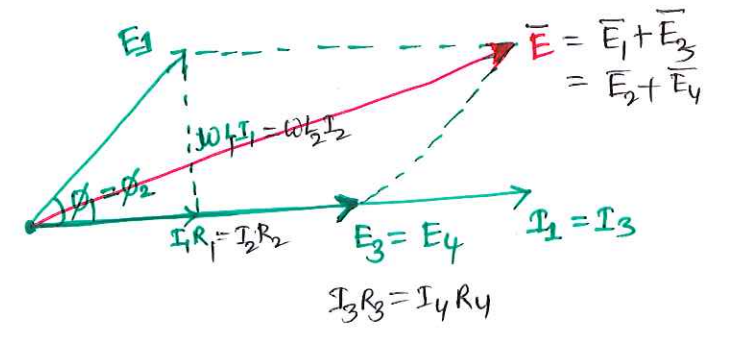
$\angle \phi_1 = \angle \phi_2$

$E_1 = I_1 Z_1 = I_1 (R_1 + j\omega L_1) = I_1 R_1 + j\omega L_1 I_1$   
 $E_2 = I_2 Z_2 = I_2 (R_2 + j\omega L_2) = I_2 R_2 + j\omega L_2 I_2$   
 $E_3 = I_3 R_3$ ;  $E_4 = I_4 R_4$

$I_1$  lags  $E_1$  by  $\phi_1$  angle.  
 $E_1$  leads  $I_1$  by  $\phi_1$  angle.  
 $I_2$  lags  $E_2$  by  $\phi_2$  angle.  
 $E_2$  leads  $I_2$  by  $\phi_2$  angle.  
 $E_3$  is in phase with  $I_3$   
 $E_4$  is in phase with  $I_4$

In all bridges current is taken as reference to draw vector or phasor diagram.

$I_3 R_3$  &  $I_4 R_4 \gg I_1 R_1$  always  
 ( $\because R_1 = \text{small impedance}$ )



$Z_1 Z_4 = Z_2 Z_3$   
 $\angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3$   
 $\phi_1 + 0 = \phi_2 + 0$   
 $\phi_1 = \phi_2$

$$(R_1 + j\omega L_1) R_4 = (R_2 + j\omega L_2) R_3$$

$$R_1 R_4 = R_2 R_3 \quad ; \quad \omega L_1 R_4 = \omega L_2 R_3$$

Unknown resistance.  $R_1 = \frac{R_2 R_3}{R_4}$  ;  $L_1 = \left(\frac{R_3}{R_4}\right) L_2$  Unknown inductance.

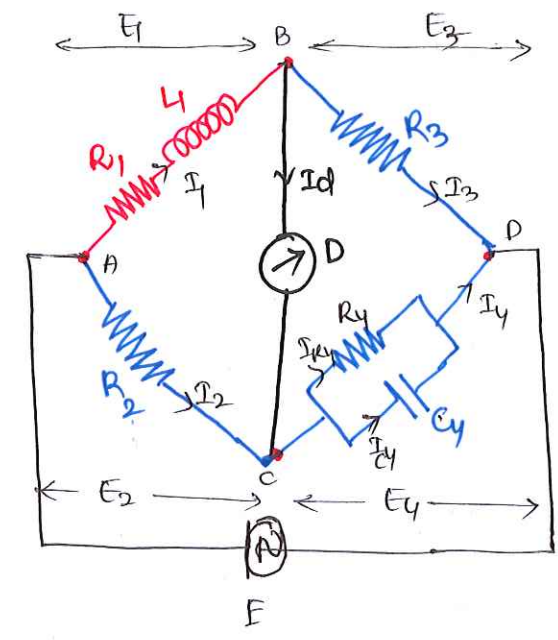
**Advantages :-**

- \* Balanced equations are simple.
- \* Balanced eqns are independent of supply frequency.
- \* No variable capacitor is present, so that **cost of bridge is lesser**

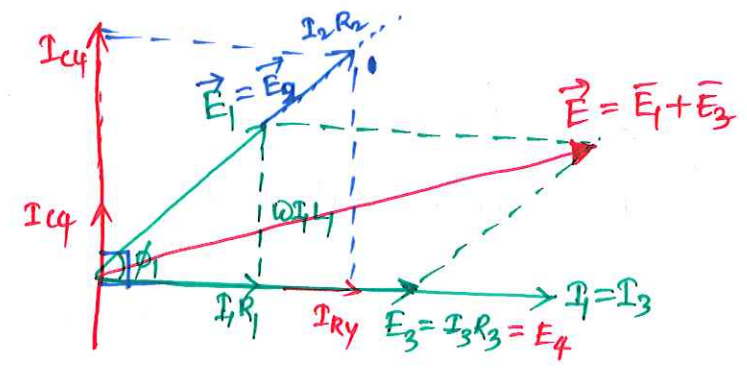
**Disadvantages :-**

- \* This bridge **cannot** suitable for measurement of **Q-factor** of coils b/c in order estimate the Q-factor, we must need a capacitor and the type of capacitor must be variable

**Maxwell's Inductance - Capacitance Bridge :-**



$E_2$  is in phase with  $I_2$   
 $E_3$  " " " "  $E_3$   
 $E_{C4}$  is lags the  $I_{C4}$  by  $90^\circ$   
 $E_{R4}$  is in phase with  $I_{R4}$   
 $E_{C4} = E_{R4} = E_4$



At Balance

$$\begin{aligned} \bar{E}_1 &= \bar{E}_2 & \bar{I}_1 &= \bar{I}_3 \\ \bar{E}_3 &= \bar{E}_4 & \bar{I}_2 &= \bar{I}_4 = \bar{I}_{C4} \neq \bar{I}_{R4} \end{aligned}$$



Bridge is Balanced  $Z_1 Z_4 = Z_3 Z_2$

$$(R_1 + j\omega L_1) \left( \frac{R_4}{1 + j\omega C_4 R_4} \right) = R_2 R_3$$

$$\frac{R_4 (R_1 + j\omega L_1) (1 - j\omega C_4 R_4)}{(1 - \omega^2 C_4 R_4)} = R_2 R_3$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega C_4 R_4 R_2 R_3$$

$$R_1 R_4 = R_2 R_3 \quad ; \quad L_1 R_4 = C_4 R_4 R_2 R_3$$

$$R_1 = \frac{R_2 R_3}{R_4}$$

$$L_1 = C_4 R_2 R_3$$

$$Q\text{-factor} \Rightarrow Q = \frac{\omega L_1}{R_1} = \omega R_4 C_4$$

$$Q = \omega R_4 C_4$$

$$R_4 \rightarrow 10^3 \quad ; \quad C_4 \rightarrow 10^{-6}$$

$$\therefore Q < 1 \quad \text{low-Q-factor.}$$

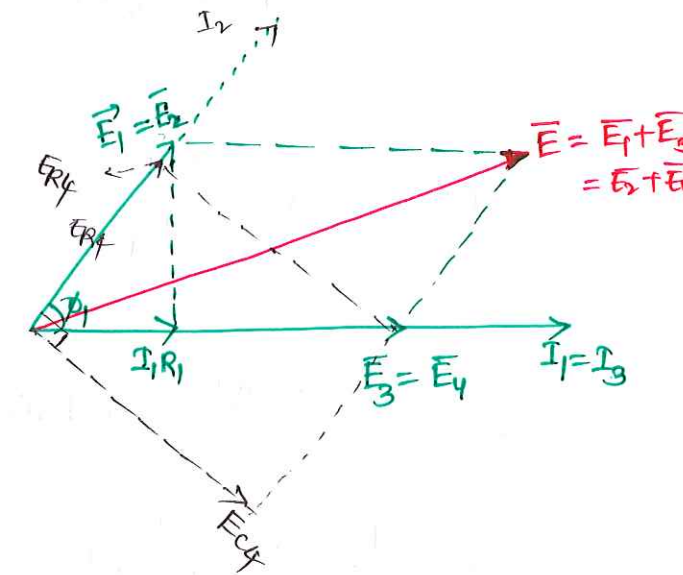
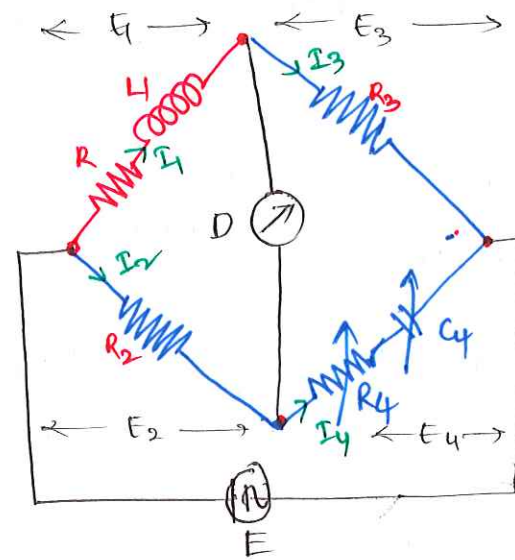
advantages:-

- \* Balanced eqns are independent.
- \* simple eqn. Suitable for measurement low Q-coils.

Disadvantages:-

- \* Cost is more b/c of variable capacitor.
- \* Cannot suitable for Q-factor of large Q-coils.

# Hay's Bridge :-



$$I_{R4} = I_{C4} = I_2 = I_4 ; I_1 = I_3$$

$$I_4 = I_{R4} = I_{C4}$$

$$\bar{E}_1 = \bar{E}_2 ; \bar{E}_4 = \bar{E}_2$$

$$E_4 = \bar{E}_{R4} + \bar{E}_{C4}$$

$\bar{E}_{C4}$  lags  $(I_{C4} = I_4 = I_2)$  by  $90^\circ$   
 $\bar{E}_{R4}$  in phase with  $(I_{R4} = I_4 = I_2)$

Bridge is under balanced condition.

$$(R_1 + j\omega L_1)(R_4 + \frac{1}{j\omega C_4}) = R_2 R_3 \Rightarrow$$

$$R_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 + \frac{R_1}{j\omega C_4} = R_2 R_3$$

$$(R_1 + j\omega L_1)(1 + j\omega C_4 R_4) = j\omega R_2 R_3 C_4$$

$$R_1 - \omega^2 C_4 R_4 L_1 + j\omega(R_1 C_4 R_4 + L_1) = j\omega R_2 R_3 C_4$$

$$R_1 - \omega^2 C_4 R_4 L_1 = 0 \Rightarrow R_1 C_4 R_4 + L_1 = R_2 R_3 C_4$$

$$\omega^2 = \frac{R_1}{R_4 C_4 L_1}$$

$$(\omega^2 C_4 R_4 L_1)(C_4 R_4) + L_1 = R_2 R_3 C_4$$

$$L_1 (1 + \omega^2 C_4^2 R_4^2) = R_2 R_3 C_4$$

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2} ; R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 C_4^2 R_4^2} \quad \text{- Hay's Bridge}$$

$$Q\text{-factor } Q = \frac{\omega L_1}{R_1} = \frac{\omega (R_2 R_3 C_4)}{\omega^2 (R_2 R_3 R_4 C_4^2)} = \frac{1}{\omega C_4 R_4}$$

$$\therefore Q = \frac{1}{\omega R_4 C_4} ; L_1 = \frac{R_2 R_3 C_4}{1 + \frac{1}{Q^2}} ; R_1 = \frac{(R_2 R_3 / R_4)}{1 + Q^2}$$

$$Q \propto \frac{1}{C_4 R_4} \propto \frac{1}{10^3 \times 10^6} \Rightarrow Q > 10 \Rightarrow \text{practically high } Q.$$

advantages:- By selecting high  $Q \Rightarrow \frac{1}{Q^2} = \omega^2 R_4^2 C_4^2 \lll 1$ .

$$\therefore L_1 = R_2 R_3 C_4 ; R_1 = \omega^2 R_2 R_3 R_4 C_4^2$$

$$Q^2 = \frac{1}{\omega^2 C_4^2 R_4^2}$$

$$R_1 = \frac{R_2 R_3}{R_4 Q^2}$$

$$R_1 = \frac{R_2 R_3 R_4 C_4^2}{R_4^2 C_4^2 Q^2}$$

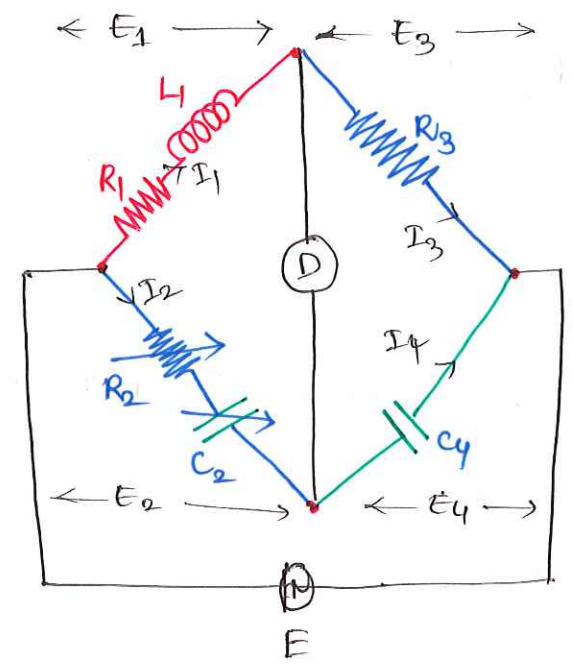
Disadvantages:-

- \* Balanced eqns are complex & dependent on supply frequency.
- \* Cost of the bridge is more b/c of variable capacitor.
- \* Best suitable for high  $Q$ -factor measurement.
- \* By selecting large value of  $Q$ , we can make the balanced equations independent of supply frequency.



## Owen's Bridge :- (medium-Q-bridge).

→ Bridge consists of two capacitors



Balanced condition.

$$(R_1 + j\omega L_1) \left( \frac{1}{j\omega C_4} \right) = R_3 \left( R_2 + \frac{1}{j\omega C_2} \right)$$

$$(R_1 + j\omega L_1) C_2 = R_3 C_4 (1 + j\omega R_2 C_2)$$

$$R_1 C_2 + j\omega L_1 C_2 = R_3 C_4 + j\omega R_2 C_2 R_3 C_4$$

$$R_1 C_2 = R_3 C_4 ; L_1 C_2 = R_2 C_2 R_3 C_4$$

$$\boxed{R_1 = \frac{R_3 C_4}{C_2}} ; \boxed{L_1 = R_2 R_3 C_4}$$

at Bridge, balance condition

$$I_1 = I_3 ; I_2 = I_4 = I_{R_2} = I_{C_2}$$

Q-factor

$$Q = \frac{\omega L_1}{R_1} = \frac{\omega (R_2 R_3 C_4)}{\left( \frac{R_3 C_4}{C_2} \right)}$$

$$\boxed{Q = \omega R_2 C_2}$$

∴ medium-Q-factor.

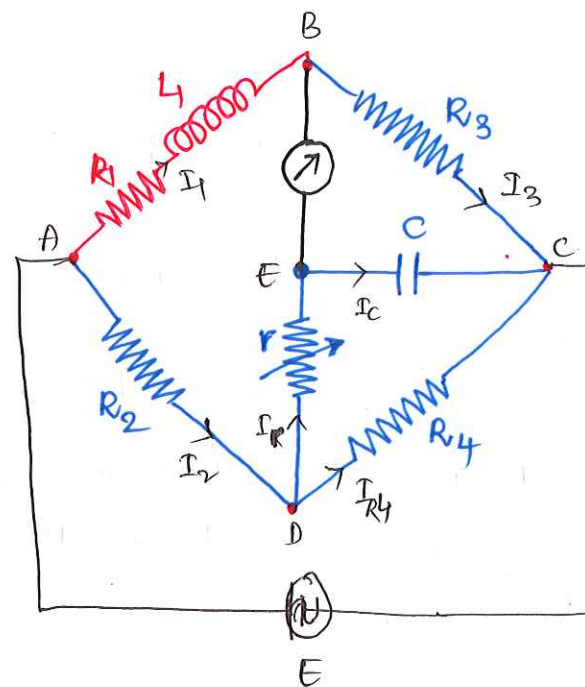
$$(1 < Q < 10)$$

advantages :- (Coven's bridge) & disadvantages

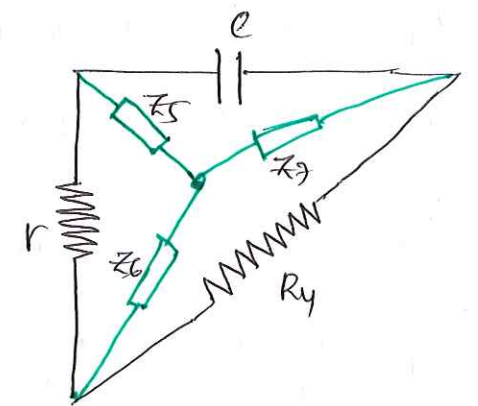
- \* Balanced eqns are simple & independent of supply frequency.
- \* Suitable for measuring medium Q-factor coils, (only).
- \* Cost of the bridge is more b/c of variable capacitance.

Aderson Bridge :- (5-point Bridge)

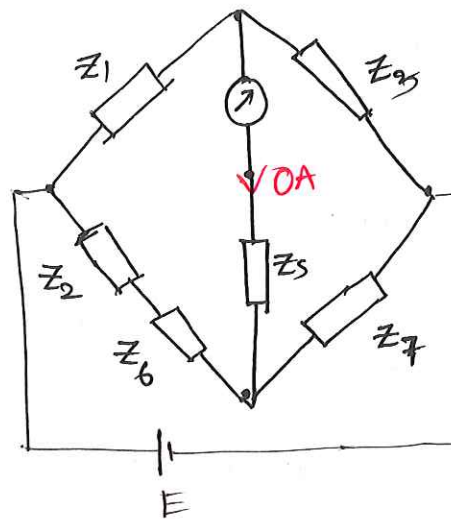
- \* Fixed capacitor ( $\therefore$  not possible to measure Q-factor of high)
- \* two variables  $r, R_4 \Rightarrow$  can obtain faster balance.  
( $\therefore$  can measure <sup>very</sup> low Q-factor).



To analyse this bridge convert  $\Delta$ -n/w into Y-n/w



$$Z_5 = \frac{\delta \cdot \frac{1}{j\omega C}}{\delta + R_4 + \frac{1}{j\omega}} ; Z_6 = \frac{\delta R_4}{\delta + R_4 + \frac{1}{j\omega}} ; Z_7 = \frac{R_4 \cdot \frac{1}{j\omega C}}{R_4 + \delta + \frac{1}{j\omega}}$$



At balance :- (63)

$$(Z_1)(Z_3) = (Z_2)(Z_4)$$

$$(R_1 + j\omega L_1) \frac{R_4}{[1 + j\omega(R_4 + \delta)]} = R_3 \left[ R_2 + \frac{r R_4}{r + R_4 + j\omega C} \right]$$

$$\frac{R_4 (R_1 + j\omega L_1)}{[1 + j\omega(R_4 + \delta)]} = \frac{R_3 (R_2 [(\delta + R_4)j\omega C + 1] + \delta R_4)}{[\delta + j\omega(R_4 + \delta)]}$$

$$(R_1 R_4 + j\omega L_1 R_4) = R_3 \left[ (R_2 \delta + R_2 R_4)j\omega C + R_2 + j\delta R_4 \omega C \right]$$

$$\Rightarrow j(R_3 R_2 \delta + R_2 R_3 R_4) \omega C + R_2 R_3 + j\omega \delta R_4 R_3 C$$

$$\therefore R_1 = \frac{R_2 R_3}{R_4}$$

$$L_1 R_4 = C R_2 R_3 (\delta + R_4) + C \delta R_3 R_4$$

$$L_1 = \frac{C R_3}{R_4} [R_2 (\delta + R_4) + \delta R_4]$$

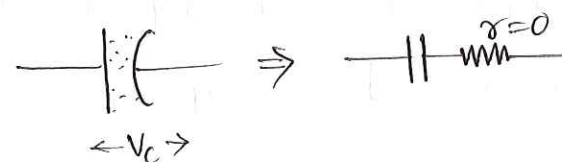
$$\therefore L_1 = \frac{C R_3}{R_4} [\delta (R_2 + R_4) + R_2 R_4]$$



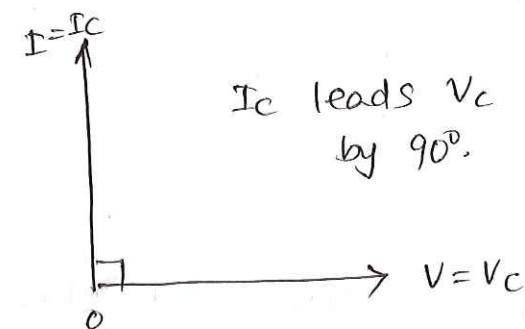
Q:- which one of the following factors is used to measure the quality of capacitive instruments.

- (a) Q-factor
- (b) D-factor  $\Rightarrow$  dissipation factor.

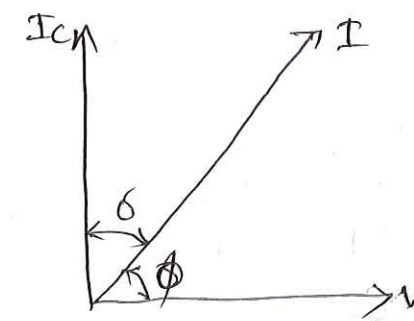
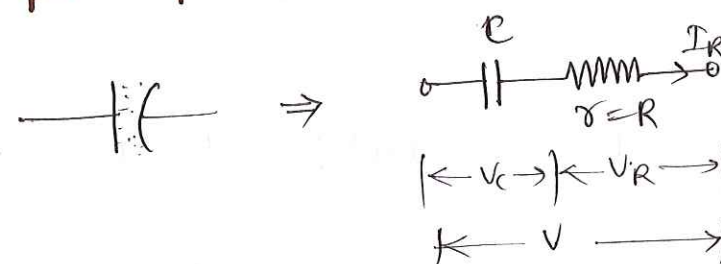
**pure capacitor :-**



No power loss  $\Rightarrow$  Ploss = 0



**Impure capacitor :-**



$I_c$  leads  $V$  by  $\phi$ .  
 $Pf = \cos \phi = \sin \delta$ .



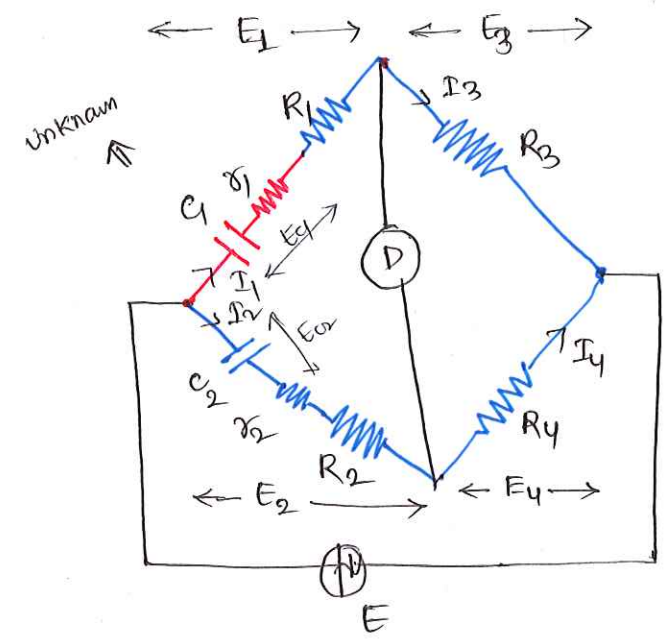




- capacitance measurement → Desauty's Bridge. (only pure C), δ-factor
- modified Desauty's Bridge. (pure & impure)
- Schering Bridge

Modified Desauty's Bridge :-

modified desauty's bridge is used to measure the capacitance & defactor (tan δ = DF) of impure capacitor as well as pure capacitance.



$V_{c1}$  lags  $I_1$  by  $90^\circ$ .  
 $E_{c1}$  is lags  $I_1$  by  $\phi_1$   
 $\bar{E}_1 = \bar{V}_{c1} + I_1 \delta_1$   
 $E_1$  is lags  $I_1$  by  $\phi_1^*$   
 $(\phi > \phi_1^*)$ .

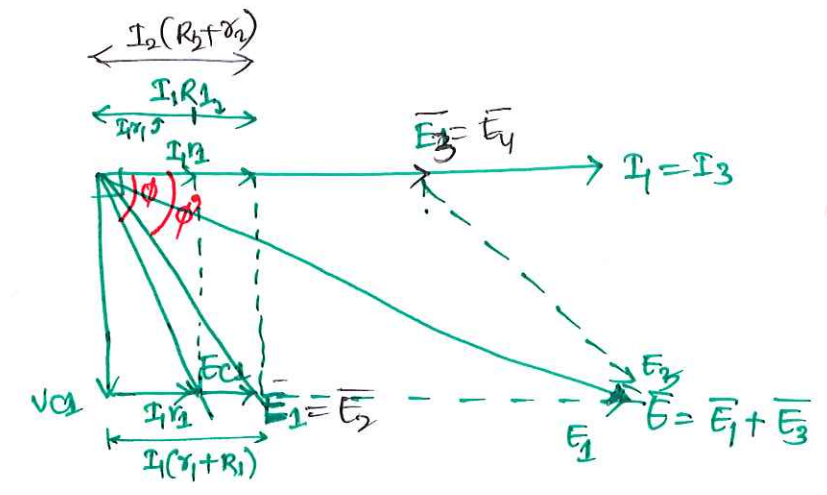
$E_1 = E_{c1} + I_1 R_1$

Bridge is under balance

$I_1 = I_{c1} = I_3$

$E_4 = E_{c2}$

$Z_1 Z_4 = Z_2 Z_3$



$V_{c1} = V_{c2}$   
 $I$

$(R_1 + \delta_1 + \frac{1}{j\omega C_1})(R_4) = (R_3) (R_2 + \delta_2 + \frac{1}{j\omega C_2})$

$[1 + j\omega C_1(\delta_1 + R_1)] R_4 C_2 = R_3 C_2 [1 + j\omega(R_2 + \delta_2)]$

$$R_4 C_2 = R_3 C_1$$

$$(R_1 + \delta_1) R_4 = (R_2 + \delta_2) R_3$$

$$C_1 = \frac{R_4 C_2}{R_3}$$

$$\delta_1 = R_1 = \left( \frac{(R_2 + \delta_2) R_3}{R_4} - R_1 \right)$$

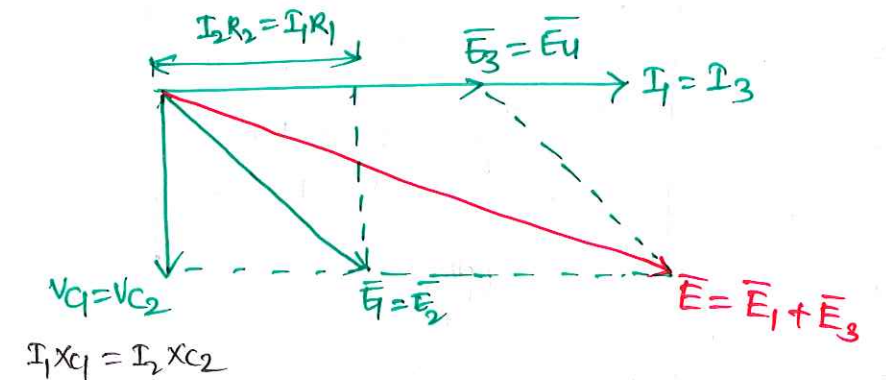
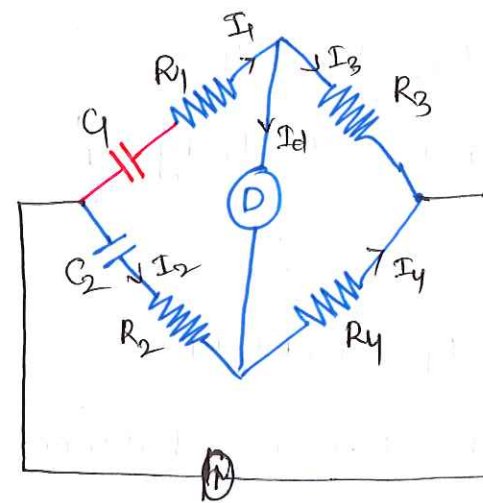
$$C_1 = C_2 \frac{R_4}{R_3}$$

$$\delta_1 = \frac{(R_2 + \delta_2) R_3}{R_4} - R_1$$

D-factor  $DF = \tan \delta = \omega C \delta_1$ .

put  $\delta_1 = 0$ , it will become desauty's bridge.

### Desauty's Bridge :-



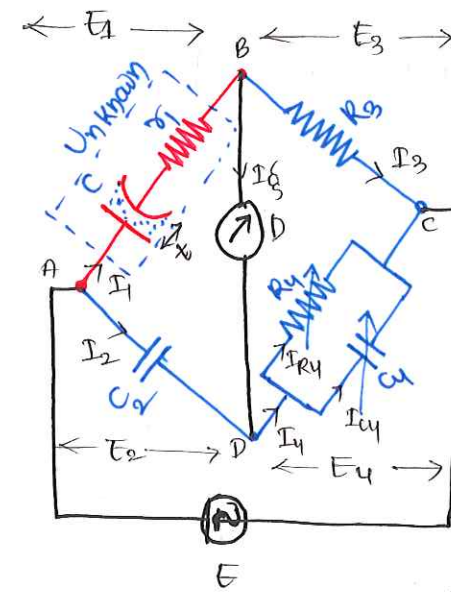
$$C_1 = C_2 \frac{R_4}{R_3}$$

$$R_1 = \frac{R_2 R_3}{R_4}$$

$$DF = 0$$

## Schering Bridge :- (3 capacitance bridge)

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### Applications :-

$C_2 \rightarrow$  standard compressed air capacitor (loss less capacitor).

- \* It is used to measure the capacitance
- \* It is also used to measure the capacitance of underground cables
- \* To check the healthy-ness of bushings used in transformers.
- \* To check the healthy-ness of insulators, used in power systems.
- \* To estimate the thickness of dielectric (strength) sheet.
- \* To estimate the permittivity of dielectric " "
- \* To estimate the dielectric power loss.
- \* To estimate the power-factor of lossy capacitor and loss-angle.

Bridge is under-balanced condition  $Z_1 Z_4 = Z_2 Z_3$

$$\left( R_1 + \frac{1}{j\omega C_1} \right) \left( R_4 \parallel \frac{1}{j\omega C_4} \right) = R_3 \left( \frac{1}{j\omega C_2} \right)$$

$$(1 + j\omega C_1 R_1) (1 + j\omega C_4 R_4) \cdot \frac{1}{j\omega C_4} = j\omega C_1 \frac{1}{j\omega C_2} R_3$$

$$(1 - \omega^2 C_1 R_1 C_4 R_4) + j\omega (C_1 R_1 + C_4 R_4) = j\omega C_1 R_3$$



$$C_1 = C_2 \frac{R_4}{R_3}$$

$$R_1 = \frac{C_4}{C_2} R_3$$

$$\text{D-Factor} \Rightarrow \tan \delta = \text{D-Factor} = \omega R_1 C_1 = \omega \cdot \frac{C_4}{C_2} R_3 \cdot C_2 \cdot \frac{R_4}{R_3}$$

$$\text{D-Factor} = \omega R_4 C_4$$

$$\text{For circular plates} \Rightarrow A = \frac{\pi d^2}{4} ; C_1 = \frac{A \epsilon}{t}$$

$d = 2r = \text{diameter}$ .

$$\therefore t = \frac{A \epsilon}{C_1} = \frac{\pi d^2 \epsilon}{4 C_1}$$

$$\epsilon = \frac{t C_1}{A}$$

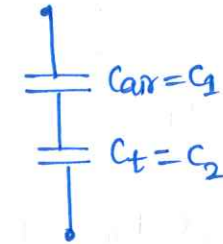
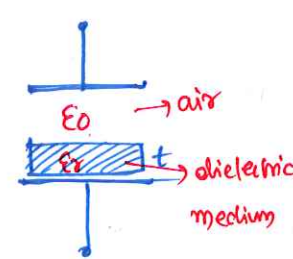
$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

$$\text{Loss pf} = \cos \phi = \sin \delta = \frac{r_1}{\left| (r_1 + \frac{1}{j\omega C_1}) \right|} = \frac{r_1}{\sqrt{r_1^2 + \frac{1}{\omega^2 C_1^2}}}$$

\* for <sup>very</sup> small  $\delta$  angle,  $\delta \approx \sin \delta \approx \tan \delta \approx \cos \phi$

$$\therefore \text{Loss pf} = \text{D-Factor}$$

In all capacitive bridges every junction (or) node to the earth (or) a capacitance is formed is known as earth capacitance (or) stray-capacitance b/c of electrostatic-field effect. To eliminate the effect of effect capacitance **wagner earthing device** is used.

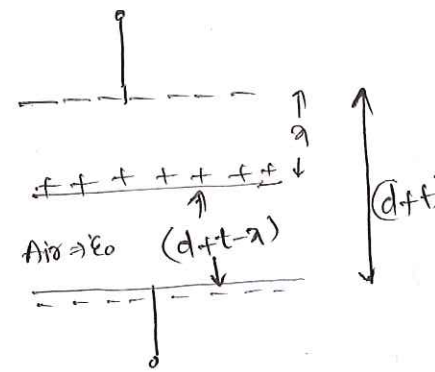


$$C_1 = \frac{A E_o}{d} ; C_2 = \frac{A E_r E_o}{t}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{A E_o}{d} \cdot \frac{A E_r E_o}{t}}{\frac{A E_o}{d} + \frac{A E_r E_o}{t}}$$

$$= \frac{A E_o E_r / dt}{(t + d E_r) dt}$$

$$C_{eq1} = \frac{A E_o E_r}{t + d E_r}$$



$$C_{eq2} = \frac{A E_o}{(d+t-x)}$$

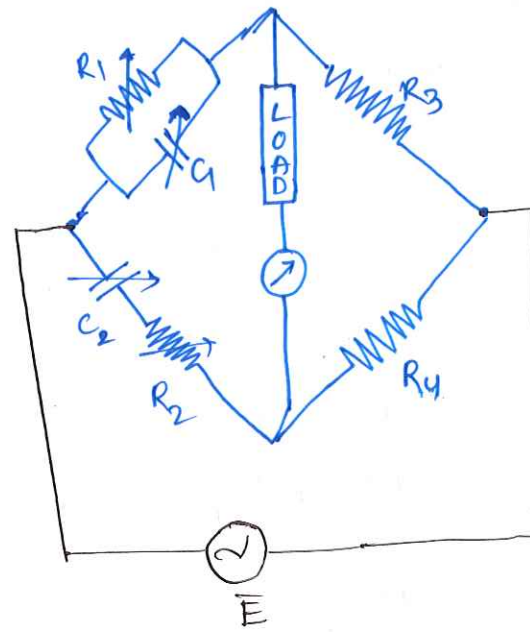
$$\textcircled{1} = \textcircled{2}$$

$$\frac{A E_o E_r}{t + d E_r} = \frac{A E_o}{d + t - x}$$

$$d E_r + (t-x) E_r = t + d E_r$$

$$\boxed{E_r = \frac{t}{(t-x)}} \Rightarrow$$

## Wein's - Robinson Bridge $\rightarrow$ (Sensitive for harmonics).



Balanced condition.

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left( R_1 \parallel \frac{1}{j\omega C_1} \right) R_4 = \left( R_2 + \frac{1}{j\omega C_2} \right) R_3$$

$$\frac{R_1 R_4}{(1 + j\omega C_1 R_1)} = \frac{(1 + j\omega C_2 R_2) R_3}{j\omega C_2}$$

$$j\omega R_1 C_2 R_4 = R_3 (1 + j\omega C_2 R_2) (1 + j\omega C_1 R_1)$$

$$j\omega R_1 C_2 R_4 = (R_3 - \omega^2 R_1 R_2 R_3 C_1 C_2) + j\omega (R_1 R_3 C_1 + R_2 R_3 C_2)$$

$$R_3 = \omega^2 R_1 R_2 R_3 C_1 C_2$$

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \Rightarrow f = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}}$$

If identical capacitors & resistors  
 $C_1 = C_2 = C$  ;  $R_1 = R_2 = R$

$$f = \frac{1}{2\pi RC}$$

### Applications :-

- \* It is used to measure the unknown frequency.
- \* It is also used to measure the unknown capacitance.
- \* It is used as frequency determining element in a

Harmonic distortion analyzer.

- \* It is used as frequency determining element in audio frequency & high frequency signals.



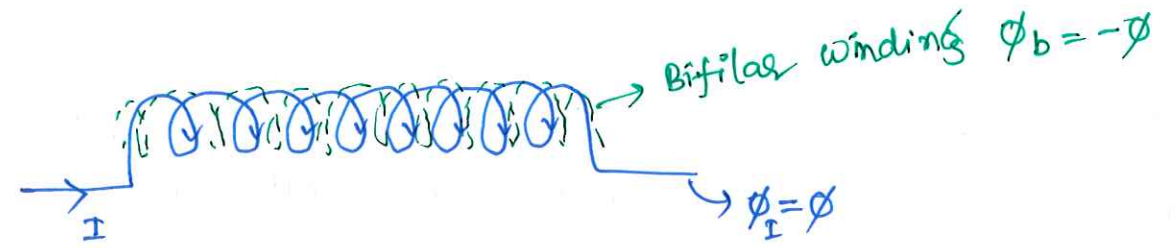
\* It is also used as frequency determining element in Notch filter applications.

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Note :-

1. This is a harmonic sensitive bridge, in order to obtain balance condition, usually a low pass filter is connected in series with detector, so that the harmonics may not be flowing through the detector.
2. Wein's bridge is easily balanced for sinusoidal waves only and it may not get balance for other signals.

## Wire-wound resistors :-

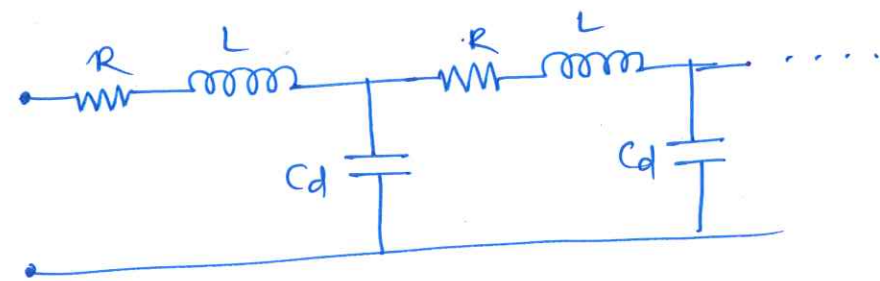


All wire-wound resistors will exhibit inductive property when we use using at high frequency in the order of several Kilo-hertz (i.e. in electronic radios like) to MHz.

To reduce this effect a winding which is placed on wire-wound resistors, which can produce the flux equal in magnitude but opposite in direction that of flux produced by wire-wound resistors is known as **Bifilar wdg.**

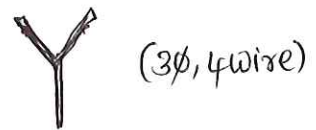
Equivalent ckt of wire wound resistor  $\Downarrow$

$C_d$  = distributed capacitor (or) interturn capacitor.



# Measurement of POWER

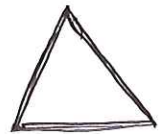
$$\text{Insulation Level} = \sqrt{2} \times 1.6 \times V_{ph}$$



(3 $\phi$ , 4 wire)

Low-voltage ( $V_{ph} = \frac{V_L}{\sqrt{3}}$ )  
high-current ( $I_{ph} = I_L$ )

\* Less insulation



(3 $\phi$ , 3 wire)

high-voltage ( $V_{ph} = V_L$ )  
Low-current ( $I_{ph} = \frac{I_L}{\sqrt{3}}$ )

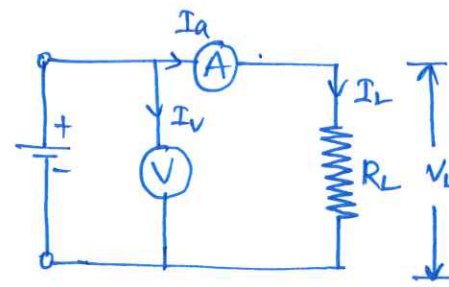
\* More insulation



## DC power measurement

(71)

### 1. voltmeter-ammeter method :-



$$P_{\text{True}} = V_L I_L = I_L^2 R_L$$

$$P_{\text{meas}} = (V)(A)$$

$$= (I_a R_a + I_L R_L) I_a$$

$$P_m = I_a^2 R_a + I_a^2 R_L \quad (\because I_a = I_L)$$

$$\text{Error} = P_m - P_T = (I_a^2 R_a + I_L^2 R_L) - (I_L^2 R_L) = I_a^2 R_a$$

$$\text{Error} = +ve \Rightarrow (\text{measured power}) > (\text{true power}).$$

$$\% \text{Error} = \frac{P_m - P_T}{P_T} \times 100 = \frac{I_a^2 R_a}{I_L^2 R_L} = \frac{V_a R_a}{V_L} = \frac{R_a}{R_L} \times 100$$

$$\therefore \% \text{Error} = \frac{R_a}{R_L} \times 100$$

errors can be measured by (i) selecting  $R_a \rightarrow 0$   
(good ammeter)

(ii)  $R_L \uparrow$ , high resistive load.

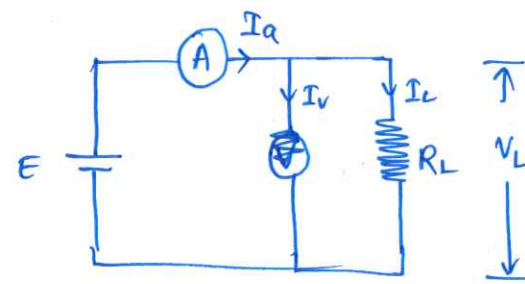
Voltmeter-ammeter method is best suitable for measurement of power in a high resistive load (or) low load current, the produced error is positive b/c of ammeter connected on load-side.

Ideally ammeter resistance is zero, practically it is not so. so that it includes the power consumed by ammeter.

Q:- If ammeter-voltmeter method is used to measure the DC power the produced error is

- (a) +ve
- (b) -ve
- (c) zero.

## 2. Ammeter-voltmeter method :-



$$I_a = I_v + I_L$$

$$V_L = I_L R_L$$

$$P_T = V_L I_L = I_L^2 R_L$$

$$P_m = (V)(A)$$

$$P_m = I_a \cdot V_L$$

$$= (I_L + I_v) V_L$$

$$P_m = V_L I_L + V_L I_v = I_L^2 R_L + \frac{V_L^2}{R_v}$$

$$\begin{aligned} \text{Error} &= P_m - P_T \\ &= \left( I_L^2 R_L + \frac{V_L^2}{R_v} \right) - (I_L^2 R_L) \end{aligned}$$

$$\text{error} = \frac{V_L^2}{R_v}$$

$$\% \text{ Error} = \frac{P_m - P_T}{P_T} \times 100 = \frac{\frac{V_L^2}{R_v}}{V_L I_L} \times 100 = \frac{V_L}{R_v I_L} \times 100 = \left( \frac{I_v}{I_L} \right) \times 100$$

$$\% \text{ Error} = \frac{V_L^2 / R_v}{V_L^2 / R_L} = \frac{R_L}{R_v} \times 100$$

$$\therefore \% \text{ Error} = \frac{R_L}{R_v} \times 100 = \frac{I_v}{I_L} \times 100$$

In order to get low error ... select (i) Low resistive loads (or) high load current

(ii) high ( $\infty$ ) or (m $\rightarrow$ ) resistance voltmeter.

To get equal error in both (A-V), (V-A) method of DC power measure of power.

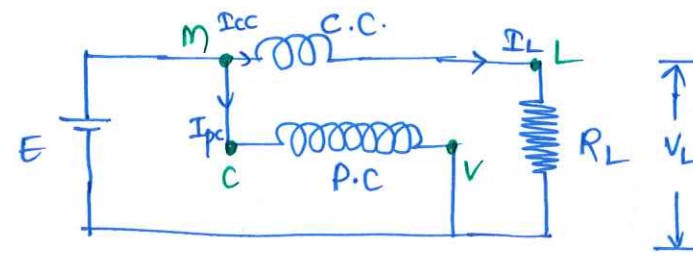
$$\% \text{ Error in (A-V)} = \% \text{ Error in (V-A)}$$

$$\frac{R_a}{R_L} = \frac{R_L}{R_v} \Rightarrow R_L^2 = R_a R_v$$

$$\therefore R_L = \sqrt{R_a R_v}$$

Ammeter-voltmeter method is best suitable for measurement of power in a low-resistive loads (or) high load currents, produced error is positive b/c of voltmeter. Ideally voltmeter resistance is  $\infty$  practically it is not possible. So that it includes the power consumed by voltmeter.

V-A method  $\equiv$  (EDM) or (EMMC) power measurement :- (MC-short connection).



MC-short connection

$$P_T = V_L I_L = I_L^2 R_L = \frac{V_L^2}{R_L}$$

$$I_{CC} = I_L$$

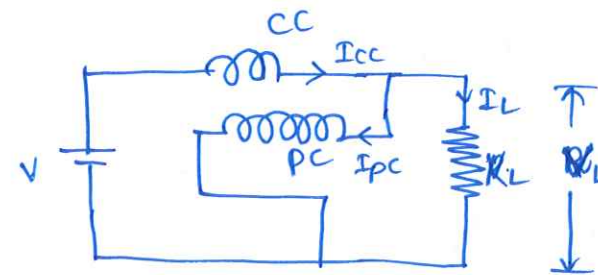
$$P_m = V_L I_L + I_{CC}^2 R_{CC}$$

$$P_m = P_T + I_{CC}^2 R_{CC}$$

$$P_m = P_T + I_L^2 R_{CC} ; (\because I_{CC} = I_L)$$

$$\text{Error} = P_m - P_T = I_L^2 R_{CC} = +ve \Rightarrow (P_m) > (P_T)$$

A-V method  $\equiv$  LC-short connection of EDC power measurement :-



LC-short connection.

$$P_m = V_L I_L + \frac{V_{PC}^2}{R_{PC}}$$

$$P_m = (V_L) (I_{CC}) = V_L (I_L + I_{PC})$$

$$\therefore P_m = P_T + \frac{V_{PC}^2}{R_{PC}}$$

$$\therefore \text{error} = +ve (\because P_m > P_T)$$

Equal error in both LC & MC short connection.

$$I_L^2 R_{CC} = \frac{V_L^2}{R_{PC}}$$

$$\therefore \left(\frac{V_L}{I_L}\right)^2 = R_{CC} \cdot R_{PC}$$

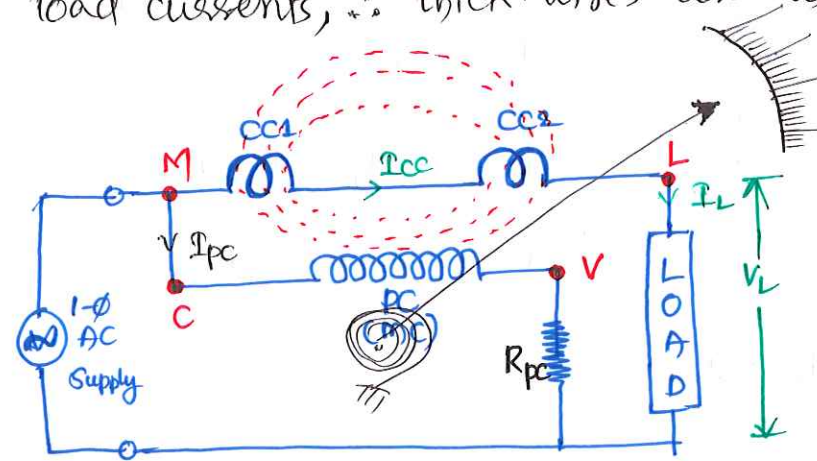
$$\boxed{\frac{V_L}{I_L} = \sqrt{R_{CC} \cdot R_{PC}}}$$



# 1- $\phi$ AC POWER measurement :-

## 1- $\phi$ AC power measurement by EDM wattmeter :-

EDM-type wattmeter works on the principle of change in mutual inductance. It will consist of two coils, one of them is known as fixed coil usually split up into two parts in order to obtain uniform-symmetrical flux distribution, this coil is always connected in series with the load so that it is designed to carry large load currents,  $\therefore$  thick-wires are used.



CC  $\rightarrow$  Fixed coils  
(Current coils)  
PC  $\rightarrow$  pressure coil  
(Moving coils).

### Assumptions :-

1. voltage across the current coil is zero ( $V_{cc} = 0$ ).

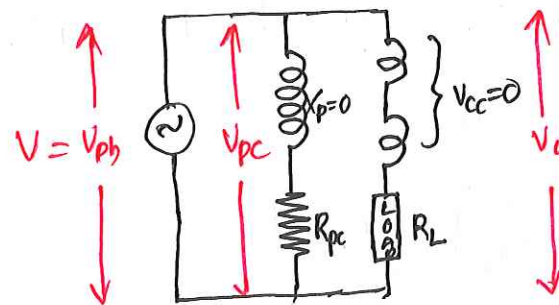
i.e. pure inductor  $\Rightarrow$  current coil.  $\therefore V_{supply} = V_L = V_{ph} = V_{pc}$

2. pressure coil is assumed as pure resistive.

$$Z_{pc} = R_{pc} + jX_{pc} = R_{pc} \quad (\because X_{pc} = 0)$$

$\therefore I_{pc}$  is in phase with ( $V_{pc} = V_L$ ).

$$I_{pc} = \frac{V_{pc}}{R_p} = \frac{V_L}{R_p}$$

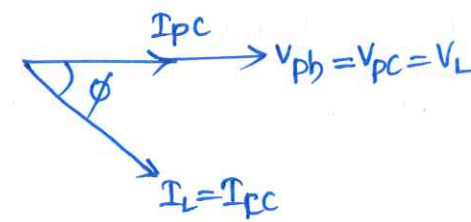


Fixed coil is also known as field coil (or) current coil. (73)

Another coil is known as moving coil always connected in parallel with the load, it is energised by supply voltage so that it is also called as pressure coil (or) potential coil (or) voltage coil, It is designed to carry small currents, so that always for pc, thin-wires are used. So that weight of the moving system is less. So that torque-to-weight ratio increases, sensitivity also increases.

In the analysis of EDM type wattmeter the two assumptions are importance.

⇒ Assume lagging Load.  $\phi (V_L I_L) \Rightarrow \phi (I_{pc}, I_{cc})$ .



$$T_d = I_{fc} \cdot I_{pc} \cdot \cos(I_{fc}, I_{pc}) \cdot \frac{dm}{d\theta}$$

$$T_d = I_{pc} \cdot I_{pc} \cos(I_{pc}, I_{cc}) \frac{dm}{d\theta}$$

$$= I_L \cdot \frac{V_{pc}}{R_{pc}} \cos\phi \frac{dm}{d\theta}$$

$$T_d = \frac{V_L \cdot I_L \cos\phi}{R_{pc}} \frac{dm}{d\theta}$$

$$T_d = \frac{(V_L I_L \cos\phi)}{R_{pc}} \left( \frac{dm}{d\theta} \right)$$

$$\therefore \boxed{T_d \propto (V_L I_L \cos\phi)} \cdot \frac{dm}{d\theta} = \text{constant}$$

maintain.

∴ (deflection torque)  $\propto$  (true power in load).

(deflection)  $\propto$  (AC real power)

$$P_T = V_L I_L \cos\phi$$

Spring Control  $\Rightarrow T_c = K_c \theta$ .

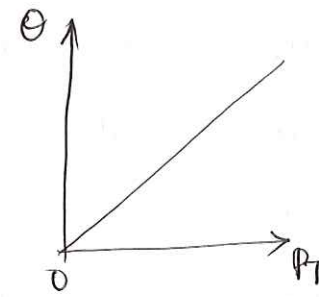
∴ at steady-state  $T_d = T_c$

$$K_c \theta = \frac{P_T}{R_{pc}} \frac{dm}{d\theta}$$

$$\therefore \boxed{\theta = \frac{P_T}{K_c R_{pc}} \frac{dm}{d\theta}}$$



scale  $\Rightarrow$  Linear



$\theta \Rightarrow (-45^\circ \text{ to } 45^\circ)$

(refer EDM type meter/instrument).

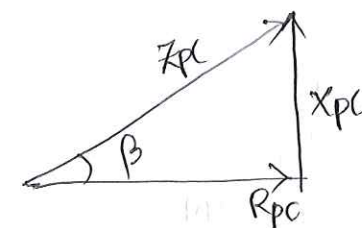
Errors :- (in EDM type wattmeter)

1. error due to **pressure coil inductance**
2. error due to interturn-capacitance of pressure coil.
3. Error due to **stray-magnetic field**. b/c of weak operating field, it can be eliminated by providing **astatic arrangement**
4. More internal heating problem.  $\therefore$  More temperature error.
5. Error due to **coil connections** (i.e. MC-short & LC-short).
6. Error due to current coil resistance (**power loss in current coil**)  
i.e.  $P_{\text{LOSS}} = I_{\text{CC}}^2 R_{\text{CC}} = I_{\text{L}}^2 R_{\text{CC}}$
7. Error due to **power-factor of the load**. A general purpose EDM type wattmeter, if it is used in a low pf loads always, it gives less deflection, even though current-coil & pressure-coils are fully excited.

Error Due to Pressure Coil Inductance for Lagging Loads :-

$$\text{Let } Z_{\text{PC}} = R_{\text{PC}} + jX_{\text{PC}} ; |Z_{\text{PC}}| = \sqrt{R_{\text{PC}}^2 + X_{\text{PC}}^2}$$

$$\text{Let } \beta = \text{impedance angle of PC} = \tan^{-1} \left( \frac{X_{\text{PC}}}{R_{\text{PC}}} \right)$$





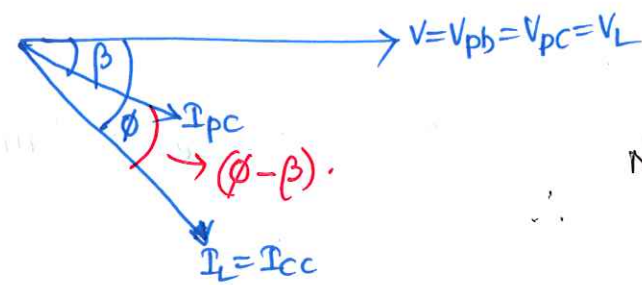
$\therefore I_{pc}$  lags  $V_{pb} = V_{pc} = V_L$  by  $\beta$  ( $V_{cc} = 0$ ;  $CC \rightarrow$  pure inductive). (7A)

$I_L$  lags  $V_L$  by  $\phi$  (Load pf angle).

$$I_{pc} = \frac{V_{pc}}{Z_{pc}} = \frac{V_{pc}}{\sqrt{R_{pc}^2 + X_{pc}^2}} \quad \angle = \beta$$

$$\cos \beta = \frac{R_{pc}}{Z_{pc}} \Rightarrow Z_{pc} = \frac{R_{pc}}{\cos \beta}$$

$$I_{pc} = \frac{V_{pc}}{Z_{pc}} = \frac{V_{pc}}{\left(\frac{R_{pc}}{\cos \beta}\right)} = \left(\frac{V_{pc}}{R_{pc}}\right) \cos \beta = \frac{V_L \cos \beta}{R_{pc}} = I_{mc}$$



New torque eqn

$$T_d' = I_{pc} \cdot I_{mc} \cos(I_{pc}, I_{mc}) \frac{dm}{d\theta}$$

$$T_d' = I_L \cdot \frac{V_L \cos \beta}{R_{pc}} \cdot \cos(\phi - \beta) \frac{dm}{d\theta}$$

$$\therefore T_d' = \frac{V_L I_L \cos \beta \cos(\phi - \beta)}{R_{pc}} \left(\frac{dm}{d\theta}\right)$$

$$T_d' = \frac{(V_L I_L \cos \phi) \left(\frac{dm}{d\theta}\right)}{(R_{pc})} \left[ \frac{(\cos \beta) \cos(\phi - \beta)}{(\cos \phi)} \right]$$

$$T_d' = T_d \left( \frac{\cos \beta \cdot \cos(\phi - \beta)}{\cos \phi} \right)$$

$T_d' = T_d$  (measured) ;  $T_d = T_d$  (true) .  $T_d \propto P$ .

$$\therefore P_T = V_L I_L \cos \phi ; \quad P_m = \frac{V_L I_L \cos \beta \cdot \cos(\phi - \beta)}{1} \times \frac{\cos \beta}{\cos \phi}$$

$$P_m = P_T \cdot \left( \frac{\cos \beta \cdot \cos(\phi - \beta)}{\cos \phi} \right)$$

$$(CF) P_m = P_T$$

$\therefore$  (Correction factor) (measured value) = (true value).

$$\therefore P_m = (V_L I_L \cos \phi) \left( \frac{\cos \beta \cos(\phi - \beta)}{\cos \phi} \right) = P_T \left( \frac{\cos \beta \cos(\phi - \beta)}{\cos \phi} \right)$$

$$\therefore CF = \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)}$$

$$P_T = P_m \times \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)}$$

for lagging loads.

Always  $CF < 1$ ; Correction factor less than unity.

If  $\beta \uparrow \Rightarrow (\phi - \beta) \downarrow \Rightarrow \cos(\phi - \beta) \uparrow \Rightarrow CF \downarrow \Rightarrow P_m \uparrow$   
(i.e.  $P_m > P_T$ ).

% Error :-

$$\begin{aligned} \% \text{ error} &= \frac{P_m - P_T}{P_T} \\ &= \frac{\frac{P_T}{CF} - P_T}{P_T} = \frac{1 - CF}{CF} \times 100 \\ &= \frac{1 - \frac{\cos \phi}{\cos \beta \cos(\phi - \beta)}}{\frac{\cos \phi}{\cos \beta \cos(\phi - \beta)}} \times 100 \\ &= \frac{\cos \beta \cos(\phi - \beta) - \cos \phi}{\cos \phi} \\ &= \frac{\cos \beta [\cos \phi \cos \beta + \sin \phi \sin \beta] - \cos \phi}{\cos \phi} \\ &= \frac{\cos^2 \beta + \tan \phi \sin \beta \cos \beta - 1}{\cos \phi} \\ &= \tan \phi \sin \beta \cos \beta - 1 + \cos^2 \beta \end{aligned}$$

$$\frac{1}{CF} = \frac{1 + \tan\phi \tan\beta}{1 + \tan^2\beta} = 1 + \tan\phi \tan\beta \quad \tan^2\beta \ll 1$$

$$\therefore \% \text{ Error} = \left(\frac{1}{CF} - 1\right) \times 100 = (1 + \tan\phi \tan\beta - 1) \times 100$$

$$\% \text{ Error} = +\tan\phi \tan\beta \times 100$$

if  $\beta = 0 \Rightarrow \text{Error} = 0$ ;  $\therefore \beta \uparrow \Rightarrow \tan\beta \uparrow \Rightarrow \text{error} \uparrow$

The produced error is positive  $\Rightarrow (P_{\text{measured}} > P_{\text{true}})$ .

if load pf  $\cos\phi$  is low. i.e.  $\phi = \text{high} \Rightarrow \tan\phi$  is  $\uparrow$  (more).

$\therefore$  Error also increases.  $\Rightarrow (P_m > P_T) \Rightarrow$  wrong reading.

1. Error due to pressure coil inductance produced is always positive. So that  $P_m > P_T$ , so that  $(CF < 1)$  always, Always the instruments reads more in a lagging loads.

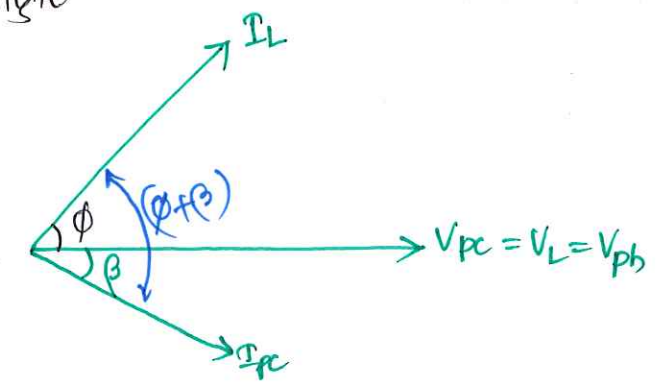
2. A general purpose EPM type wattmeter if it is used in a low pf loads, the instruments always reads more, increases appreciably as the increase in impedance angle of pressure coil.

Error due Pressure coil inductance for Leading loads

$I_L$  leads  $V_{pc} = V_L$  by  $\phi$  angle.

$$T_d = \frac{I_L I_{pc} \cos(\phi + \beta)}{R} \cdot \frac{dM}{d\theta}$$

$$T_d = \frac{V_L I_L \cos(\phi + \beta) \cos\beta}{R_{pc}} \frac{dM}{d\theta}$$





$$CF = \frac{P_T}{P_m} = \frac{\cos\phi}{\cos\beta \cos(\phi+\beta)} \quad ; \text{ Leading loads}$$

$$\beta \uparrow \Rightarrow (\phi+\beta) \uparrow \Rightarrow \cos(\phi+\beta) \downarrow \Rightarrow CF \uparrow \text{ i.e. } (CF > 1)$$

Always correction factor greater than for leading loads.

$$\therefore \% \text{ Error} = \left( \frac{1}{CF} - 1 \right) \times 100$$

$$= \left( \frac{\cos\beta \cos(\phi+\beta)}{\cos\phi} - 1 \right) \times 100$$

$$= (1 - \tan\phi \tan\beta - 1) \times 100$$

$$\% \text{ Error} = -(\tan\phi \tan\beta) \times 100$$

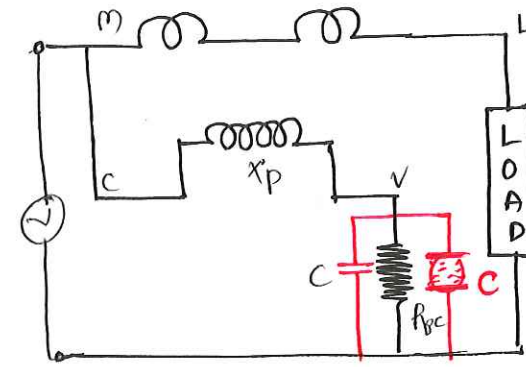
Error  $\Rightarrow$  -ve  $\Rightarrow$  ( $P_{measured} < P_{true \text{ power}}$ )

$$\beta \uparrow \Rightarrow \tan\beta \uparrow \Rightarrow \text{error} \uparrow \Rightarrow \uparrow CF > 1 \text{ (always)} \Rightarrow P_m < P_t$$

Error due to pressure coil inductance in a leading load is -ve, ( $P_m < P_t$ )  $\therefore$  CF always greater than unity ( $CF > 1$ )  
 $\therefore$  As  $\beta \uparrow$  always instrument reads lesser.

Error due to inter turn capacitance of pressure coil always the instrument reads lesser for lagging loads & more for leading loads.

Remedy :- (for effect of pressure coil inductance).



$$Z_{pc}(\text{total}) = j\omega L_p + R_p \parallel \frac{1}{j\omega C}$$

$$= j\omega L_p + \frac{R_{pc}}{1 + j\omega C R_{pc}}$$

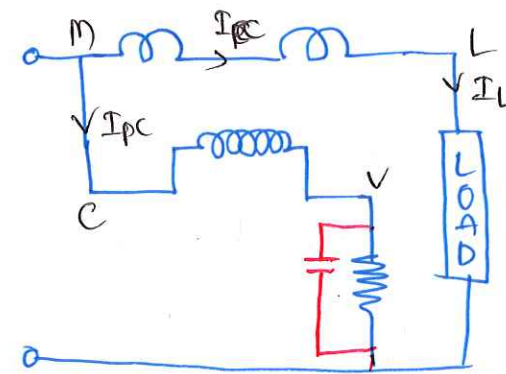
$$= j\omega L_p + \frac{R_{pc} (1 - j\omega C R_p)}{(1 + j\omega C R_{pc})(1 - j\omega C R_p)}$$

$$C = 0.41 \frac{L_{pc}}{R_{pc}^2}$$

Error due inductive reactance of pressure-coil can be reduced (or) eliminated by connecting a capacitor in parallel with  $R_{pc}$ , whose value  $0.41 \times \frac{L_{pc}}{R_{pc}^2}$

Error due to coil connections :- ( $V_{drop}$  across CC  $\approx 0$ ), neglected.  $\therefore V_p = V_L = V_{ph}$

MC-short connection



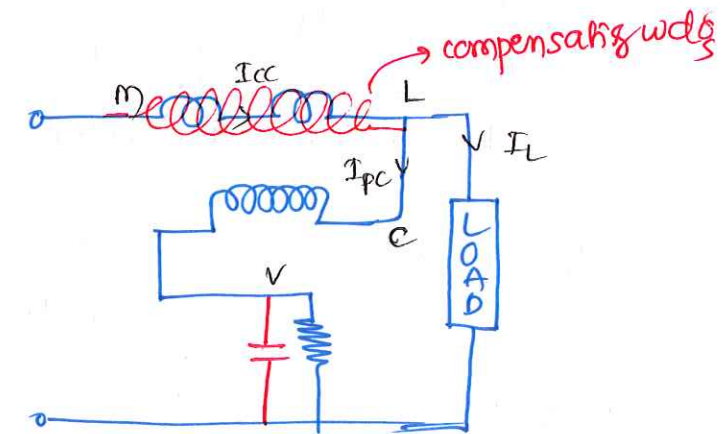
$$I_{cc} = I_L$$

$$P_{m(\text{MC-short})} = V_{pc} I_{cc} \cos(\phi_{pc}, I_{cc})$$

$$P_{m(\text{MC-S})} = V_L I_L \cos \phi$$

$$P_{m(\text{MC-short})} = P_T$$

LC-short connection.



$$I_{cc} = (I_L + I_{pc}) ; V_{pc} = V_L = V_{ph}$$

$$P_{m(\text{LC-short})} = V_{pc} \cdot I_{cc} \cos(\phi_{pc}, I_{cc})$$

$$= V_L \cdot (I_L + I_{pc}) \cos \phi$$

$$= V_L I_L \cos \phi + V_L I_{pc} \cos \phi$$

$$P_{m(\text{LC-short})} = P_T + \text{error}$$

$$\therefore P_m(\text{LC}) > P_T$$

$$\phi_{cc} \propto I_{cc}$$

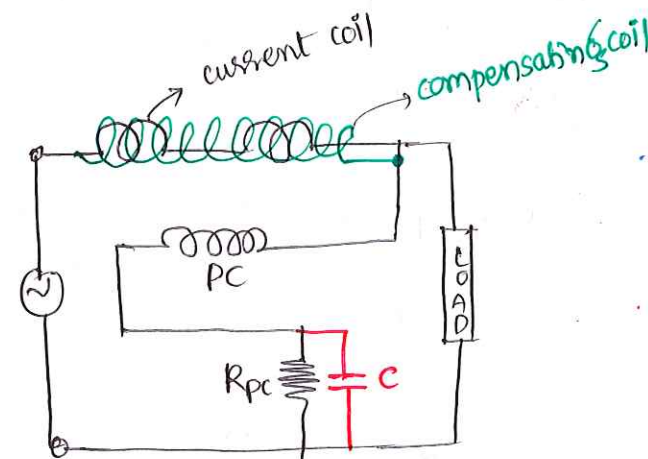
$$\phi_{cc} \propto (I_L + I_{pc}) \Rightarrow \text{LC-connection}$$

$$\phi_{pc} \propto I_{pc}$$

$$\phi_{cc} \propto (\phi_L + \phi_{pc}) \Rightarrow \text{LC-connection.}$$

Extra flux is produced due to pressure coil current produced by current coil... must be cancelled.

$\therefore$  one wdg needed to cancel it i.e. called compensating wdg which must be placed on cc that must carry pc current means it is connected in series with pressure coil.



$$\phi_{comp} = -\phi_{pc}$$

$$\begin{aligned} \therefore \phi_{cc_{net}} &= (\phi_L + \phi_{pc}) + \phi_{comp} \\ &= \phi_L + \phi_{pc} - \phi_{pc} \\ &= \phi_L. \end{aligned}$$

Fig: LC-short wattmeter

In a LC short connection extra compensating wdg is required in order to eliminate the power consumed by pressure coil that means the flux produced in a current coil due to pressure coil current can be eliminated. The compensating coil is placed on current coil & connected in series with the pressure coil, it is placed in such manner that the flux produced in a current coil due to pc current can be cancelled.



## LPF wattmeter :-

(77)

$$(i) \quad \Theta \propto V_{pc} I_{cc} \cos(\phi)$$

$$\Theta \propto V_L I_L \cos\phi$$

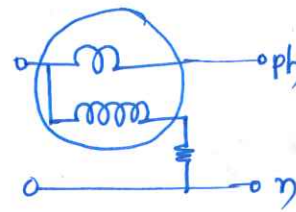
$$Pf \downarrow = \cos \downarrow \Rightarrow \Theta \downarrow$$

An ordinary EDM type wattmeter if we use using at LPF Loads the instrument will give the wrong reading & produced deflection is small even though pc & cc are fully excited.

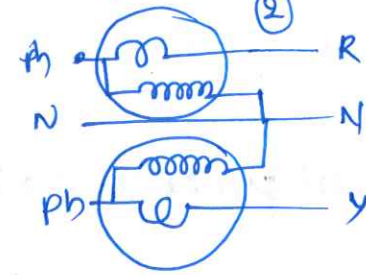
$$(ii) \quad \% \text{ Error} = (\tan\phi \cdot \tan\beta) \times 100$$



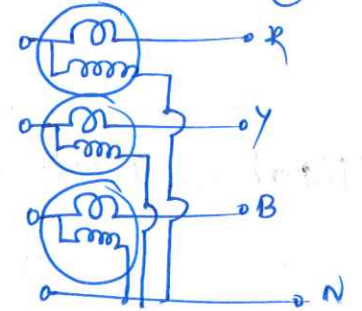
1- $\phi$ , 2-wire system ①



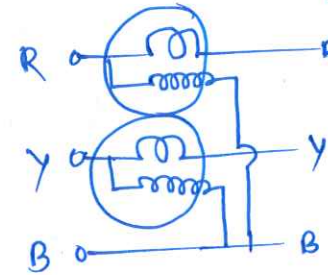
2- $\phi$ , 3-wire ②



3 $\phi$ , 4-wire ③



3- $\phi$ , 3-wire, Unbalanced } min-wattmeters ②.  
balanced }



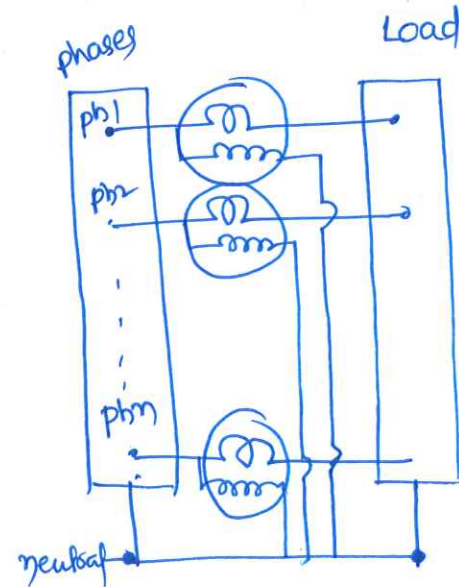
5- $\phi$ , 6-wire  $\Rightarrow$  5 $\omega$

9- $\phi$ , 10-wire  $\Rightarrow$  9 $\omega$

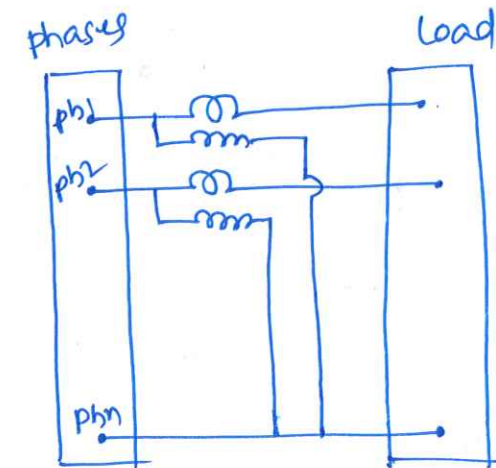
9- $\phi$ , 9-wire  $\Rightarrow$  8 $\omega$ .

Balanced - 1 $\omega$   
Unbalanced - 3 $\omega$

BLONDEL'S Theorem :-



$\therefore$   $n$ -wattmeters.  
(neutral is available)



$(n-1)$ -wattmeters  
(no neutral is available)

In a  $n$ -phase system if there is a neutral point is available the no. of wattmeters required are  $n$  to measure real power, if there is no neutral point is available  $\therefore$  the no. of wattmeters are required  $\therefore$  is  $(n-1)$ .

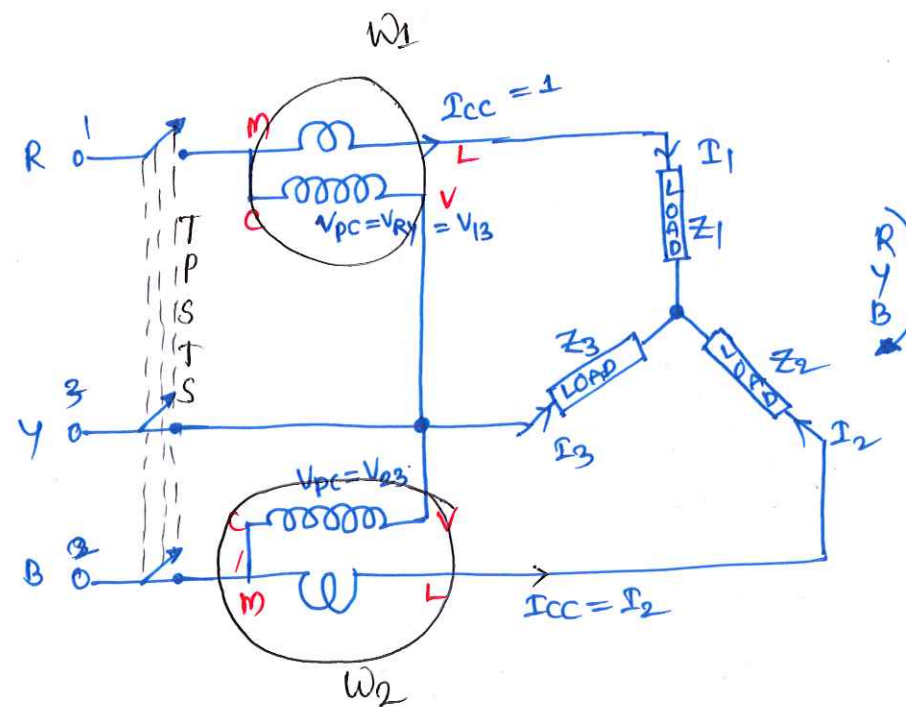


Total power consumed by load =  $\Sigma$  wattmeter readings used in measurement.

Total real (or) 3 $\phi$  real power measurement :-

(or)

3- $\phi$  real power measurement by 2-wattmeter method :-  
 (or)  
 3 $\phi$  reactive power measurement by " " "  
 (or)  
 pf (or) phase angle of load " " " " " :-



$$\begin{aligned} i_1 + i_2 + i_3 &= 0 \\ i_1 + i_2 &= -i_3 \end{aligned}$$

Let  $v_1, v_2, v_3$  are instantaneous voltages,  
 $i_1, i_2, i_3$  are instantaneous currents.

$$W_1 = v_{13} i_1 = (v_1 - v_3) i_1 = v_1 i_1 - v_3 i_1$$

$$W_2 = v_{23} i_2 = (v_2 - v_3) i_2 = v_2 i_2 - v_3 i_2$$

$$\Sigma P = W_1 + W_2 = \text{Total} = v_1 i_1 + i_2 v_2 - v_3 (i_1 + i_2)$$

$$= v_1 i_1 + v_2 i_2 - v_3 (-i_3)$$

$$P = W_1 + W_2 = v_1 i_1 + v_2 i_2 + v_3 i_3 \quad \text{Hence Blondel's theorem proved.}$$

$$P = [v_1, v_2, v_3] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = [v][i]^T$$

(79)

Instantaneous power balance theory has been satisfied by Blondell's theorem.

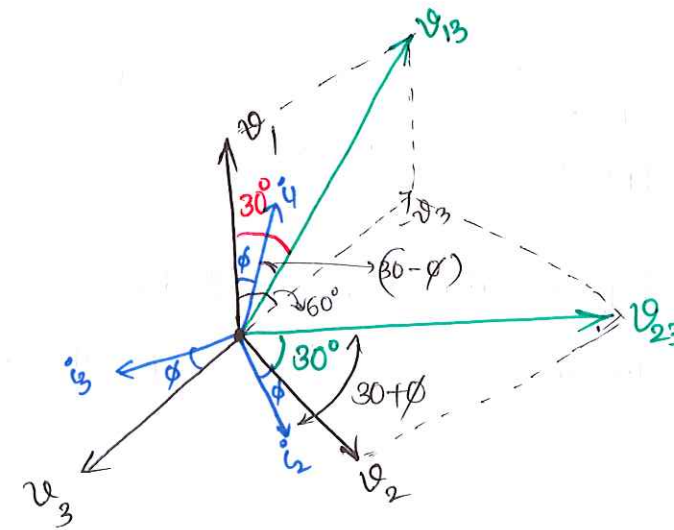
Assume Balanced Load  $\Rightarrow z_1 = z_2 = z_3 = Z \angle \phi$

Let  $v_1 = v_2 = v_3 = v_{ph}$  ;  $I_1 = I_2 = I_3 = I_L = I_{ph}$ .

$$v_{ph} = \frac{V_L}{\sqrt{3}} ; v_{13} = v_{23} = v_{12} = V_L$$

$$P_1 = v_{13} i_1 \cos(\angle v_{13}, i_1) = \sqrt{3} V_{ph} I_{ph} \cos(30 + \phi)$$

$$P_2 = v_{23} i_2 \cos(\angle v_{23}, i_2) = \sqrt{3} V_{ph} I_{ph} \cos(30 - \phi)$$



$$\therefore \left. \begin{aligned} \omega_1 &= \sqrt{3} V_{ph} I_{ph} \cos(30 - \phi) \\ \omega_2 &= \sqrt{3} V_{ph} I_{ph} \cos(30 + \phi) \end{aligned} \right\} P = \omega_1 + \omega_2 = 3 V_{ph} I_{ph} \cos \phi = \text{Real power}$$

$$\omega_1 + \omega_2 = \sqrt{3} V_{ph} I_{ph} (2 \cos 30^\circ \cos \phi) = (\sqrt{3} V_{ph} I_{ph}) (\sqrt{3} \cos \phi) \quad \text{--- (1)}$$

$$\omega_1 - \omega_2 = \sqrt{3} V_{ph} I_{ph} (2 \sin 30^\circ \sin \phi) = (\sqrt{3} V_{ph} I_{ph}) (\sin \phi) \quad \text{--- (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{\sin \phi}{\sqrt{3} \cos \phi} = \frac{\omega_1 - \omega_2}{\omega_2 + \omega_1} \Rightarrow \tan \phi = \frac{\sqrt{3} (\omega_1 - \omega_2)}{(\omega_1 + \omega_2)}$$

$$P.f = \cos \phi = \cos \left[ \tan^{-1} \left( \frac{\sqrt{3} (\omega_1 - \omega_2)}{(\omega_1 + \omega_2)} \right) \right]$$

$$W_1 - W_2 = (\sqrt{3} V_{ph} I_{ph}) \sin \phi$$

$$= \frac{\sqrt{3}}{\sqrt{3}} (\sqrt{3} V_{ph} I_{ph}) \sin \phi$$

$$(W_1 - W_2) = \frac{3 V_{ph} I_{ph} \sin \phi}{\sqrt{3}}$$

$$Q_{\text{total}} = \text{Reactive power} = \sqrt{3} (W_1 - W_2) = 3 V_{ph} I_{ph} \sin \phi$$

$W_1, W_2 \Rightarrow$  all simple readings <sup>(no units)</sup>, but  
 $W_1 + W_2 \Rightarrow$  real power (KW) = P  
 $\sqrt{3} (W_1 - W_2) \Rightarrow$  reactive power (KVAR) = Q

$$\phi = \text{phase angle} = \tan^{-1} \left( \frac{\sqrt{3} (W_1 - W_2)}{(W_1 + W_2)} \right) = \tan^{-1} \left( \frac{Q}{P} \right)$$

$$\text{Pf} = \cos \phi = \cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+\frac{Q^2}{P^2}}} = \frac{P}{\sqrt{P^2+Q^2}} = \frac{P}{S}$$

Q: In a 3 $\phi$  power measurement by 2-wattmeter method, the wattmeter readings are equal in magnitude but opposite in sign then, nature of load.

- sol:
- (a) pure resistive
  - (b) pure inductive <sup>operating at UPF</sup>
  - (c) ~~capacitive~~
  - (d) All

Q: In the above problem the two wattmeter readings are equal in magnitude with same sign. then the nature of load

(c)

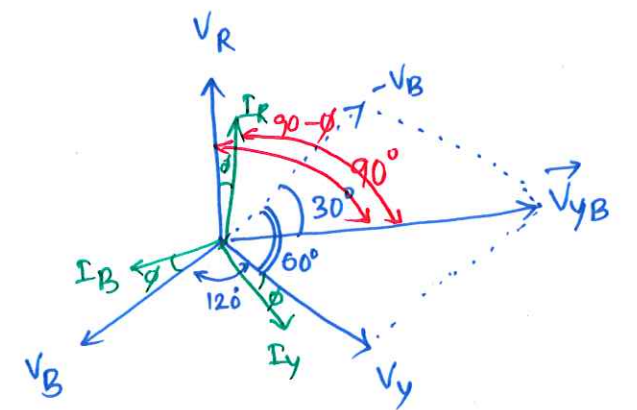
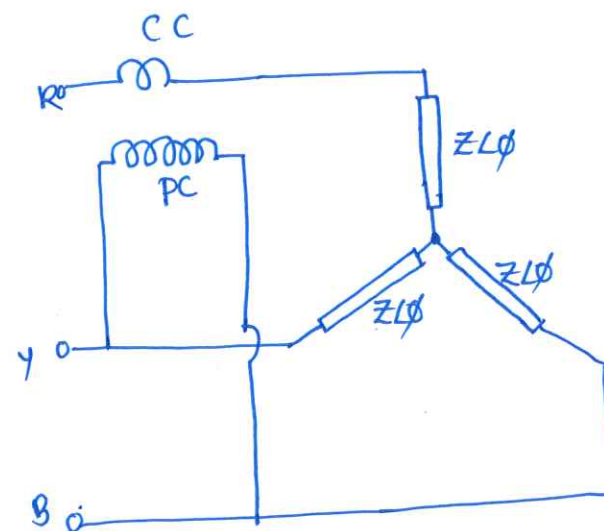


S.No.	$\phi$	$\text{pf} = \cos\phi$	$P_1 = \sqrt{3} V_{ph} I_{ph} \cos(90-\phi)$	$P_2 = \sqrt{3} V_{ph} I_{ph} \cos(30+\phi)$	relation b/w $P_1$ & $P_2$
1.	$0^\circ$	UPF	$\frac{3}{2} V_{ph} I_{ph}$	$\frac{3}{2} V_{ph} I_{ph}$	$P_1 = P_2$
2.	$30^\circ$	0.86 lag	$\sqrt{3} V_{ph} I_{ph}$	$\frac{\sqrt{3}}{2} V_{ph} I_{ph}$	$P_2 = \frac{P_1}{2}$
3.	$45^\circ$	0.707 lag	$(\frac{1.06}{\sqrt{2}}) V_{ph} I_{ph}$	$0.44 V_{ph} I_{ph}$	$P_1 = \frac{P_2}{3.99}$
4.	$60^\circ$	0.5 lag	$\frac{3}{2} V_{ph} I_{ph}$	0	$P_1$ - reads $P_2 = 0$
5.	$75^\circ$	0.25 lag	$\frac{\sqrt{3}}{2} V_{ph} I_{ph}$	$-0.44 V_{ph} I_{ph}$	$P_1$ - reads $P_2$ - (-ve) read
6.	$90^\circ$	ZPF (0)	$\frac{\sqrt{3}}{2} V_{ph} I_{ph}$	$-\frac{\sqrt{3}}{2} V_{ph} I_{ph}$	$P_1 = -P_2$

$\text{pf} \Rightarrow (0 \text{ to } 0.5)$   
 $\phi \Rightarrow (60^\circ \text{ to } 90^\circ)$

$\Rightarrow$  One of the meter readings shows  $\vec{-ve}$  value.

Measurement of  $3\phi$  Reactive power by - single (w) method :-



$$\begin{aligned}
 (w) &= W_{pc} I_{cc} \cos(\angle V_{pc}, I_{cc}) \\
 &= V_{yB} I_R \cos(\angle V_{yB}, I_R) \\
 &= (\sqrt{3} V_{ph}) I_{ph} \cos(90-\phi) \\
 &= \sqrt{3} V_{ph} I_{ph} \sin\phi
 \end{aligned}$$

$$\begin{aligned} \text{Total reactive power } Q &= 3V_{ph} I_{ph} \sin\phi \\ &= \sqrt{3} (\sqrt{3} V_{ph} I_{ph} \sin\phi) \\ &= \sqrt{3} (\text{Wattmeter reading}). \\ Q &= \sqrt{3} W \end{aligned}$$

---

1- $\phi$  Energy meter diagram

## Measurement of Energy

(81)

$$\text{Energy} = \int_0^t \text{power} dt = \int_0^t v i dt = V i t \quad \begin{array}{l} \text{volt-amp-sec.} \\ \text{(or)} \\ \text{Watt-sec} \end{array}$$

$$\text{energy} = \frac{V i t}{1000} \text{ kW-sec} \Rightarrow (t \rightarrow \text{time is in seconds})$$

$$= \frac{V i t}{1000 \times 60 \times 60} \Rightarrow (t \rightarrow \text{is in seconds})$$

$$= \frac{V i t}{36 \times 10^5} \text{ kWh} \Rightarrow (t \rightarrow \text{sec})$$

$$1 \text{ sec} = \frac{1}{3600} \text{ Hr}$$

$$= \frac{V i t}{6 \times 10^4} \text{ kW minute} \Rightarrow (t \rightarrow \text{minutes}).$$

Energy-meter  $\Rightarrow$  Integrating type instrument

- (i) Driving torque.
- (ii) Braking torque.
- (iii) Registering mechanism.

### Driving torque

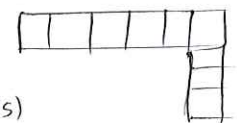
The torque which is required to revolve the disc. It can be obtained by using electromagnetic induction effect.

### Braking torque

The torque which is required to revolve the disc at constant speed, braking torque can be obtained by placing the permanent magnet meter.

### Registering mechanism. (Gear train system)

Wastak & Kotari (Centals)



RM is obtained by gear-train mechanism in the analog meters. In the digital meters counters are used. Energy meter working principle similar to transformer action.

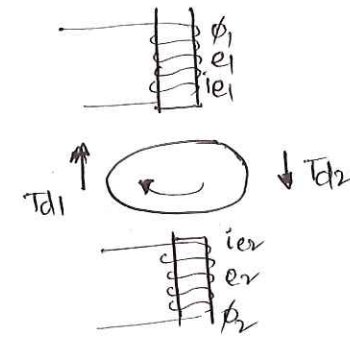
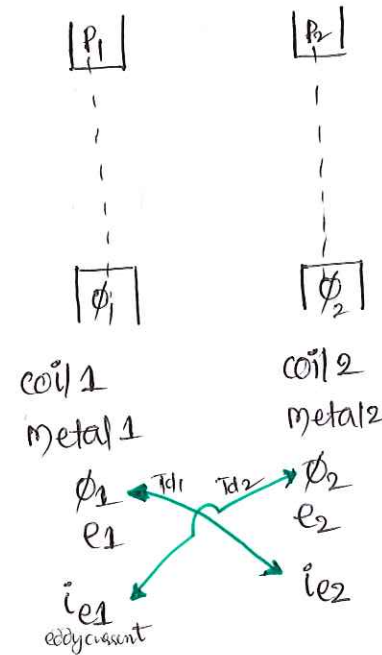


Energy meter working principle  $\Rightarrow$  similar to that of 1- $\phi$  Induction motor.

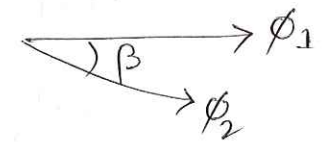
(or) " " transformer

$\Rightarrow$  it works on the theory of induction instrument.

### Theory of Induction Instruments :-



Let  $\phi_1 = \phi_{m1} \sin \omega t$   
 $\phi_2 = \phi_{m2} \sin(\omega t - \beta)$   
 $\phi_2$  lags  $\phi_1$  by  $\beta$ .



$\therefore \text{emf} \propto \frac{d\phi}{dt}$

$e_1 \propto \frac{d\phi_1}{dt} \propto \frac{d(\phi_m \sin \omega t)}{dt}$

$e_1 \propto \phi_m \omega \cos \omega t$

$e_1 = -\phi_m \omega \cos \omega t$

$e_1 = \phi_m \omega \sin(\omega t - 90^\circ)$

$e_1$  lags  $\phi_1$  by  $90^\circ$ .

Let  $Z_e \Rightarrow$  eddy impedance path offered by metallic disc.

$Z_e = r_e + jx_e$

$|Z_e| = \sqrt{r_e^2 + x_e^2}$  ;  $\alpha = \tan^{-1}\left(\frac{x_e}{r_e}\right)$

$\alpha \rightarrow$  eddy impedance angle.

$i_{e1} = \frac{e_1}{Z_e}$

$\therefore i_{e1}$  is in lag  $e_1$  by  $\alpha$ .

Similarly coil 2

$$e_2 = -\frac{d\phi_2}{dt} = -\omega\phi_{m2} \cos(\omega t - \beta)$$

$$e_2 = +\omega\phi_{m2} \sin(\omega t - \beta - 90^\circ)$$

$$e_2 \text{ lags } \phi_2 \text{ by } 90^\circ; \quad i_{e2} = \frac{e_2}{Z_e}$$

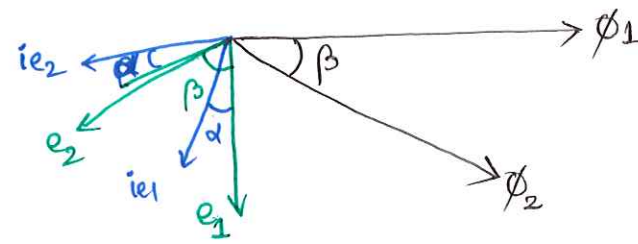
$i_{e2}$  lags  $e_2$  by  $\alpha'$  ( $\because$  eddy path assumed is same for  $i_{e1}$  &  $i_{e2}$ )

$$\therefore (\text{torque})_1 \propto \phi_1 i_{e2} \cos(\phi_1, i_{e2})$$

$$(\text{torque})_2 \propto \phi_2 i_{e1} \cos(\phi_2, i_{e1})$$

$$\angle(\phi_1, i_{e2}) = 90 + \beta + \alpha$$

$$\angle(\phi_2, i_{e1}) = 90 - \beta + \alpha$$



$$(\text{net torque}) \propto (T_{d1} \sim T_{d2})$$

$$T_d \propto (\phi_1 \phi_2 \cos(\alpha) \sin(\beta))$$

If eddy impedance path is pure resistive ( $\alpha = 0$ ),  $\therefore Z_e = R_e$ .

$$\therefore T_d \propto [\phi_1 \phi_2 \sin(\beta)]. \quad \phi_1 = \frac{\phi_{m1}}{\sqrt{2}}; \quad \phi_2 = \frac{\phi_{m2}}{\sqrt{2}}$$

$\therefore$  (produced) driving torque always depends on

- (i) magnitudes of two interacting fluxes.
- (ii) the sine of angle b/w the two fluxes.

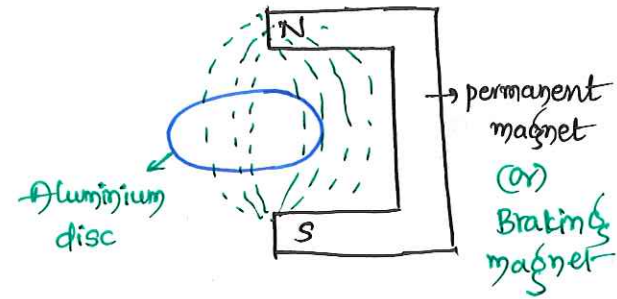
$$T_d \propto \phi_1 \phi_2 \sin(\phi_1, \phi_2)$$

$$\phi_1 = \phi_{s1}$$

$$T_d \propto \phi_{s1} \phi_{se} \sin(\phi_{s1}, \phi_{se})$$

$$\phi_2 = \phi_{se}$$

## Braking Torque :-



$$\text{emf} \propto n \cdot \phi$$

$n \rightarrow$  speed of disc

$\phi \rightarrow$  flux produced by braking magnet

$$\therefore (\text{emf}) = k n \phi$$

$k = \text{constant};$

Let  $r_e \Rightarrow$  eddy resistance path of A

$\therefore$  Aluminium disc offers some resistance  $\dots r_e \dots$  some current produced by emf (by BM) called eddy current (this is different from earlier eddy currents).

$$i_e = \frac{\text{emf}}{r_e} = \frac{k n \phi}{r_e}$$

$$T_{\text{braking}} \propto \phi \cdot i_e$$

$$T_B \propto \left( \phi \frac{k n \phi}{r_e} \right)$$

$$T_B \propto \left( \frac{\phi^2 n}{r_e} \right) \Rightarrow T_B = k' \frac{\phi^2 n}{r_e} \Rightarrow n \propto \frac{r_e T_B}{\phi^2}$$

$$T_{\text{braking}} \propto (\text{Speed of disc}).$$

$$n \propto \frac{r_e}{\phi^2} \quad (T_B = \text{const}).$$

(speed of disc)  $\propto$  (eddy resistance path).

$$r_e = r_{e0} (1 + \alpha \Delta t)$$

if  $\Delta t \uparrow$  (i.e.) by overloading  $\therefore r_e \uparrow \Rightarrow n \uparrow \Rightarrow \text{speed} \uparrow$   
cost  $\uparrow$



(shunt magnet  $\Rightarrow$  pressure coil  $\Rightarrow \phi_{sh} \propto V_{sh} \propto V_{supply}$ )

(series magnet  $\Rightarrow$  current coil  $\Rightarrow \phi_{se} \propto I_{se} \propto I_{Load}$ )

$\hookrightarrow$  splitted into two parts

(i) to avoid magnetic saturation.

(ii) to get uniform field

### Assumptions :-

1. Voltage across the series coil is neglected.

$$\therefore V_{ph} = V_L \quad \& \quad \phi_{sh} \propto (V_{ph} = V_L)$$

$\phi_{se} \propto I_{Load}$  current.

2.  $\phi_{sh}$  lags  $V_{ph} = V_L$  by  $90^\circ$ ... i.e. shunt coil is pure inductive.

practically not possible exactly  $90^\circ$ . to get near  $90^\circ$ ...  
pure inductive material will be placed in shunt magnet called  
copper shading bands also called as lag adjustment.

$\hookrightarrow$  movable type ...  $\therefore$  we can adjust the phase angle to  $90^\circ$ .

$$\phi_{sh}' = \phi_{sh} + \phi_{cu} \quad \therefore \phi_{sh}' \text{ lags } V_{sh} = V_{ph} = V_L \text{ by } 90^\circ$$

$$\therefore T_d \propto \phi_{sh} \phi_{se} \sin(\phi_{sh}, \phi_{se})$$

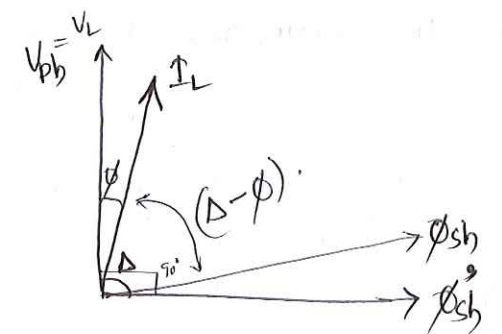
$$T_d \propto V_L I_L \sin(\phi_{sh}, \phi_{se})$$

$$\therefore T_d \propto V_L I_L \sin(\Delta - \phi)$$

$$T_d \propto V_L I_L \sin(90 - \phi)$$

$$T_d \propto V_L I_L \cos \phi$$

$$T_d \propto (\text{AC power})$$



ideally  $\Delta = 90^\circ$

At steady state.  $T_D = T_B$

$\therefore$  (AC power)  $\propto$  (speed of disc)

$\int$  (speed of disc) dt  $\propto$   $\int$  (AC power) dt

$\therefore$  **No. of revolution  $\propto$  Energy consumed**

no. of revolution =  $K \times$  Energy consumption:

$K =$  meter constant. i.e. in rev/kWh

$$K = \frac{\text{no. of revolutions}}{\text{Energy consumed}}$$

The no. of revolutions made by the disc to consume one kWh energy is known as meter constant.. units are rev/kWh.

Creeping :-

is the slow rotation of the disc under no-load condition and slightly higher speed under lightly loaded and loaded conditions.

Reasons :-

Speed  $\propto \frac{1}{(\text{dist. b/w centre of disc \& braebis magnet})}$ .

- (i) The **main reason** for creeping is the **Over-compensation** provided in order to overcome the static friction offered by aluminium disc.
- (ii) **over-voltages** across shunt magnet.
- (iii) mechanical vibrations etc.
- (iv) Due to excessive rise in temperatures -

Remedy for creeping :- Creeping can be reduced by making diametrically two holes (opposite) on the aluminium disc. (24)

(creep error always positive) i.e. high speed; more no. of revolutions.

calculated value of revolutions  $\Rightarrow$  is true value ( $A_T$ )

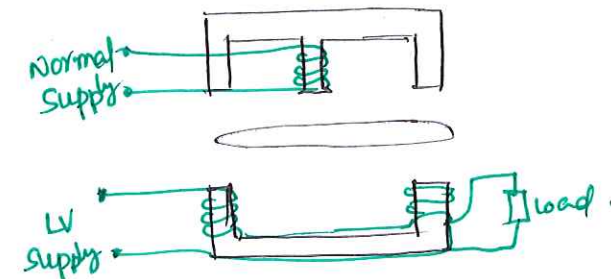
shown by meter value  $\Rightarrow$  is measure value ( $A_m$ )

$$\text{Error} = \frac{A_m - A_T}{A_T} \times 100 \quad \Rightarrow +ve \Rightarrow \text{meter is fast} \\ \Rightarrow -ve \Rightarrow \text{meter is slow.}$$

### phantom loading :-

In order to avoid the unnecessary power loss during the testing of energy meter, a special arrangement called phantom loading is used as shown in figure.

In this arrangement, pressure coil (or) shunt coil is excited by normal supply voltage but the series coil (or) current coil a separate source supplying rated current at low voltage  $\therefore$  the losses are minimized.

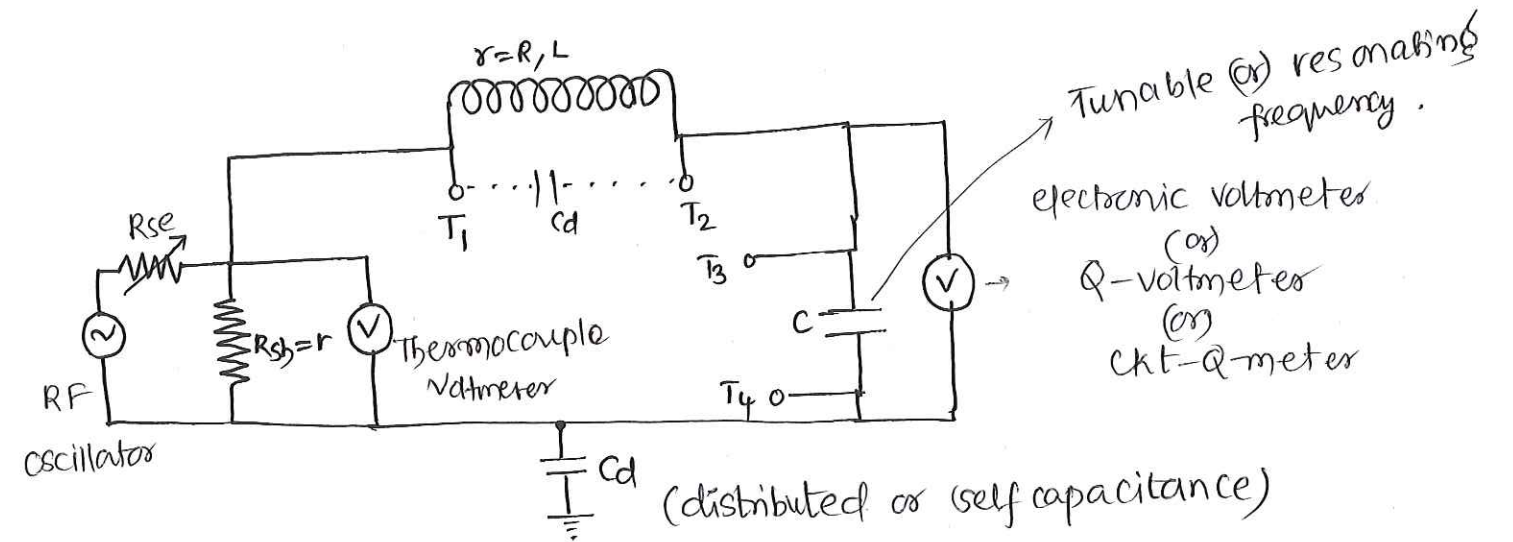




By moving B magnet inward (or) outwards, we can control the speed of Aluminium disc. By moving the braking magnet closer to the centre of Al. disc, cutting flux will be more  $\therefore$  speed of the disc is less more, By moving the braking magnet away from the centre of the disc the cutting of flux is more so that the speed of Disc is lesser.



## Q-meters



at Balance (i.e. Resonance)  $|X_L| = |X_C| \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{L(C+C_d)}}$$

$$\frac{f_{02}}{f_{01}} = \left( \frac{C_d + C_1}{C_d + C_2} \right)^2 \Rightarrow C_d = \frac{C_1 - 4C_2}{3} ; \text{ if } f_{02} = 2f_{01}$$

$$\text{if } f_{02} = n f_{01} \Rightarrow \boxed{C_d = \frac{C_1 - n^2 C_2}{(n^2 - 1)}}$$

$$\text{true Q-factor} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C} = Q_T$$

Two types of errors :-

1. Error due to distributed capacitance of the coil.
2. Error due to shunt resistance of the ckt.

Error due to Cd  $Q_T = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C}$

$$Q_m = \frac{1}{\omega_0 C R} = \frac{1}{\omega_0 (C + C_d) R}$$



$$\frac{Q_T}{Q_m} = \frac{c + Cd}{c} = \left(1 + \frac{Cd}{c}\right) \Rightarrow Q_T = \left(1 + \frac{Cd}{c}\right) Q_m$$

$$(or) \quad Cd = c \left(\frac{Q_T}{Q_m} - 1\right) ; \quad \frac{Q_m}{Q_T} = \left(\frac{c}{c + Cd}\right)$$

$$\frac{Q_m - Q_T}{Q_T} = \frac{c - c - Cd}{c + Cd}$$

$$Q_{measured} < Q_{true}$$

$$\therefore \% \text{ error} = \frac{-Cd}{c + Cd} \times 100 \quad \leftarrow \text{error is negative}$$

If  $Cd = 0 \Rightarrow \% \text{ error} = 0$ ;

Error due to  $R_{sh}$  :-

$$Q_m = \frac{\omega_0 L}{(R + R_{sh})} = \frac{\omega_0 L}{(R + r)}$$

$$R_{sh} = r = \left(\frac{Q_T}{Q_m} - 1\right) R$$

$$\frac{Q_T}{Q_m} = 1 + \frac{R_{sh}}{R} \Rightarrow Q_T = \left(1 + \frac{r}{R}\right) Q_m$$

$$\% \text{ error} = \left(\frac{Q_m}{Q_T} - 1\right) \times 100 = \frac{-r}{(R + r)} \times 100 \Rightarrow \text{error is -ve}$$

$\therefore Q_{measured}$  is less than  $Q_{true}$ .

Applications :-

1. It is used to measure the  $Q$ -factor of the coil.

$$Q_T = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} ; \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

2. Used to measure the  $Cd$  (i.e. distributed (or) self-capacitance) of coil.

$$Cd = \frac{C_1 - \eta^2 C_2}{\eta^2 - 1} ; \quad f_{o2} = \eta f_{o1}$$

3. Used to measure the unknown capacitance. connected in ckt across the terminals  $T_3$  &  $T_4$

$$f_{o1} = \frac{1}{2\pi \sqrt{L(C_1 + Cd)}} ; \quad f_{o2} = \frac{1}{2\pi \sqrt{L(C_2 + Cd)}} ; \quad C_2 = \frac{C_1 + C}{\eta^2 - 1}$$

$$f_{02} = \frac{1}{2\pi \sqrt{L(C_2 + C_x + C_d)}} \quad ; \quad f_{01} = f_{02}$$

$$C_2 + C_x = C_1 \Rightarrow C_x = C_1 - C_2$$

After connecting the unknown capacitance  $C_x$  across the terminals of  $T_3$  &  $T_4$  shown in ckt, vary the tunable capacitor to a new value of  $C_2$  in order to obtain same resonating frequency as in the step 1 process, that means without connecting unknown capacitance.

4. To find the inductance of coil.

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C}$$

5. We can find the resistance of the coil. ( $R_{\text{coil}} = R$ )

$$Q_{\text{eff}} = Q_m \left(1 + \frac{\gamma}{R_v}\right) \quad ; \quad \gamma_{\text{shunt}} \quad ; \quad Q = \frac{\omega_0 L}{R}$$

6. Bandwidth (signal can be reproduced in this range)

$$Q = \frac{\omega_0}{\text{BW}} \Rightarrow \text{BW} = \frac{\omega_0}{Q_0}$$

Standardisation:- (Potentiometer) The process of adjusting the working current so as to match the voltage drop across a portion of sliding wire against a standard reference source is known as standardisation.

$$\frac{\text{Standard voltage}}{(\text{Length obtained in cm}) \left(\frac{\rho}{\text{cm}}\right)} = \text{working current.}$$

\* If the potentiometer has been calibrated once, its working current is never changed.

→ The standardisation of AC potentiometer is done by (Zener source) using dc standard sources & transfer instruments.

## POTENTIOMETER

Potentiometer is basically length comparison device b/c of null balance condition it has good accuracy. (87)

It consists of

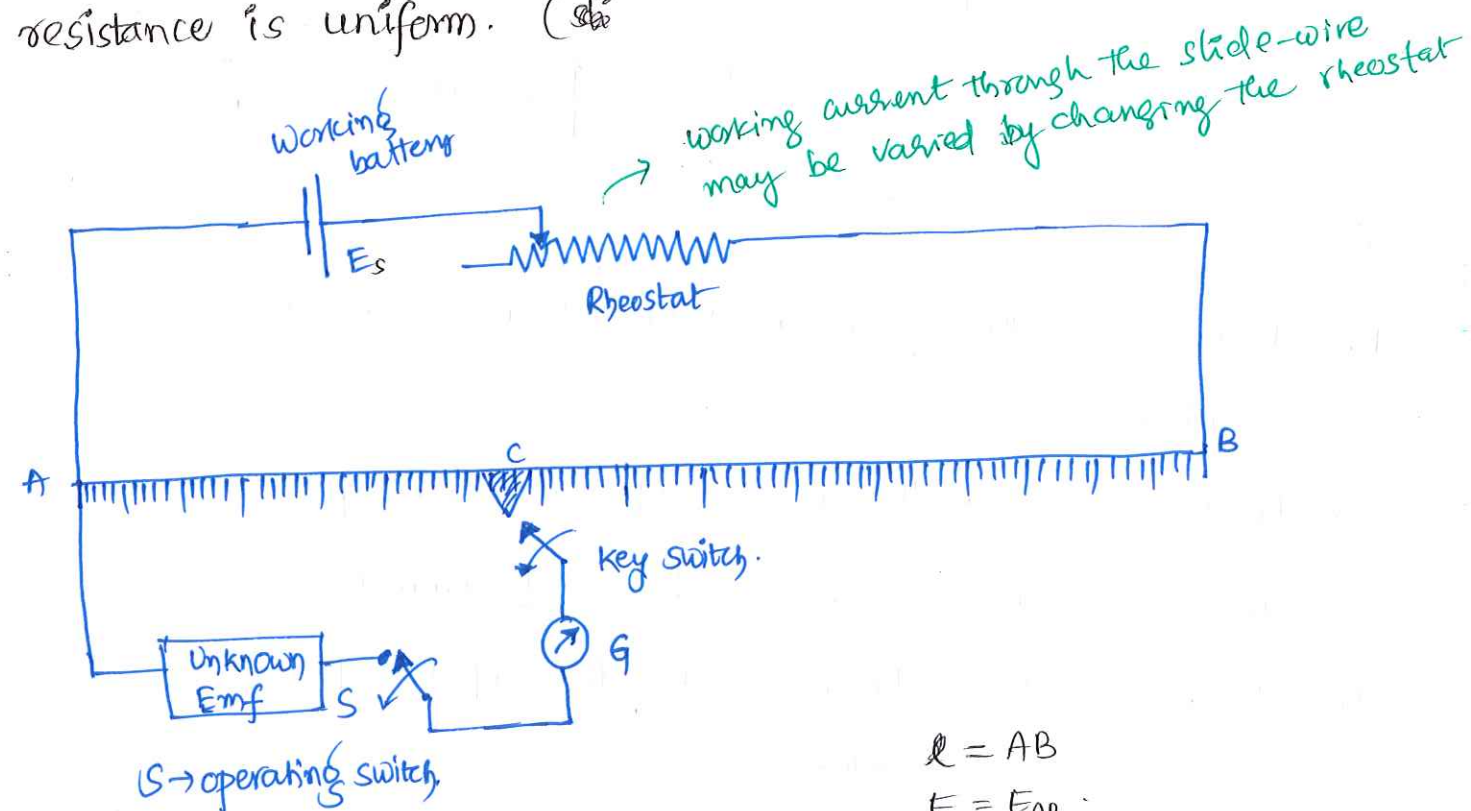
(\*) Working battery, can supply working current it flowing through rheostat and slide-wire.

The working current can be adjusted by using rheostat setting so that the current flowing through the slide-wire is also adjusted so that the voltage drop across the slide wire is also adjusted.

slide wire is prepared by platinum, silver, gold alloy..

sliding contact is prepared by copper, silver, gold alloy...

slide-wire has uniform area of cross section, so that offered resistance is uniform. (88)



$$l = AB$$

$$E = E_{AB}$$

$$l_1 = AC$$

length ratio = Voltage ratio = resistance ratio.

$$\frac{l_{AB}}{l_{AC}} = \frac{E_{AB}}{E_{AC}} = \frac{R_{AB}}{R_{AC}}$$



To find the unknown emf connected in the ckt as shown in the below figure, it consists of operating switch 's' is in the closed position and the galvanometer key switch 'K' is in the open position, keep on adjusting the working current by using rheostat setting at one particular instant, if the galvanometer key-switch is closed, galvanometer shows zero deflection is known as null-balance condition. Finding of emf (unknown) is nothing but calculating of voltage drop across AC portion of length of the slide wire.

potentiometer  $\Rightarrow$  voltage divider,  
resistive divider n/o;  
length comparison instrument.  
passive transducer (no source internally to slide).



### Applications :-

1. To measure the unknown emf, to measure unknown resistance
2. It is used in the process of calibration of voltmeters.
3. It is used in process of calibration of ammeters. <sup>even</sup> <sup>ammeters</sup> <sup>Wattmeters also.</sup>
4. It is also used to measure the input displacement in the order of several mm.
5. Potentiometer is a passive transducer. B/c which will convert displacement into change of resistance (or) voltage.

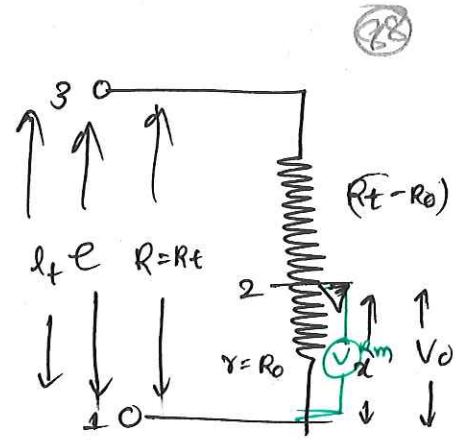
Potentiometer is a LINEAR device.

$$\frac{x}{l_t} = \frac{V_o}{e} = \frac{R_o}{R_t} \Rightarrow V_o = \left(\frac{e}{l_t}\right) x$$

$V_o = Kx \Rightarrow$  Apply Laplace.

$$V_o(s) = K X(s)$$

$$T/F = \frac{V_o(s)}{X(s)} = K s^0$$

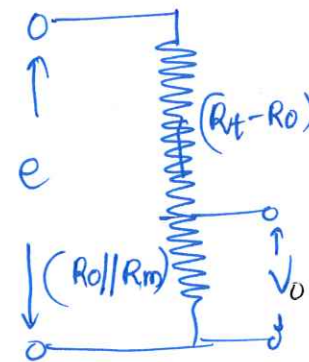
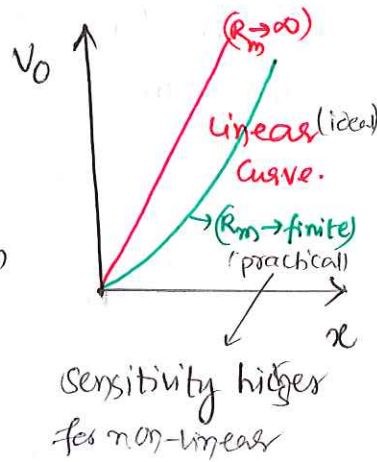


$\therefore$  potentiometer is a zero order system.

$$\therefore V_o \propto x$$

$$S = \frac{\Delta O/P}{\Delta I/P} = \frac{\Delta R}{x} \text{ } \Omega/\text{mm}$$

$$S = \frac{\Delta V}{x} \text{ Volt/mm}$$



$$V_o = \frac{\left(\frac{R_o R_m}{R_o + R_m}\right) e}{\left(\frac{R_o R_m}{R_o + R_m} + (R_t - R_o)\right)} = \frac{R_o R_m e}{R_o R_m + R_o R_t - R_o^2 + R_t R_m - R_o R_m}$$

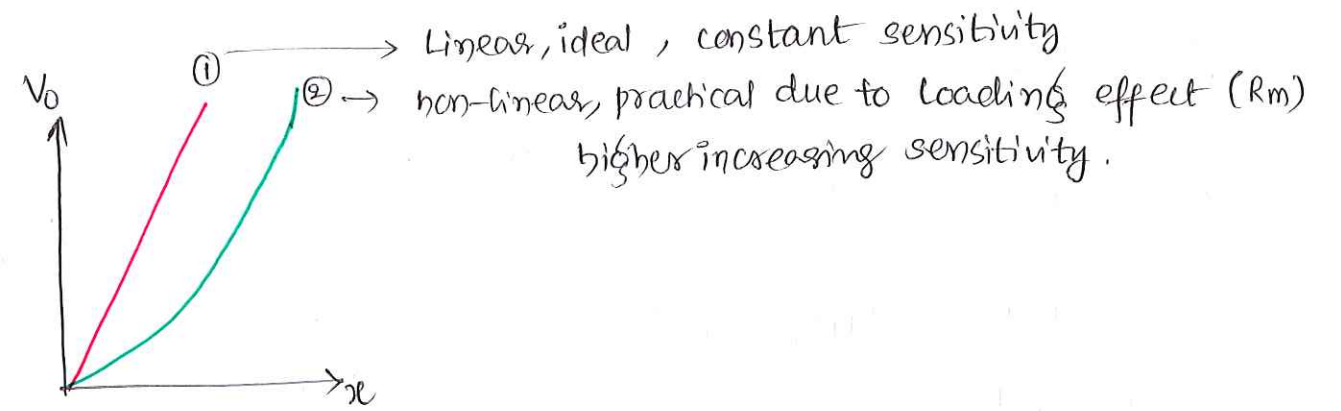
$$V_o = \frac{R_o R_m e}{R_o(R_t - R_o) + R_t R_m}$$

$$V_o = \frac{R_o e}{R_t + \frac{R_o}{R_m} (R_t - R_o)}$$

loading effect.

if  $R_m = R_v = \infty \Omega \therefore V_o = \frac{R_o e}{R_t + 0}$

$$\therefore V_o = e \left(\frac{R_o}{R_t}\right)$$



Sensitivity & linearity are conflicting requirements because as the linearity decreases, (non-linearity increases) the sensitivity will increase it is because of loading effect of voltage meter (since voltmeter doesn't have  $\infty$  but has finite resistance).

Potentiometer → comparing instrument  
 → Null type instrument (no power consumption & independent of deflection).  
 (or) balance type.

→ Measurement of comparison methods are capable of high accuracy b/c the result doesn't depend on actual deflection of a pointer, as the case of deflection methods. But it depends only upon the accuracy with which the voltage of the reference is known.

→ At balance (or) null point, no current flows & hence no power is consumed in the ckt containing the unknown emf when the instrument is at balance point. Thus, the determination of unknown voltage of a source/battery is independent of its (source's) resistance.



# CRO

Mainly consists of CRT (Cathode Ray tube).

main components of CRT.

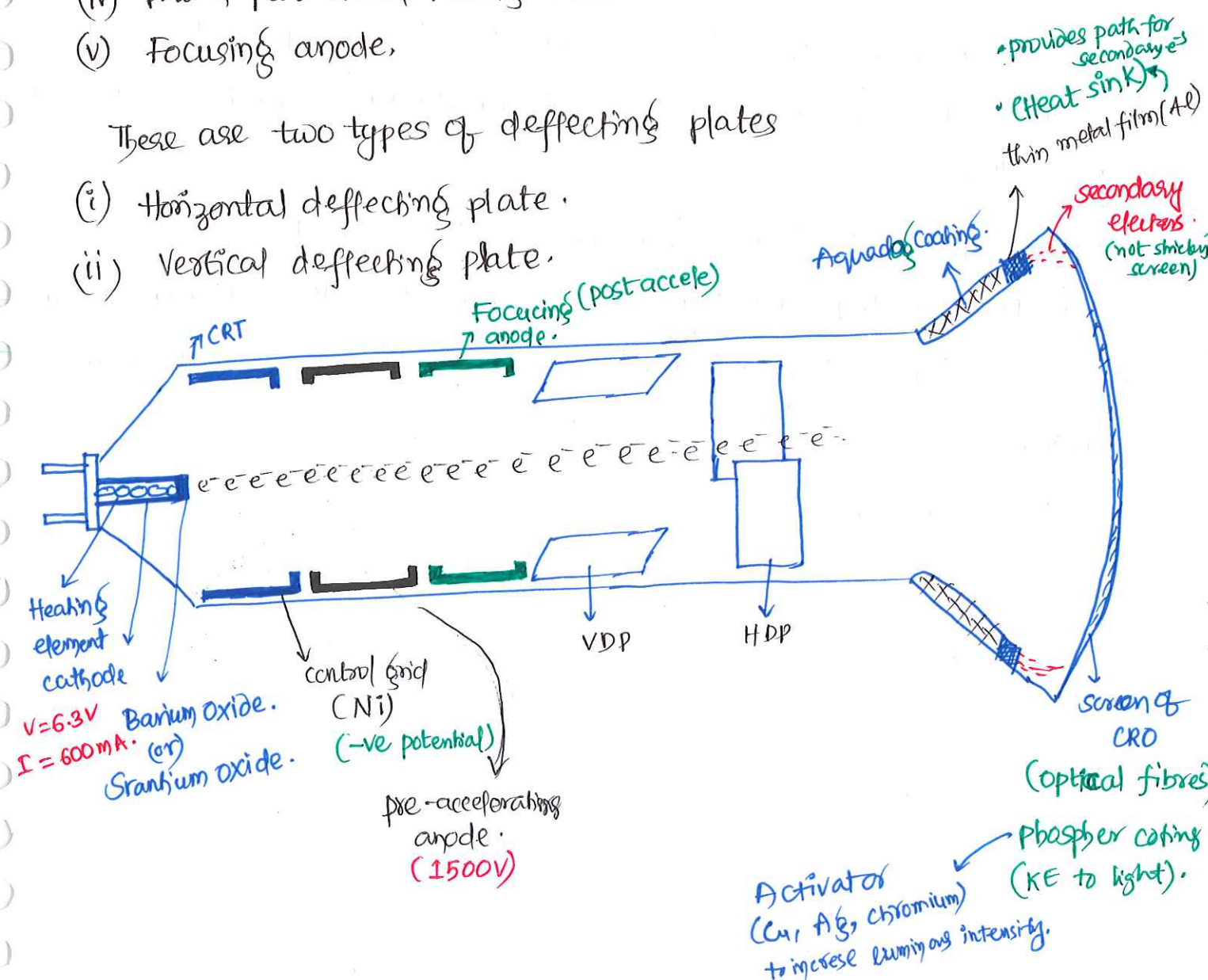
- (i) Electron Gun.
- (ii) Deflecting plates.
- (iii) Screen of CRO

Electron Gun consists of

- (i) heater element.
- (ii) Cathode
- (iii) Control grid
- (iv) pre & post accelerating anode.
- (v) Focusing anode.

These use two types of deflecting plates

- (i) Horizontal deflecting plate.
- (ii) Vertical deflecting plate.



$$\frac{1}{2}mv^2 = KE, \quad v \rightarrow \text{velocity of electron.}$$

$$KE = W.D = Vq \Rightarrow V = \text{applied voltage}, \quad q = e = 1.6 \times 10^{-19} \text{ C}$$

$$\therefore v = \sqrt{\frac{2eV}{m}} \quad ; \quad m = \text{mass of electron} = 9.31 \times 10^{-31} \text{ kg.}$$

The purpose of heating element, it is used to heat-up the cathode. The required voltage is around 8.3V and the required current is around 0.6A = 600mA.

Cathode, which is cylindrical shape, at the end of which a layer of barium oxide (or) strontium oxide is deposited in order to high emission of electrons at moderate temperature.

control grid, which is in cylindrical shape usually prepared by nickel material and always connected to -ve potential, it is used to control the intensity of electron beam which is released from cathode.

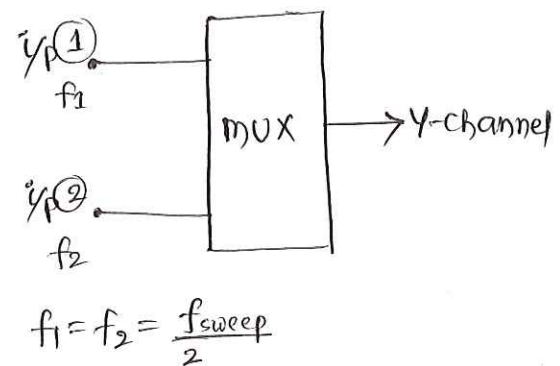
pre & post accelerating anodes, these are the anodes responsible for imparting the acceleration to the electron beam. in order to maintain constant velocity. so their voltage is 1500V.

Focusing anode





## Dual trace CRO :-



In dual trace CRO, the two signals given to the two input channels, it can be displayed on the screen of CRO, which is not possible in the normal CRO. In this, horizontal deflecting plate is connected to output of sweep generator and vertical deflecting plate is connected to two input signals

with the help of multiplexer ckt, the 2- $\gamma$ p signal frequencies are half of the frequency of sweep wave generator. So that these two signals are alternatively shifts at very high switching rate so that two  $\gamma$ p waveforms & two  $\gamma$ p waveforms displayed on the screen of CRO bc of high persistence.

Screen of CRO is made up of optical fibre. Inside the screen a layer of phosphor coating is deposited. The main function of phosphor is to convert the KE into light energy.

Some of the **actuators** are added in order to increase the luminous efficiency (it is the measure of intensity). To increase the persistence of phosphor. To increase the spectral emission.

Eg: Cu, Si, Chromium.

On the non-viewing side of CRO a thin metal of aluminium is deposited. It serves as

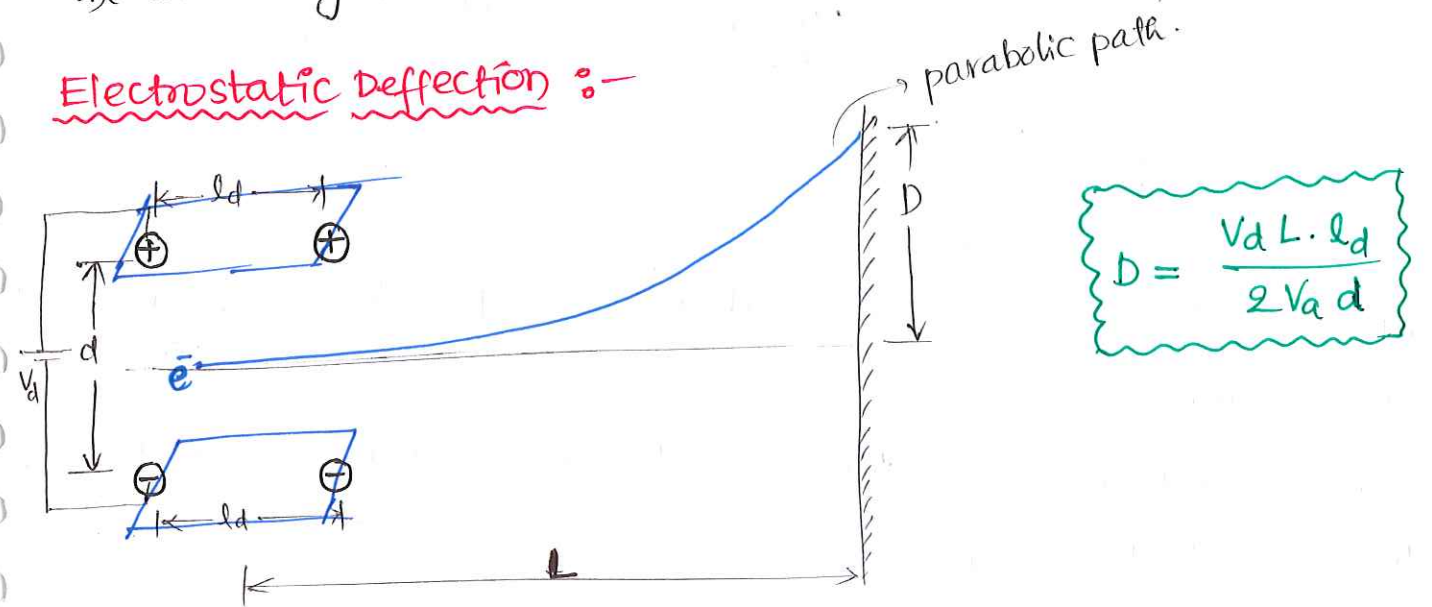
(i) heat sink  $\rightarrow$  It will absorb the heat generated during the collision of electrons.

(ii) It reflect the light scattered from phosphor back towards the screen.

(ii) It provides the conducting path for secondary emission electrons from screen to aquadag. (91)

Aquadag :- Is an aqueous solution of graphite, which is deposited on the inside of the screen. It is used to collect the secondary emitted electrons.

Electrostatic Deflection :-



- Let
- $l_d \rightarrow$  Length of the deflecting plates
  - $d \rightarrow$  separation distance b/w two plates
  - $D \rightarrow$  electrostatic deflection
  - $L \rightarrow$  distance of screen of CRT from deflecting plates
  - $V_d \rightarrow$  potential difference b/w the two deflecting plates.
  - $V_a \rightarrow$  accelerating potential.

$$S^{\circ}(\text{sensitivity}) = \frac{\Delta O/P}{\Delta i/p} = \frac{D}{V_d} = \frac{L l_d}{2 d V_a}$$

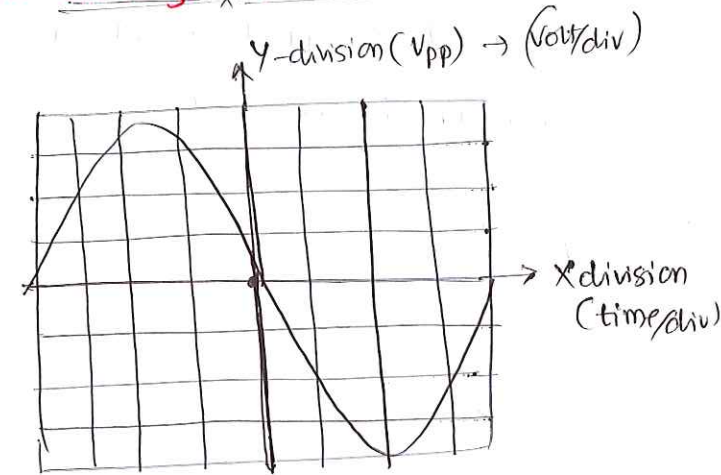
Deflection factor = scaling factor =  $\frac{1}{\text{sensitivity}}$  = Inverse sensitivity

$$DF = \frac{\Delta i/p}{\Delta O/P} = \frac{V_d}{D} = \frac{2 d V_a}{L l_d}$$



## Measurements by CRO

### 1. Voltage measurement:-



$$V_p = V_m = \sqrt{2} V_{RMS}$$

$$V_{pp} = 2V_p = 2\sqrt{2} V_{RMS}$$

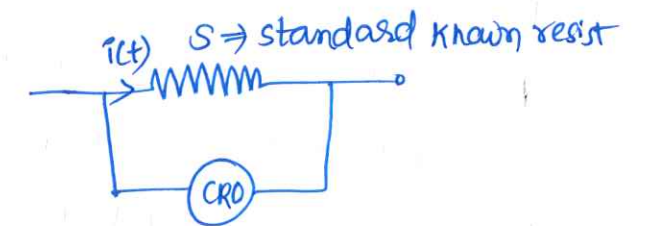
$$\therefore V_{RMS} = \frac{V_{pp}}{2\sqrt{2}}$$

CRO is voltage control device, it never display the current waveform.

CRO cannot measure the current directly but we can measure the current indirectly.

### (2) Current measurement,

The current waveform is passed through known standard resistance and observe the voltage waveform across standard resistance.



$$I_{RMS} = \frac{V_{RMS}}{S} = \frac{V_{pp}}{2\sqrt{2} S}$$

\*\*\*

The input impedance of CRO is in the order of  $10\text{ M}\Omega$  so that loading effect of CRO is negligible.

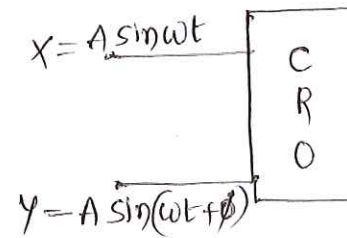
1. Ordinary voltmeter  $\Rightarrow$   $Z_{i,p}$   $\Rightarrow$   $\text{K}\Omega$
2. CRO  $\Rightarrow$   $10\text{ M}\Omega$
3. Modern multimeter  $\Rightarrow$   $(10-40\text{ M}\Omega)$   $\rightarrow$  Loading effect is almost negligible.  
(preferred choice of volt. measurement  $5727\Omega$ ).



### 3. phase angle measurement

(a2)

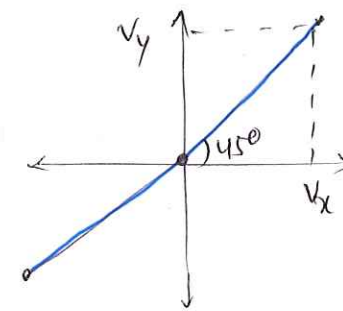
By operating CRO in dual mode (or) X-Y mode we can measure the phase angle for that the two sinusoidal signals are having equal amplitude but having a phase difference given to two X- & Y-channels of CRO so that we can obtain the pattern on the CRO screen is known as Lissajous patterns.



case (i)  $\phi = 0^\circ$

$$X = A \sin \omega t$$

$$Y = A \sin(\omega t + 0) \Rightarrow Y = X = mX.$$



$$\therefore \tan \theta = m = \text{slope} = 1$$

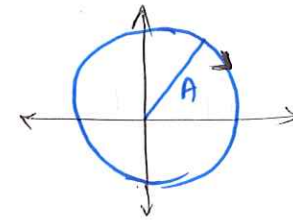
$$\therefore \theta = 45^\circ = \text{phase angle.}$$

case (ii) :-  $\phi = 90^\circ$  ;  $X = A \sin \omega t$

$$Y = A \sin(\omega t + 90^\circ) = A \cos \omega t$$

$$x^2 + y^2 = A^2 \quad (\text{circle}).$$

$$\text{radius} = A.$$



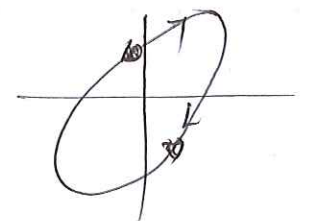
case (iii) :-  $\phi = (0^\circ \text{ to } 90^\circ) = 30^\circ$

$$X = A \sin \omega t \Rightarrow A \cos \omega t = \sqrt{1 - x^2}$$

$$Y = A \sin(\omega t + 30^\circ) = A (\sin \omega t \cos 30^\circ + \sin 30^\circ \cos \omega t)$$

$$Y = \frac{A}{2} (\sqrt{3} \sin \omega t + \cos \omega t)$$

$$Y = \frac{1}{2} (\sqrt{3} X + \sqrt{1 - x^2})$$



$$(Y - \frac{\sqrt{3}}{2} X) = \sqrt{1 - x^2} \Rightarrow Y^2 - \frac{3}{2} X^2 - 2\sqrt{3} XY = 1 - x^2$$

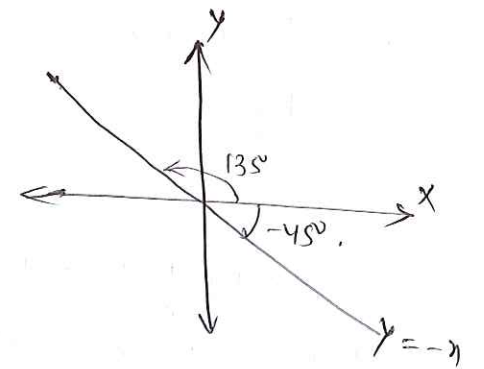
$$Y^2 - \frac{X^2}{2} - 2\sqrt{3} XY = 1 \quad (\text{ellipse}).$$

case (iv) :-  $\phi = 180^\circ$

$$X = A \sin \omega t$$

$$Y = A \sin(\omega t + 180^\circ) = -A \sin \omega t$$

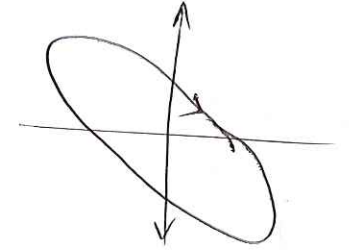
$$Y = -X$$



case (v) :-  $\phi = (90^\circ \text{ to } 180^\circ) = 120^\circ \text{ (say)}$

$$X = A \sin \omega t$$

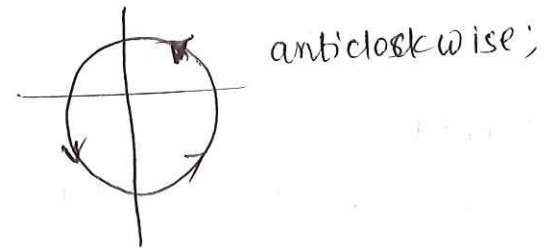
$$Y = A \sin(\omega t + \phi) \Rightarrow \text{(ellipse)}$$



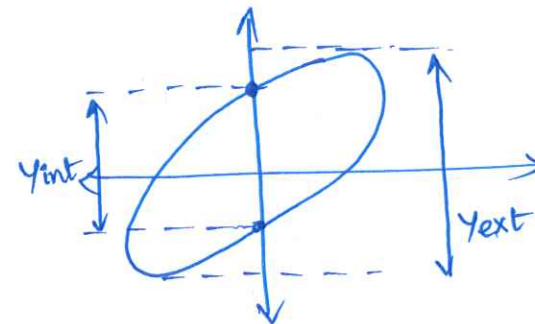
case (vi) :-  $\phi = 270^\circ$

$$X = A \sin \omega t$$

$$Y = A \sin(\omega t + 270^\circ) = -A \cos \omega t$$



Note :- If the phase angle is lying in b/w  $0^\circ$  to  $180^\circ$  the direction of rotation of pattern is in clockwise. If the phase angle is lying in b/w  $180^\circ$  to  $360^\circ$  the direction of rotation of pattern is in the anti-clockwise.



$$\phi = \pm \sin^{-1} \left( \frac{Y_{int}}{Y_{ext}} \right)$$

$+$   $\rightarrow$  CW (clock-wise)  
 $-$   $\rightarrow$  CCW (anti-clock)

### 4. Measurement of frequency

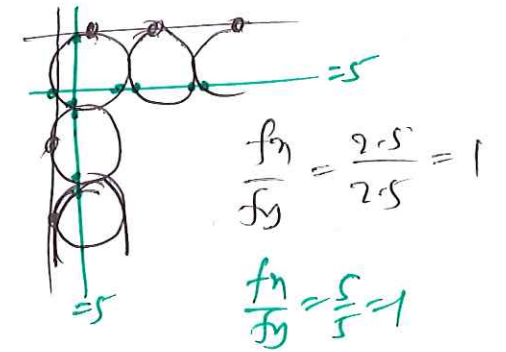
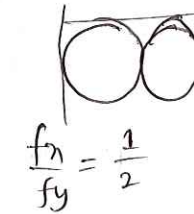
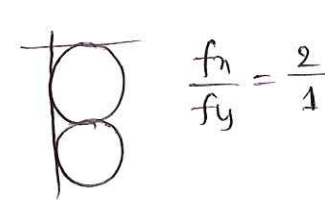
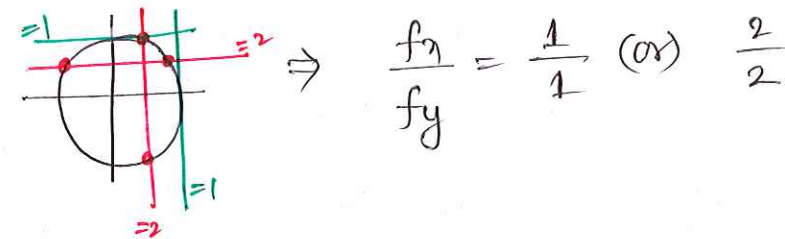
$f_x \Rightarrow$  known signal frequencies;

$f_y \Rightarrow$  unknown signal frequencies;

LP  $\rightarrow$  Lissajous figure

$$\frac{f_y}{f_x} = \frac{\text{Max no. of horizontal tangents drawn to L.P.}}{\text{Max no. of vertical tangents drawn to L.P.}} \quad (\text{or})$$

$$= \frac{\text{max. no. of horizontal intersechions drawn to L.P.}}{\text{max no. of vertical intersechions drawn to L.P.}}$$



Note :- In the CRO these are two types of amplifiers are present

(i) Horizontal amplifier,

(ii) vertical amplifier.

Horizontal amplifier, this is used to amplify the signal which is connected to horizontal deflecting plate.

Vertical amplifier, this amplifies the signal which is connected to vertical deflecting plate, this amplifier determines sensibility and bandwidth of the signal.



$$\text{The Bandwidth} = \frac{0.35}{\text{rise time}}$$

Equivalent ckt of CRO :-

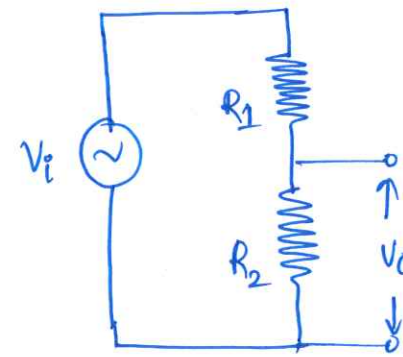
Let  $R_1 \Rightarrow$  resistance of probe connected across CRO

$C_1 \Rightarrow$  shunt capacitance (of the probe) (or) connected across probe.

$R_2 \Rightarrow$  input resistance of CRO.

$C_2 \Rightarrow$  input capacitance of CRO.

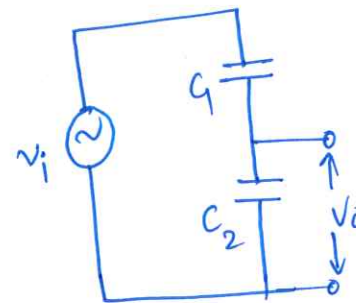
Let  $K \Rightarrow$  be the voltage divider ratio.  
(or) ~~Attenuation factor~~.



$$K = \frac{V_i}{V_o} \quad ; \quad V_o = V_i \left( \frac{R_2}{R_1 + R_2} \right)$$

$$K = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

$$R_1 = R_2(K-1)$$



$$V_o = V_i \left( \frac{C_1}{C_1 + C_2} \right) \Rightarrow K = \frac{V_i}{V_o} = 1 + \frac{C_2}{C_1}$$

$$(K-1) = \frac{C_2}{C_1} \Rightarrow C_1 = \frac{C_2}{(K-1)}$$

→ Digital voltmeters (DVM) :-

\* Calibration can be done independent of measuring  $C_{k}$ .

- \* BCD 7-segment display. \* More user friendly, can easily readable
- \* over-range facility.

\* High resolution =  $\frac{V_{FSD}}{10^N}$  volts.      Eg: (0-10V)

N = no. of bits ; (1 part in  $10^6$ )

Resolution =  $\frac{10}{10^7} = \frac{1}{10^6}$   
 = 1  $\mu$  volt. in 1 Volt range.

- \* High sensitivity.
- \* High accuracy ( $\pm 0.005\%$  of reading).

- \* Instant read-off facility available. (one time reading, switch off, on, read, off - -).
- \* More durability.

\* Input range      0.000001 to 1000.000  
                                     1.000000 to 1000.000 (7 significant figures).

$\therefore$  we can get more precise values.

- \* Input resistance 10 M $\Omega$  to 40 M $\Omega$ . }  $\therefore$  almost negligible loading effect.  
 capacitance around 40 pF }

- \* stability also good. (0.002% of reading) for 24 hours period.  
 (0.08% of reading) for 6 months period.

\* Capacity to read RMS value. (Output of DVM is always RMS value).

But

- \* Lot of components are needed
- main component needed ADC converters.
- (i) Counter (or) Basic ramp type
- (Tclk) (ii) parallel (or) flash - fastest, costlier (no. of comparators =  $2^n - 1$ ).
- ( $2^N$  Tclk) (iii) Dual slope <sup>(noise can be eliminated by two times integrating)</sup> integrating type  $\rightarrow$  slowest, cheaper, high accuracy.
- (iii) stairstepped ramp type
- (N.Tclk) (iv) Successive Approximation  $\Rightarrow$  2<sup>nd</sup> fast, moderate cost

Dual slope Integrating type ADC, most accurate ADC b/c it is less sensitive to noise & temperature changes.

time  $\Rightarrow$  (Flash type) < (SAR) < (Counter type) < (Dual slope Integ)

cost  $\Rightarrow$  (Flash) > (SAR) > (Dual slope).

### Successive Approximate type ADC :-

Always  $\rightarrow$  MSB bit is set to 1.  $\Rightarrow B_3 = 1$ .

case (i) Eg:- 4-bit converter. (if  $V_a > V_{ref}$ )  $B_3 = \frac{V_{ref}}{2^0}$  ;

$B_3$	$B_2$	$B_1$	$B_0$
$\frac{V_{ref}}{2^0}$	$\frac{V_{ref}}{2^1}$	$\frac{V_{ref}}{2^2}$	$\frac{V_{ref}}{2^3}$
$V_{ref}$	$\frac{V_{ref}}{2}$	$\frac{V_{ref}}{4}$	$\frac{V_{ref}}{8}$

case (i)

$V_{ref} = 5 \text{ Volts}$  ;

$V_a = 7.15 \text{ Volts} = 5 + 2.15$

5	$\frac{5}{2}$	$\frac{5}{2^2}$	$\frac{5}{2^3}$
---	---------------	-----------------	-----------------

$\Rightarrow 5 \quad 2.5 \quad 1.25 \quad 0.625$

$\Rightarrow 1 \quad 0 \quad 1 \quad 1$

$\Rightarrow (5 \times 1 + 0 \times 2.5 + 1 \times 1.25 + 1 \times 0.625 = 6.875)$

$\leftarrow V_a = 7.15 > V$

$V_a =$

$3.15 \Rightarrow$

$0 \quad 1 \quad 0 \quad 0$

$\Rightarrow (0 \times 5 + 1 \times 2.5 + 0 \times 1.25 + 1 \times 0.625 = 3.125)$

$V_a < V_{ref} \Rightarrow 0000.$

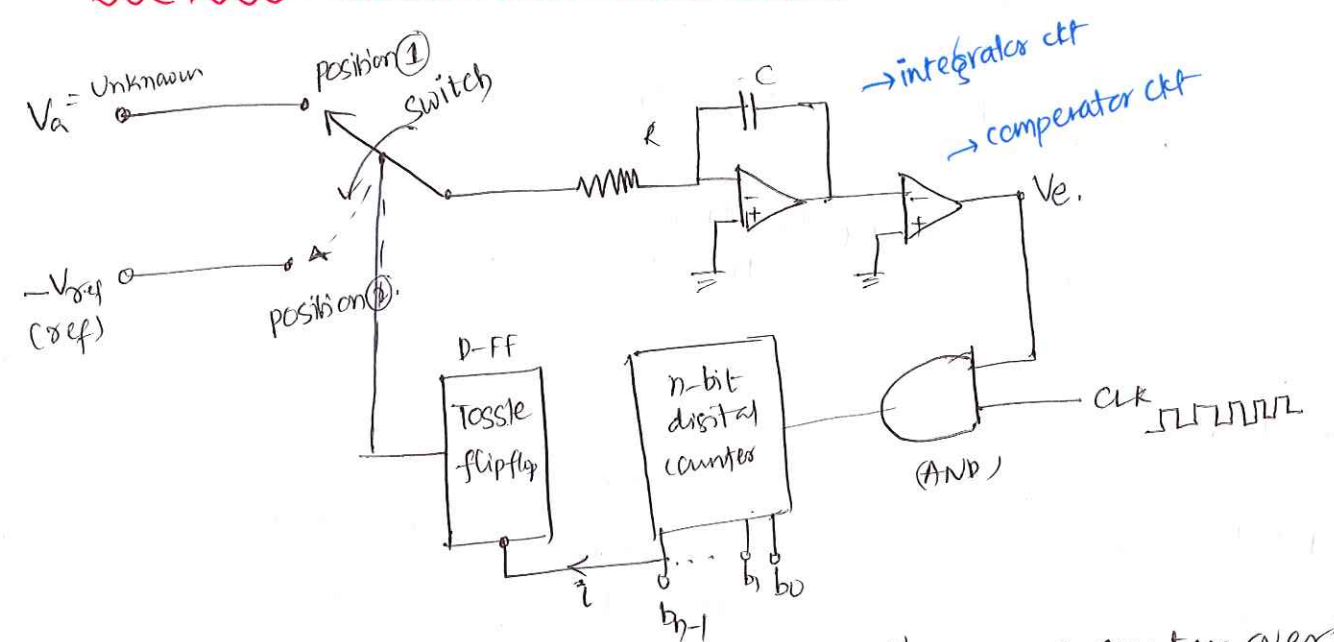
case (ii) :-  $V_a < V_{ref}$  always set  $B_3$  bit ~~to~~ to 1. ;  $B_3 = \frac{V_{ref}}{2^1}$

$B_3$	$B_2$	$B_1$	$B_0$
$\frac{V_{ref}}{2^1}$	$\frac{V_{ref}}{2^2}$	$\frac{V_{ref}}{2^3}$	$\frac{V_{ref}}{2^4}$
$\frac{5}{2}$	$\frac{5}{4}$	$\frac{5}{8}$	$\frac{5}{16}$

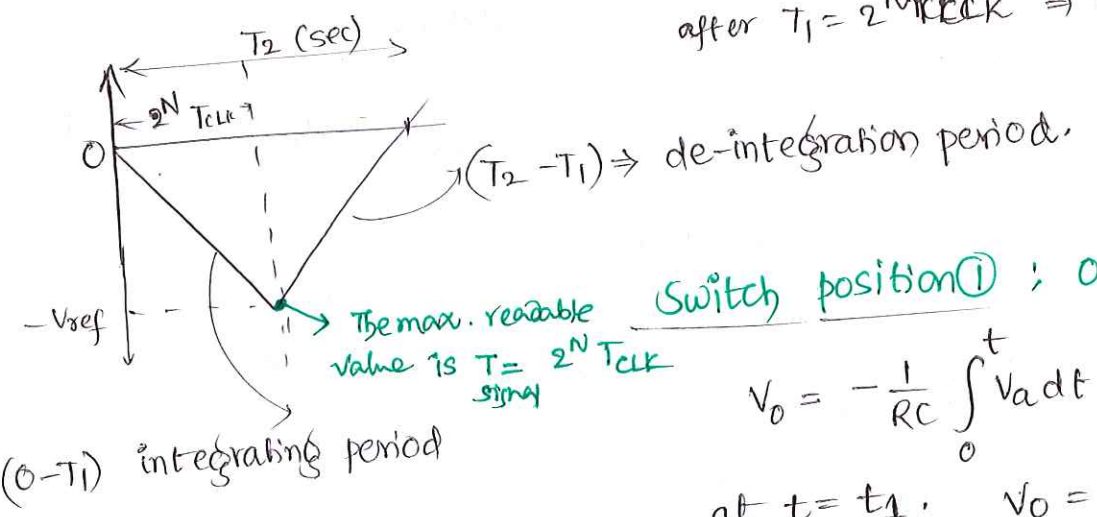


$B_3 \quad B_2 \quad B_1 \quad B_0$   
 $V_{ref} \Rightarrow 2.5 \quad 1.25 \quad 0.625 \quad 0.3125$   
 $V_a = 3.15 \Rightarrow 1 \quad 0 \quad 1 \quad 0$

Dual slope Integrating type ADC :-



after  $T_1 = 2^N T_{CLK} \Rightarrow$  counter overflows.



Switch position ① ;  $0 < t < T_1$

$$V_0 = -\frac{1}{RC} \int_0^t V_a dt = -\frac{V_a}{RC} t$$

at  $t = T_1$ ,  $V_0 = -V_{ref}$ .

$$V_0 = -\frac{V_a}{RC} \times T_1$$

Switch is at position ② :-  $T_1 < t < T_2$

$$\begin{aligned}
 V_0 &= -\frac{1}{RC} \int_{T_1}^{T_2} (-V_{ref}) dt + \left( -\frac{V_a}{RC} T_1 \right) \\
 &= -\frac{V_a}{RC} T_1 + \frac{V_{ref}}{RC} \int_{T_1}^{T_2} dt = -\frac{V_a}{RC} T_1 + \frac{V_{ref}}{RC} (T_2 - T_1)
 \end{aligned}$$

at  $t = t_2$  ;  $V_0 = 0$ .

$$V_0 = -\frac{V_a}{RC} T_1 + \frac{V_{ref}}{RC} (t - T_1)$$

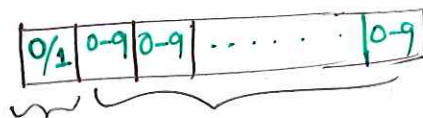
$$0 = -\frac{V_a}{RC} T_1 + \frac{V_{ref}}{RC} (T_2 - T_1)$$

$$V_a T_1 = V_{ref} (T_2 - T_1)$$

$$V_a = V_{ref} \left( \frac{T_2 - T_1}{T_1} \right)$$

$N\frac{1}{2}$  DVMS :-

Total no. of digits displayed =  $(N+1)$   
 $N \rightarrow$  no. of full digits.



Half digit  
(Left-most)

Full digits  
(N) - number

it only indicates (displace)  
 either 0 or 1, whereas all full  
 digits show from 0 to 9.

(Left most digit is Half digit).  
 \* Half digit is responsible for over  
 -range facility.

\* It can display either '0' or '1'.

\* Full digits determines the  
 resolution of display.  
 (display 0 to 9 any value)

$$\text{Resolution} = \frac{V_{FSD}}{10^N} \text{ in volts}$$

$$\text{Resolution} = \frac{I_{FSD}}{10^N} \text{ in Amp}$$

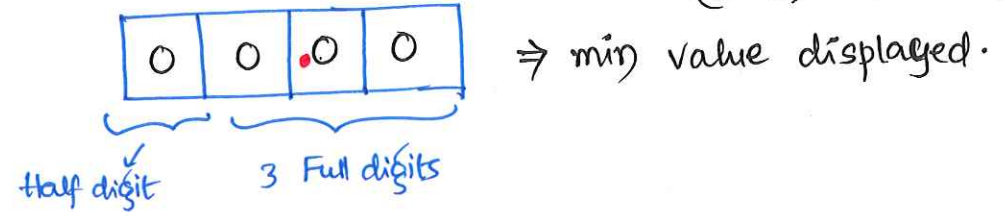
$$\text{Resolution} = \frac{1}{10^N} \text{ in fraction.}$$

Eg: 3 1/2 DVM

total no. of digits displayed = N+1 = 3+1 = 4

no. of Full digits = N = 3.

(0-10)V ⇒ V<sub>FSD</sub> = 10V.



1 0 . 0 0 ⇒ Full scale value. Resolution =  $\frac{V_{FSD}}{10^N} = \frac{10}{10^3} = 10^{-2}$

1 9 . 9 9 ⇒ Max/capability of display. = 10mVolts. =  $\frac{1}{10^2}$   
= 0.01 Volts.

Resolution of the display decides the location of the decimal position. from right end (resolution =  $\frac{1}{10^2}$  ⇒). as shown above with red dot.

over-range = (max/capability) - (Full scale value)  
= 19.99 - 10.00  
= 9.99.

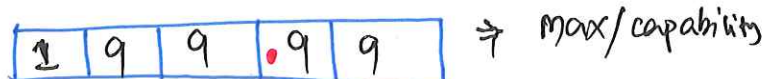
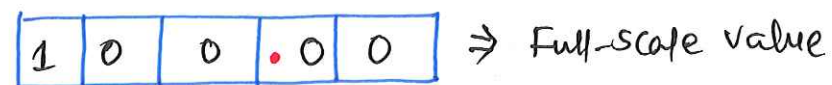
Eg: 4 1/2 DVM

total no. of digits displayed = N+1 = 4+1 = 5

no. of full digits = N = 4.

(0-100)V say

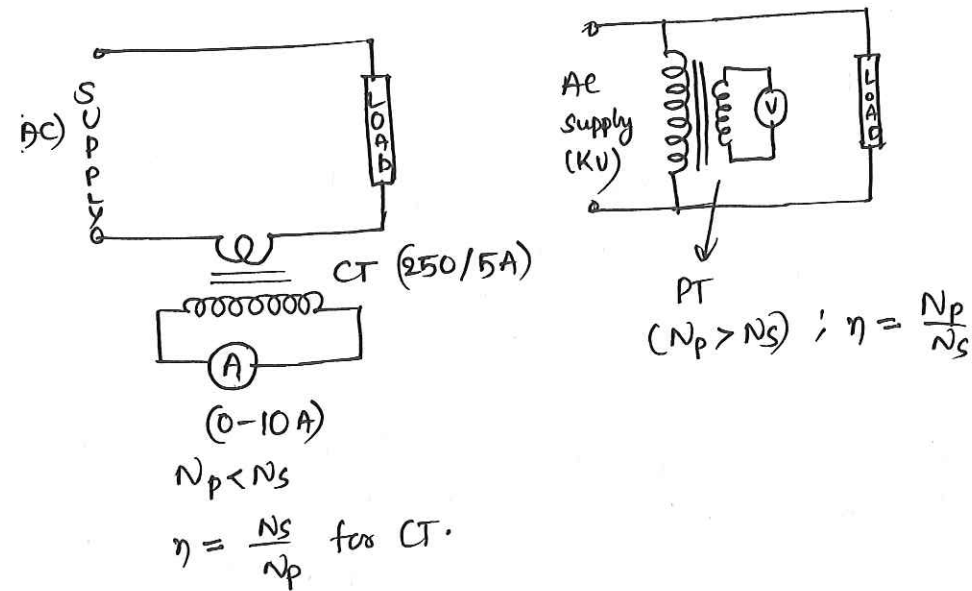
resolution =  $\frac{V_{FSD}}{10^4} = \frac{100}{10^4} = \frac{1}{10^2} = 0.01 = 0.01$



over-range  
= 199.99 - 100.00  
= 99.99 volts.



## Instrumentation Transformers



Burden :- The load which is connected on CT secondary side is known as burden, which includes the meter impedance.

$$\text{Burden} = I^2 Z_e \quad ; \quad Z_e = r_e + jx_e$$

$$|Z_e| = \sqrt{r_e^2 + x_e^2}$$

$$\Delta = \text{burden angle} = \tan^{-1} \left( \frac{x_e}{r_e} \right)$$

if Burden  $\Rightarrow$  pure resistive ;  $Z_e = r_e$  ;  $\Delta = 0^\circ$ .

transformation ratio (R) :-

It is defined as the ratio of actual primary phasor to the actual secondary phasor.

The phasor may be either voltage (or) current

$$R = \frac{V_p \text{ actual}}{V_s \text{ actual}} \quad \text{for PT}$$

$$R = \frac{I_p \text{ actual}}{I_s \text{ actual}} \quad \text{for CT.}$$

Nominal ratio ( $K_n$ ) :- It is defined as the rated primary phasor to the rated secondary phasor. (97)

$$K_n = \frac{I_p(\text{rated})}{I_s(\text{rated})} \text{ for CT}$$

$$K_n = \frac{V_p(\text{rated})}{V_s(\text{rated})} \text{ for PT.}$$

Turns ratio ( $\eta$ )

$$\eta = \frac{N_s}{N_p} \text{ for CT}$$

$$\eta = \frac{N_p}{N_s} \text{ for PT.}$$

Ratio correction factor (RCF) :-

$$RCF = \frac{R}{K_n} = \frac{\text{transformation ratio}}{\text{Nominal ratio.}}$$

There are two types of errors are present in instrument tps

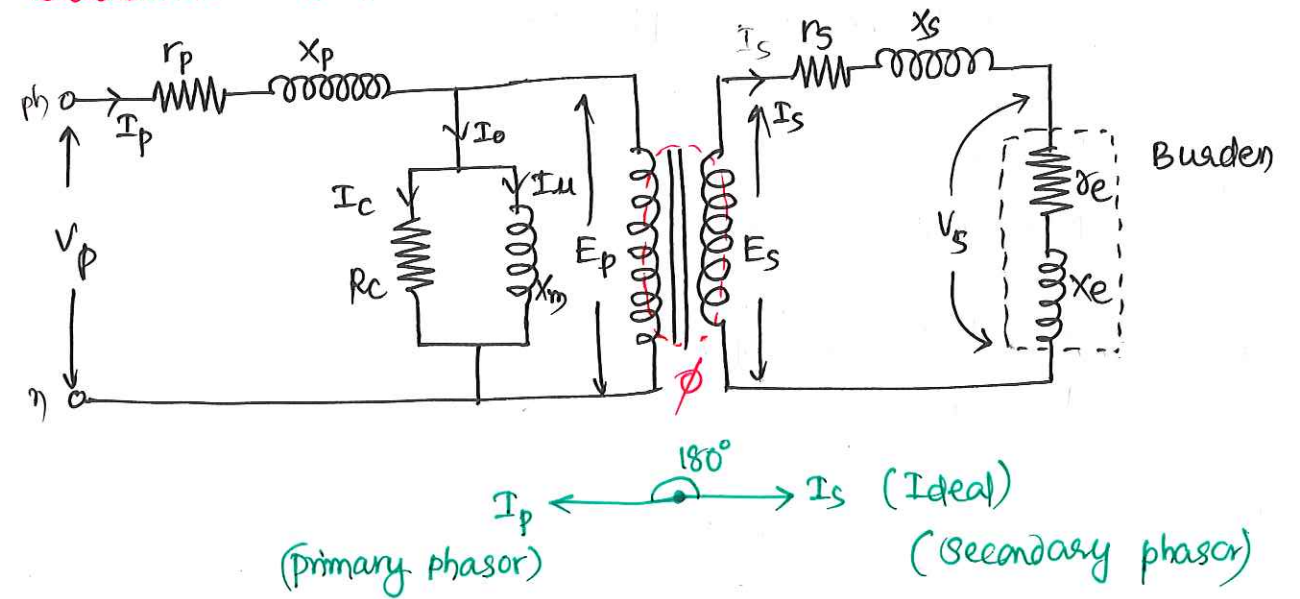
(i) Ratio error

(ii) phase angle error.

$$\% \text{ Ratio error} = \frac{K_n - R}{R} \times 100 = \frac{\text{nominal ratio} - \text{transformation ratio}}{\text{transformation ratio.}}$$

phase angle error ( $\theta$ ) =

Equivalent ckt of CT :-



practically  $(\angle I_p, I_s) \Rightarrow (\leq 180^\circ)$ .

Let  $\phi$  be the core flux (or) working flux of transformer.

$E_s \Rightarrow$  no-load secondary voltage.

$V_s \Rightarrow$  terminal voltage

$I_s \Rightarrow$  secondary rated current.

$I_0 \Rightarrow$  No load current.

Let  $\alpha$  is the angle b/w  $I_0$  & core flux.

\*\* The core flux always lags  $I_0$  by  $\alpha$ .

$I_m = I_u \Rightarrow$  magnetizing component.

$I_e = I_e \Rightarrow$  core loss component.

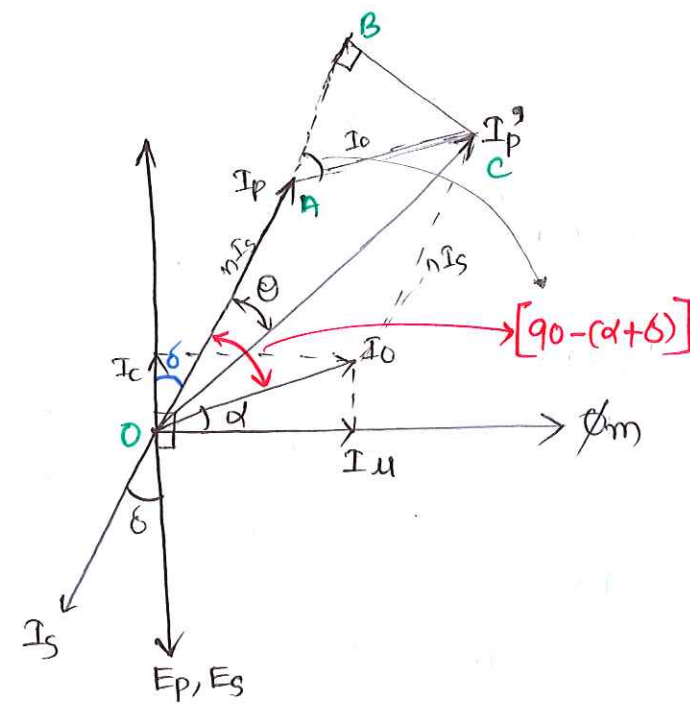
$\delta \Rightarrow$  Angle b/w  $(E_s, I_s) \quad \therefore \delta = \tan^{-1} \left( \frac{x_s + x_e}{r_s + r_e} \right)$

(includes burden angle also).

If burden is pure resistive, if  $x_s = 0 \Rightarrow \delta = 0$ .  
 $\Delta = 0$ , i.e.,  $x_e = 0$ ,

$\theta \Rightarrow (I_p \& I_s)$  phase angle of transformer.





$\Delta ABC$

$$AC^2 = BC^2 + AB^2$$

$$I_p^2 = I_0^2 \sin^2 [90 - (\alpha + \delta)] + I_0^2 \cos^2 [90 - (\alpha + \delta)]$$

$\Delta OBC$

$$OC^2 = OB^2 + BC^2$$

$$I_p^2 = (OA + AB)^2 + BC^2$$

$$I_p^2 = [\eta I_s + I_0 \sin(\alpha + \delta)]^2 + [I_0 \cos(\alpha + \delta)]^2$$

$$I_0 = \sqrt{I_m^2 + I_c^2}$$

$$I_m = I_0 \cos \alpha$$

$$I_c = I_0 \sin \alpha$$

$$I_p = \eta I_s ; \eta = \frac{I_p}{I_s} \text{ (ct)}$$

$$I_p^2 = \eta^2 I_s^2 + I_0^2 \sin^2(\alpha + \delta) + 2\eta I_s I_0 \sin(\alpha + \delta) + I_0^2 \cos^2(\alpha + \delta)$$

$$I_p = \sqrt{\eta^2 I_s^2 + I_0^2 + 2\eta I_s I_0 \sin(\alpha + \delta)}$$

$$I_p = \sqrt{(\eta I_s + I_0 \sin(\alpha + \delta))^2 + (I_0 \cos(\alpha + \delta))^2} \Rightarrow \text{Exact}$$

Case(i) if core-loss component is neglected. ( $I_c = 0$ ),  $\alpha = 0$  ( $I_0 \cong I_m$ ).

$$I_p \cong \sqrt{(\eta I_s)^2 + [I_0 \cos(\alpha + \delta)]^2} \Rightarrow \text{approximated.}$$

if burden is pure resistive;  $\Rightarrow \delta = 0$ .

$$\therefore I_p \cong \sqrt{\eta^2 I_s^2 + (I_0 \cos \alpha)^2}$$

$$I_p = \sqrt{(\eta I_s)^2 + I_m^2}$$

$I_m \Rightarrow \frac{\text{(magneto motive force)}}{\text{(primary turns)}} \Rightarrow I_m = I_m = \frac{\text{MMF}}{N_p}$

$$\therefore I_p = \sqrt{(\eta I_s)^2 + I_0^2}$$

case (ii) :-

$$\text{exact eqn} \Rightarrow I_p = \sqrt{[\eta I_s + I_0 \sin(\alpha + \delta)]^2 + I_0^2 \cos^2(\alpha + \delta)}$$

$$I_p \cong \eta I_s + I_0 \sin(\alpha + \delta) \quad ; \quad I_0^2 \cos^2(\alpha + \delta) \rightarrow \text{neglected}$$

$$R = \frac{R_p}{R_s} = \frac{\eta I_s + I_0 \sin(\alpha + \delta)}{I_s}$$

$$R = \eta + \frac{I_0}{I_s} \sin(\alpha + \delta)$$

$$= \eta + \frac{I_0}{I_s} (\sin \alpha \cos \delta + \sin \delta \cos \alpha)$$

if burden is pure resistive  $\delta = 0^\circ$ .

$$\therefore R = \eta + \frac{I_0}{I_s} (\sin \alpha \cos \delta + 0)$$

$$= \eta + \frac{(I_0 \sin \alpha) \cos \delta}{I_s}$$

$$R = \eta + \frac{I_e \cos \delta}{I_s}$$

$$I_e = I_c$$

$$R \cong \eta + \frac{I_c}{I_s}$$

$$I_p = \eta I_s \Rightarrow I_s = \frac{I_p}{\eta}$$

$$R = \eta + \frac{\eta I_c}{I_p}$$

$$R = \eta \left(1 + \frac{I_c}{I_p}\right)$$

case (iii) :-  $\theta =$  phase angle error

(99)

$$\Delta OBC \Rightarrow \tan \theta = \frac{BC}{OB} = \frac{I_0 \sin(90 - (\alpha + \delta))}{\eta I_s + I_0 \cos(90 - (\alpha + \delta))}$$

$$\because OB = OA + AB.$$

$$\therefore \boxed{\tan \theta = \frac{I_0 \cos(\alpha + \delta)}{\eta I_s + I_0 \sin(\alpha + \delta)}} \text{ exact eqn.}$$

to approximate;  $\eta I_s \gg I_0 \sin(\alpha + \delta)$

$$\boxed{\tan \theta = \frac{I_0 \cos(\alpha + \delta)}{\eta I_s}} \text{ approximate eqn}$$

when burden is pure resistive;  $\delta = 0$ .

$$\begin{aligned} \therefore \tan \theta &= \frac{I_0 \cos(\alpha + 0)}{\eta I_s} \approx \frac{I_0 \cos \alpha}{\eta I_s} \\ &= \frac{I_m}{\eta I_s} \end{aligned}$$

for very small value of  $\theta \Rightarrow \tan \theta \approx \theta$ .

$$\therefore \boxed{\theta = \frac{I_m}{\eta I_s}} \text{ radians}$$

$$\therefore \boxed{\text{phase angle error} = \theta = \frac{180}{\pi} \left( \frac{I_m}{\eta I_s} \right)} \text{ degrees.}$$



The errors produced in CT b/c of

- (i) magnetic leakage in secondary wdg.
- (ii) Flux density in the core is not a linear function of mmf.
- (iii) Error due to hysteresis & eddy current losses
- (iv) Error due to losses in the wdg of  $t/F$ .

### Remedy

In order to eliminate the both phase & ratio errors high permeability core is preferred, which ~~is~~ is prepared by

- (i) CRGO
- (ii) Silicon steel stampings
- (iii) Ferrite cores.

By using turns compensation method, we can reduce the ratio error only. ~~but~~

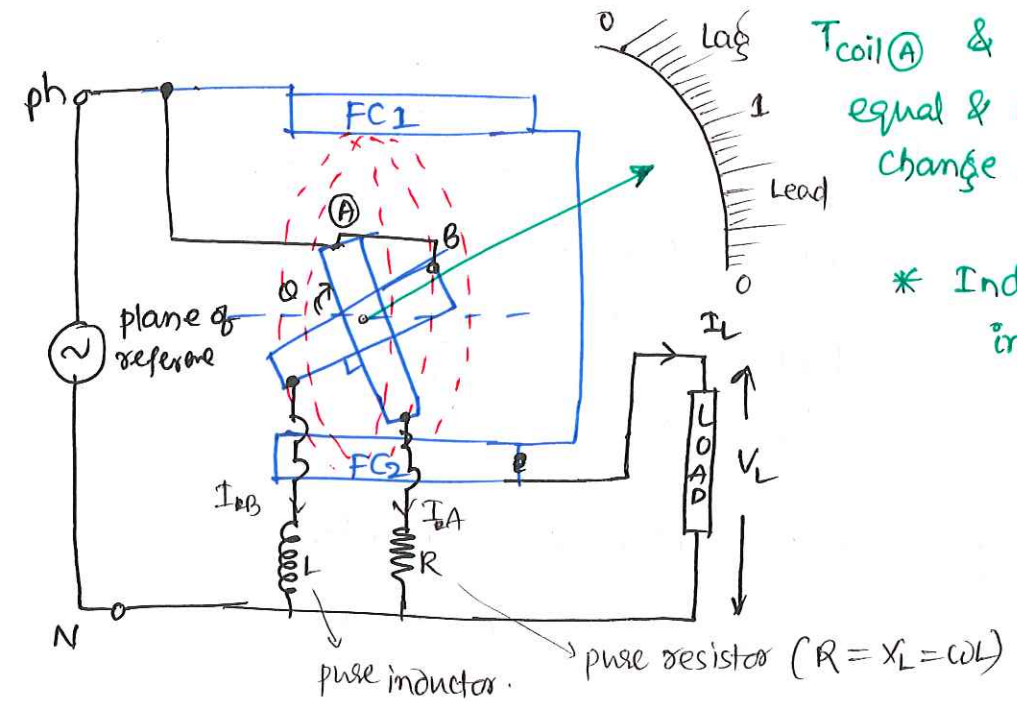
By reducing either one turn or two turn on CT secondary we can reduce the ratio error but it doesn't have any effect on phase angle error.

power factor meter

(CC/field coils)  
Fixed coils behaves like electromagnets

(1-∅ EDM type)

(Controlling torque is not required absent)



$$\cos\phi = \frac{P}{VI} \quad ; \quad \text{at S.S} \Rightarrow T_{d(A)} = T_{d(B)}$$

$$T_{dA} = I_{FC} I_{MC} \cos(\angle I_{FC}, I_{MC}) \frac{dm}{d\theta} \quad ; \quad I_{FC} = I_L$$

$$I_{MC} = \frac{V_L}{R} = I_A$$

both are pressure coils (A) & (B).

$$I_A = I_B = \frac{V_{pc}}{R} = \frac{V_L}{X_L}$$

$$\therefore T_{dA} = I_L \cdot \frac{V_L}{R} \cos\phi \cdot (-m)_{max} \sin\theta$$

$$= \frac{V_L I_L \cos\phi}{R} (-m)_{max} \sin\theta$$

$$T_{dB} = I_L \cdot I_B \cos(\angle I_{FC}, I_B) \frac{dm}{d\theta}$$

$$= I_L \cdot \frac{V_L}{\omega L} \cos(90 - \phi) (-m)_{max} \sin(\theta + 90) \quad (T_{dA} = T_{dB})$$

$$T_dA = TdB$$

$$I_L \frac{V_L}{R} \cos \phi (-\max \sin \theta) = I_L \frac{V_L}{\omega L} \cos(90 - \phi) (-\max \sin(90 + \theta))$$

$$R = \omega L;$$

$$\cos \phi \cdot \sin \theta = \sin \phi \cdot \cos \theta.$$

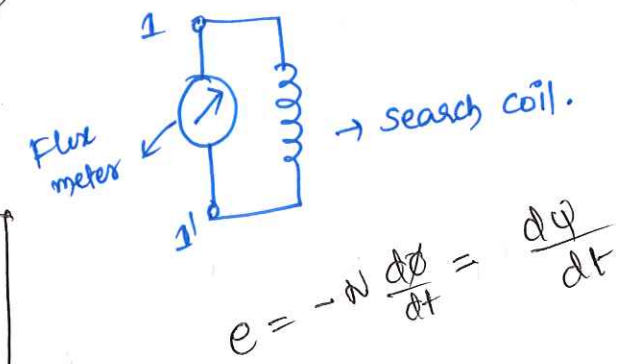
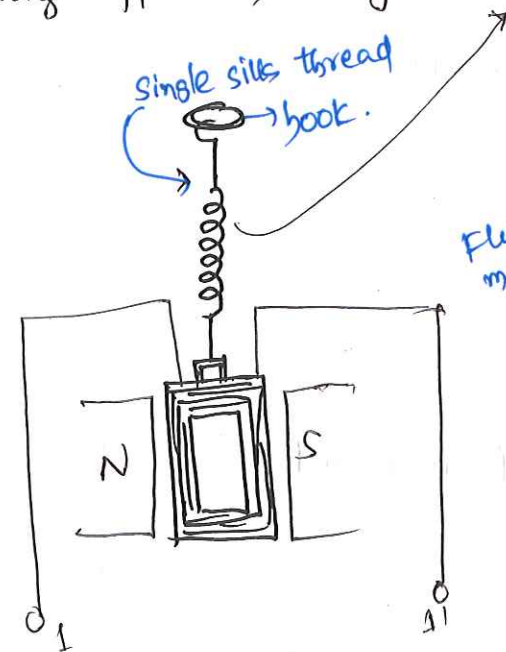
$$\tan \theta = \tan \phi$$

$$\theta = \phi$$

$\therefore$  pf angle = deflection of pointer  
= angle b/w two coils.

scale is calibrated to  $\cos \phi$ ...

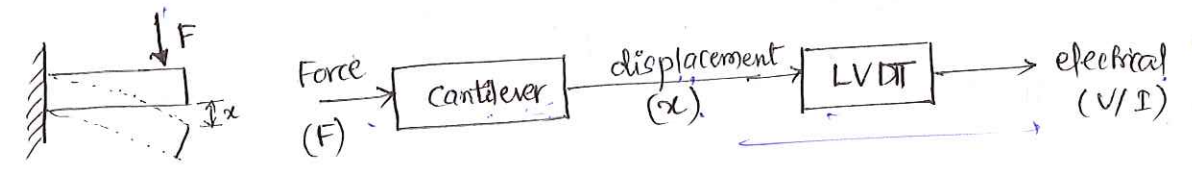
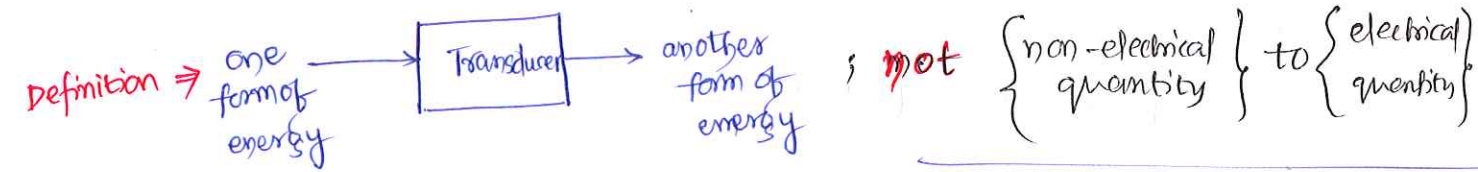
**Flux meter** is a special type of Ballistic galvanometer in which minimum controlling torque & heavy electro-magnetic damping is used. Ballistic  $\odot$  is similar to the pmmc meter. Construction of pmmc & Ballistic galvanometers are same. But these use no control springs are used b/c to obtain minimum controlling effect, very loose Helices are used.





# TRANSDUCER

• Also called as  $\Rightarrow$  pickup / sensor.



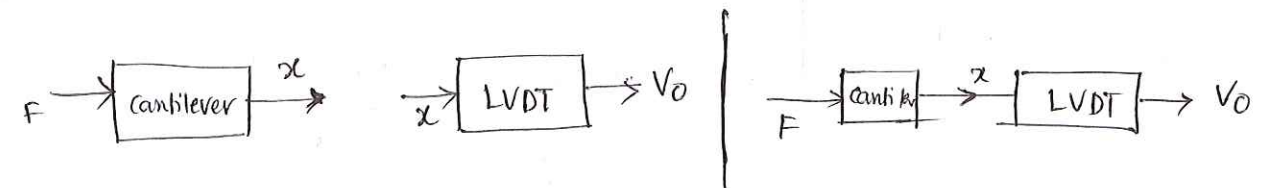
\* These are the sensing devices which convert any form of energy to electrical energy. In many practical examples non-electrical quantities into E quantity

## advantages

- (i) Transformation of signal is easier over long distance.
- (ii) Data manipulation is easy.
- (iii) processing of data is easier & power consumption is low.

## Types

1. primary and secondary transducer.



If the quantity to be measured is in direct contact with the transducer it is called primary otherwise it is secondary transducer.

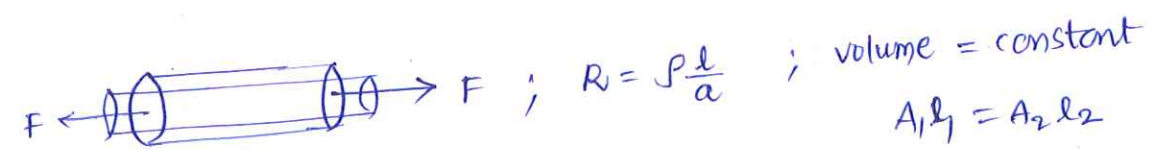
Eg:- LVDT (displacement), Thermistor, Bourdon Tube, Bellows, diaphragm — Primary.  
 LVDT (pressure) — Secondary.

2. **Active Transducer**: If a transducer is self generating i.e. output electrical signal is produced by transducer itself, it is called active.

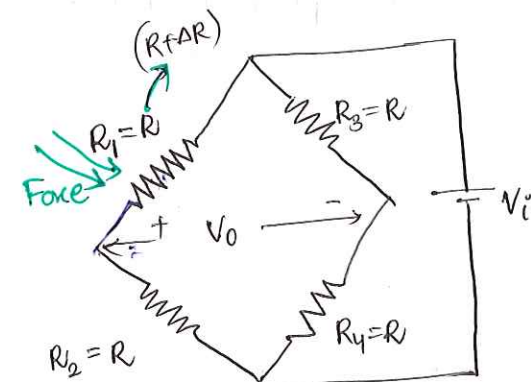
eg: Piezoelectric, photo-voltaic, solar cell, Thermocouple.

**Passive Transducer**: If the output electrical signal depends on external source then it is called passive transducer.

eg: RTD, C, RTD, LVDT, thermistor



$\therefore R' = R + \Delta R$  ;  $l_2 = l_1 + \Delta l$   
 $A_2 = A_1 + \Delta A$



$V_o = V_i \left( \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right)$

(i) If  $F=0$  ;  $R_1 R_4 = R_2 R_3$

$V_o = 0$

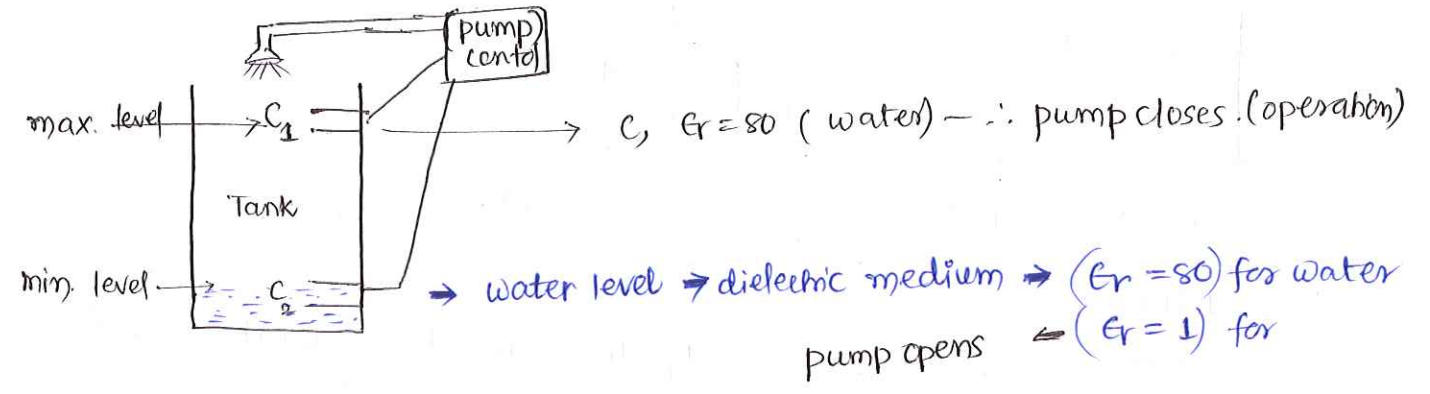
(ii) If  $F \neq 0$  ;  $R_1 R_4 \neq R_2 R_3$

$V_o \neq 0$  ;

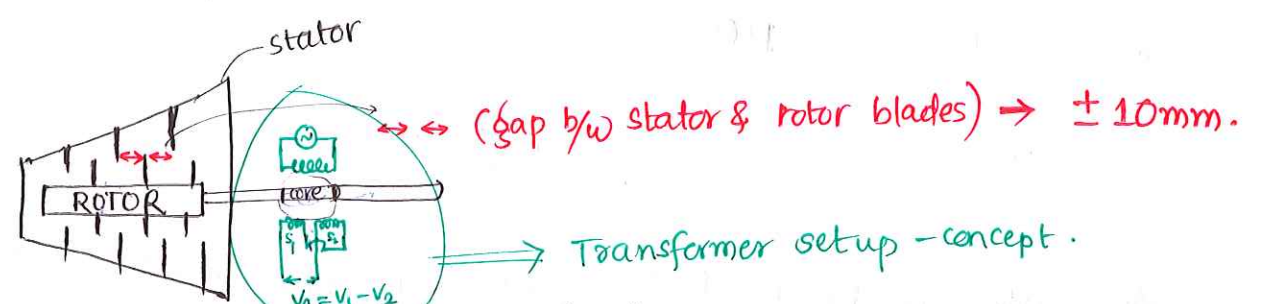
$V_o = f(F)$

→ If we use temperature... in place of Force...  $R, \rho$  will vary for  $R,$

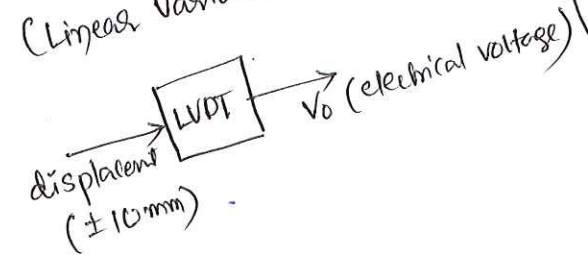
→  $C = \frac{\epsilon_0 \epsilon_r A}{d}$



→  $L = \frac{N^2 \mu_0 \mu_r A}{l}$  ;  $v = L \frac{di}{dt}$



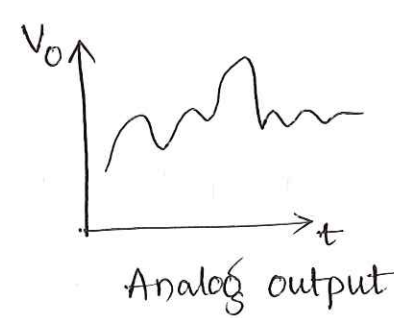
HPT - High Pressure Turbine.  
 This principle of operation is inductance variation by displacement.  
 \therefore it is called LVDT (Linear Variable Differential Transformer)



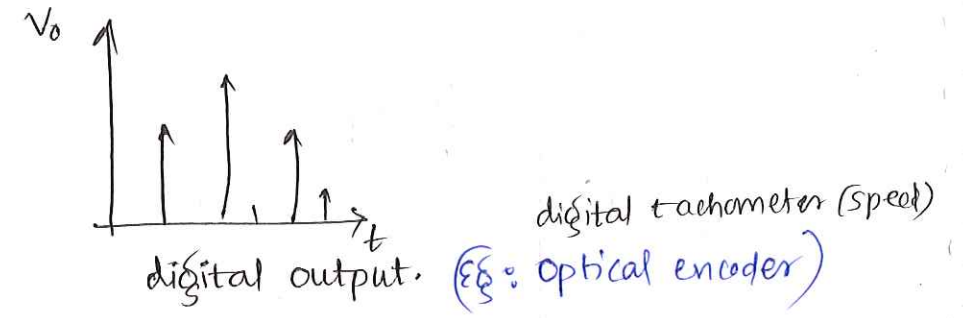
- (i) under normal condition core is exactly in b/w wdg (magnetic material) other than core non-magnetic  
 $\therefore V_0 = V_1 - V_2 = 0$
- (ii) If core shifts left or right (i.e. by rotor shift)  
 $\therefore$  flux limits changes ( $\because$  core area chg)  
 $\therefore V_0 \neq 0$ .  
 Then transducer (or sensor) will activate.



### 3. Analog & Digital Transducer :-



Eg: LVDT, C, Thermistor  
Thermocouple



If the output electrical signal is continuous then it is of analog and if it discrete then it is called digital transducer

Decimal	BCD	Gray
3	0011	0010
4	0100	0110

3-bit transition      1-bit transition (no/less transition errors)

An ADC can be used for conversion of analog into digital. This digital data is represented in gray code instead of binary. Because no. of bit transition is only one in gray code for the input data change. Hence the error is minimum.

Eg: Optical Encoder (Digital transducer) - used for measurement of Speed of shaft, has Gray code.

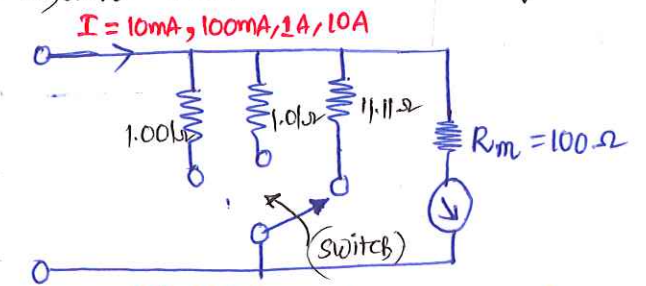
## Characteristics/Requirements of Transducers :-

1. construction must be simple & low cost.
2. power consumption should be low.
3. It should n't be effected by surrounding atmospheric condition.
4. It must have higher sensitivity & high resolution.
5. It should posses minimum amplifying devices in-side it.
6. Must be stable for wide-range of operation.

### Calibration :-

**P** An ammeter as meter resistance of  $100\Omega$  measure current upto  $10\text{mA}$ . How much of shunt resistance is required for measurement of currents

- (i)  $100\text{mA}$
- (ii)  $1\text{A}$
- (iii)  $10\text{A}$



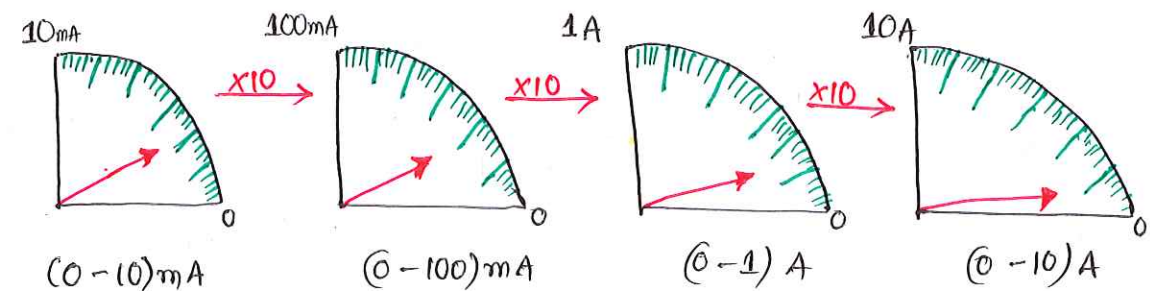
**MultiAmmeter concept.**

SOL:-

(i)  $90\text{mA} (R_{sh}) = 10\text{mA} \times 100$   
 $R_{sh} = \left(\frac{100}{9}\right) \Omega = 11.11\Omega$

(ii)  $R_{sh} = \frac{R_m}{m-1} = \frac{100}{\frac{1}{10 \times 10^{-3}} - 1}$   
 $= \frac{100}{99} \Omega = 1.01\Omega$

(iii)  $R_{sh} = \frac{100}{\frac{10}{10 \times 10^{-3}} - 1} = \frac{100}{999} \Omega = 0.1001\Omega$



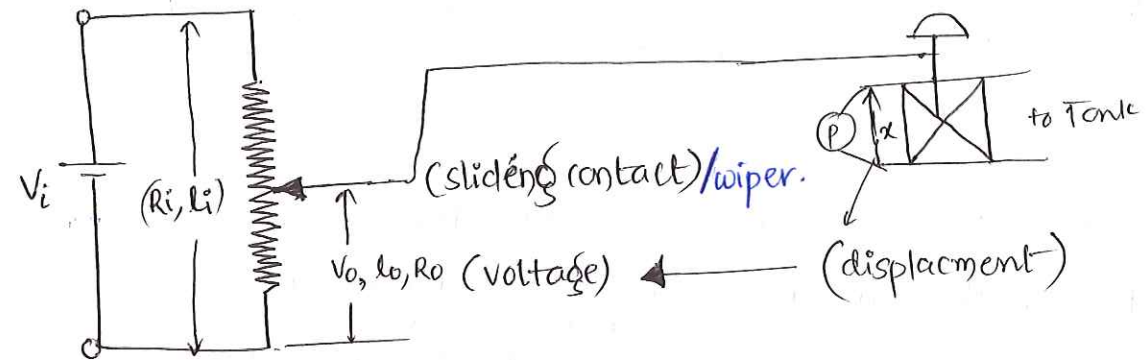
This  $\times 10$  scale manipulation is known as Calibration.

# Measurement of Displacement :-

1. potentiometer (cm)
2. LVDT (mm)
3. Hall Transducer ( $\mu\text{m}$ )

Linear device  
Zero-order system  
voltage/resistive divider n/w.  
Length comparison instrument  
passive transducer (sliding - no internal source)

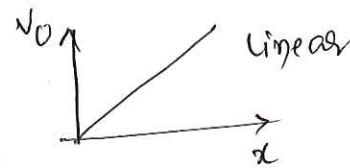
## Potentiometer Transducer :-



$$\frac{V_i}{R_i} = \frac{V_o}{R_o} \Rightarrow R_o \propto l_o \propto x; R_i \propto V_o \propto l_i$$

$$V_o = V_i \left( \frac{x}{l_i} \right) = V_i \left( \frac{l_o}{l_i} \right); \quad l_o = Kx, \quad V_i' = KV_i$$

$$\Rightarrow V_o = \left( \frac{l_o}{l_i} \right) V_i s^0$$



Zero-order system.

$$\text{Sensitivity} = \frac{V_o}{x} = \frac{V_o}{l_o} = \frac{V_i}{l_i} = s; \text{ (volt/mm)}$$

$$s = \frac{V_o}{R_o} = \frac{V_i}{R_i} \quad \forall \Omega$$

$$\text{Resolution } R = \frac{(V_o)_{\min}}{s} \quad \text{mm}$$

min. o/p voltage able to measure.



$$R = \frac{(V_o)_{min}}{S}$$

Resolution of potentiometric Transducer, mm

$(V_o)_{min}$  = min. voltage that can be measured =  $\frac{1}{5}$  (single division) value.

The min. ~~voltage~~ displacement measured by potentiometer  $\rightarrow$  Resolution  
 The max. displacement able to measure by potentiometer  $\rightarrow$  Stroke

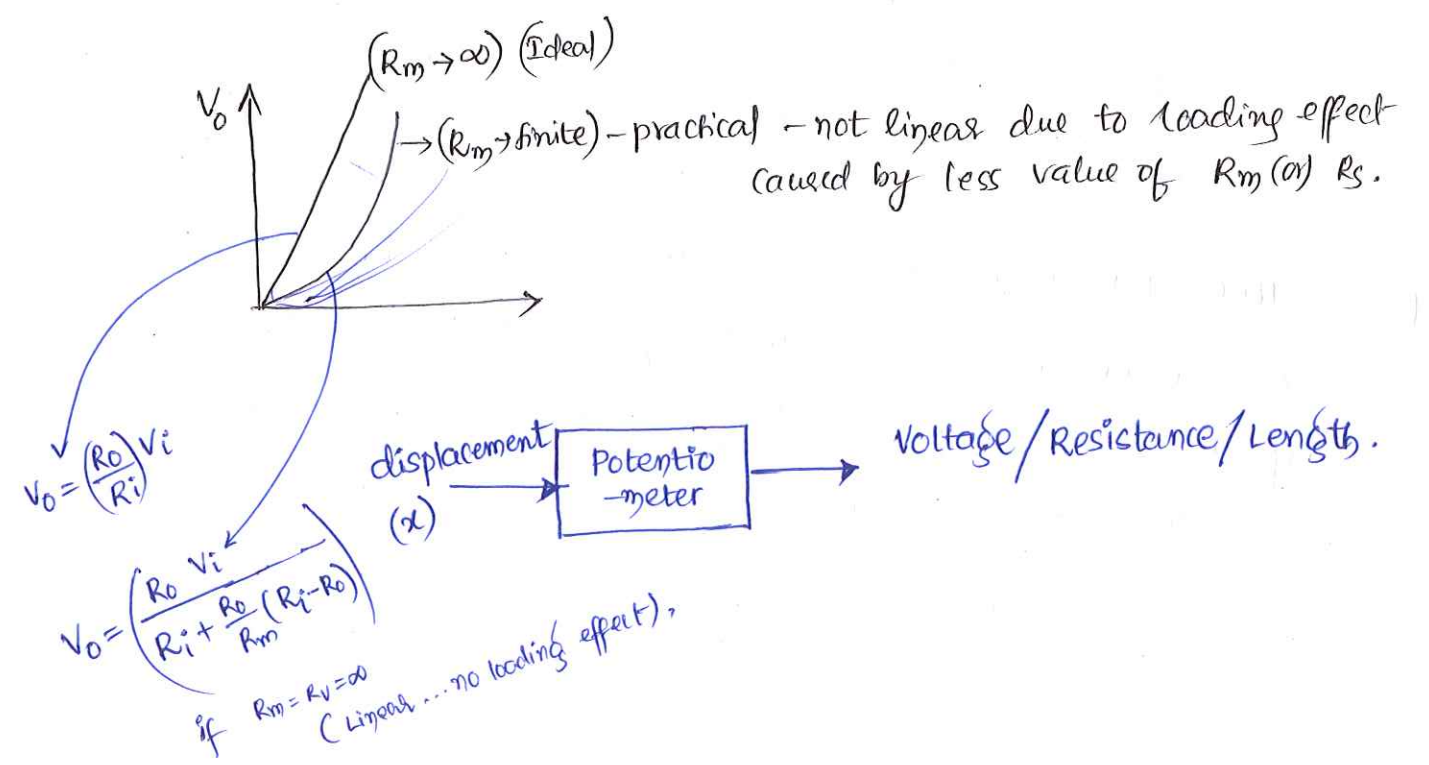
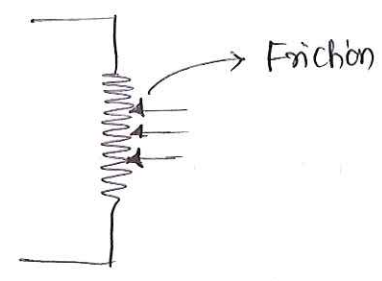
Stroke =  $l_i$  (max. displacement can be measured).

Advantages

1. Simple in construction & ~~less cost~~. High cost.
2. Linear response.
3. Higher sensitivity, higher resolution

Disadvantage

- \* 1. Friction b/w sliding contact & slide wire causes overheating, finally damages the potentiometer.
2. If the o/p voltmeter has lower sensitivity causes **Loading effect**.



P:- Potentiometer has a stroke of 20 mm; Applied with a voltage 10V, Resistance of slide wire is 200  $\Omega$ .

Find (i) o/p voltage for a displacement of 5 mm

(ii) Sensitivity

(iii) Resolution, if the o/p voltmeter having range of (0-10V) with 50 division and can be read upto  $\frac{1}{5}$ th of the divs.

Sol:-

$$V_o = \frac{5}{20} \times 10 = 2.5 \text{ V} \quad \left( V_o = \frac{x}{l} V_i \right)$$

$$\rightarrow S = \frac{V_o}{R_o} = \frac{V_i}{R_i} = \frac{10}{200} = 0.05 \text{ A (V/\Omega)}$$

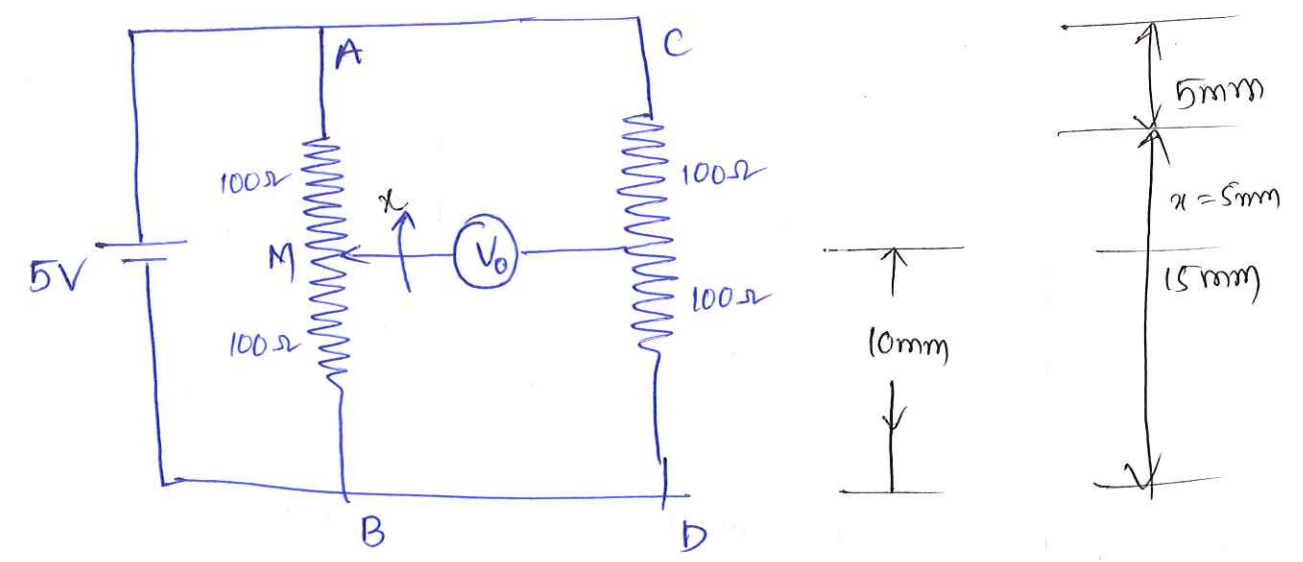
$$R = \frac{(V_o)_{\min}}{S}; \quad (V_o)_{\min} = \frac{1}{5} \times \frac{10}{50} = \frac{1}{25} = 0.04$$

$$\therefore R = \frac{1}{25 \times 0.05} = \frac{100}{125} = \frac{4}{5} = 0.8 \text{ } \underline{\underline{\Omega}}$$

$$\rightarrow S = \frac{V_o}{x} = \frac{V_o}{l_o} = \frac{V_i}{l_i} = \frac{10}{20 \times 10^3} = 0.5 \times \text{V/mm}$$

$$R = \frac{(V_o)_{\min}}{S} = \frac{1}{25 \times 0.5} = 0.08 \text{ mm.}$$

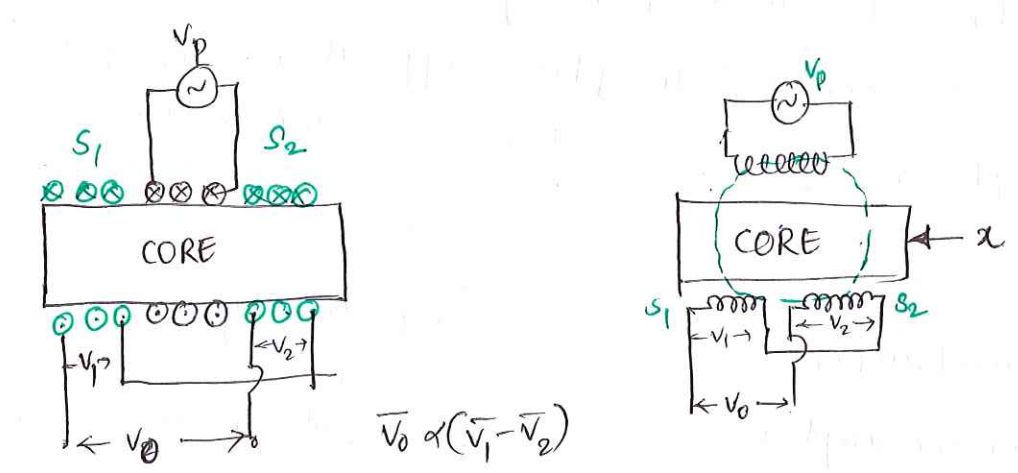
P:- Two potentiometers are arranged in the bridge format as shown in figure. Initially bridge is balanced now by measuring displacement of 5 mm find the o/p voltage if the slide wire is moved towards 'A'. Assume length slide wire ( $l_i = 20 \text{ mm}$ )



$20\text{mm} \rightarrow 200\Omega$   
 $5\text{mm} \rightarrow 50\Omega$   
 $15\text{mm} \rightarrow 150\Omega$

$\uparrow x \quad V_0 = V_i \left( \frac{150}{150+50} - \frac{100}{100+100} \right) = V_i \left( \frac{3}{4} - \frac{1}{2} \right)$   
 $\downarrow x \quad V_0 = V_i \left( \frac{50}{150+50} - \frac{100}{100+100} \right) = V_i \left( \frac{1}{4} - \frac{1}{2} \right)$

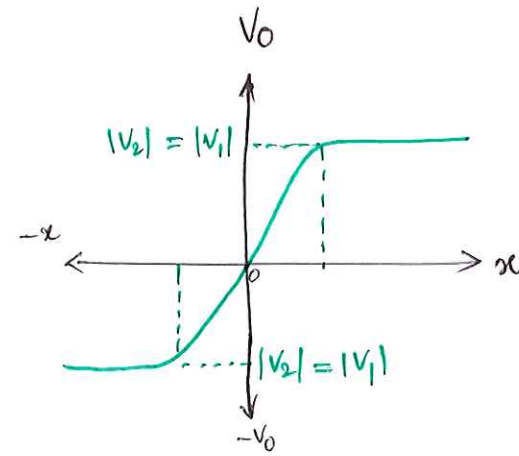
## LVDT Linear Voltage Differential Transformer



$V_0 \propto (V_1 - V_2)$

- (i) If  $x = 0$ ; core at centre; both flux linkages of two wdgs are same;  $\therefore V_0 = V_1 - V_2 = 0$ .
- (ii) If  $x \neq 0$ ; core moves/shift to left/right;  $V_0 \neq 0$ .

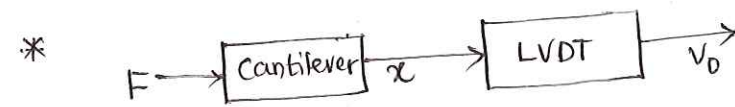




(or)

Sensitivity =  $\frac{V_o}{x}$  volts/mm

\* No moving & sliding contacts...  $\therefore$  no friction & other moving losses.  
But affected by external fields & vibrations.



$$S_c = \frac{x}{F} ; S_{LVDT} = \frac{V_o}{x}$$

$$(S_{overall}) = \frac{V_o}{F} = \frac{V_o}{x} \cdot \frac{x}{F} = S_c \cdot S_{LVDT}$$

In place of LVDT we can use potentiometer also but the main disadvantage of potentiometer is sliding frictional errors which is not there in LVDT.

LVDT consisting of single primary & two identical secondary windings which are connected in phase opposition. Output voltage is ~~with~~ their phasor differences of two secondary voltages.

### Advantages (LVDT)

1. Simple in construction
2. Less power consumption
3. Linear o/p.
4. High accuracy, high resolution & sensitivity.
5. No need of amplification of o/p voltage.

### Disadvantages (of LVDT)

1. affected by external magnetic fields
2. Output produces error if vibrations are present.

### Applications of LVDT

- 1. Measures displacement more accurately within 'mm' range.
- 2. Used as a secondary transducer for measurement of force, pressure ...etc along with primary transducer.

Q:- A cantilever has a sensitivity of 2mm/N is connected to an LVDT has a sensitivity of 5V/mm, Find overall sensitivity, The smallest force able to measure if the o/p voltage is measured by a voltmeter which has range of (0-10)V with 100 divisions & it can be read upto  $\frac{1}{4}$ th of the division.

Sol:-

$$S_0 = 2 \times 5 = 10 \text{ V/N} = S_c \cdot S_{LVDT}$$

$$\frac{1}{4} \times (V_o)_{min} = \frac{1}{4} \times \frac{1}{100} \times 10 = \frac{1}{40} = 0.025 \text{ volts.}$$

$$(min) \text{ force} = \frac{(V_o)_{min}}{S_0} = \frac{0.025}{10} = 0.0025 = 2.5 \text{ mN.} = \text{Resolution.}$$

↓  
Lowest value of input we can measure

### Hall Pick-Up Transducer:-

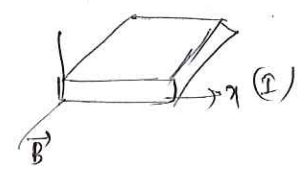
(If we hold the specimen, inside it experiences force, due to Hall effect).

$(F_H, E_H)$  or  $(\vec{I} \times \vec{B})$  external applied  
(induced Hall)

$F_H \rightarrow$  (displaces charges & upper plate & lower plate)  $\rightarrow E_H$  (field ...)  
(measured voltage)  $V_H$

$$R_H = \frac{1}{ne} = \frac{1}{\rho V}$$

$$V_H = \frac{BI}{ne(w)} = \frac{BIR_H}{w}; \quad w = \text{width of specimen along } \vec{B} \text{ direction}$$

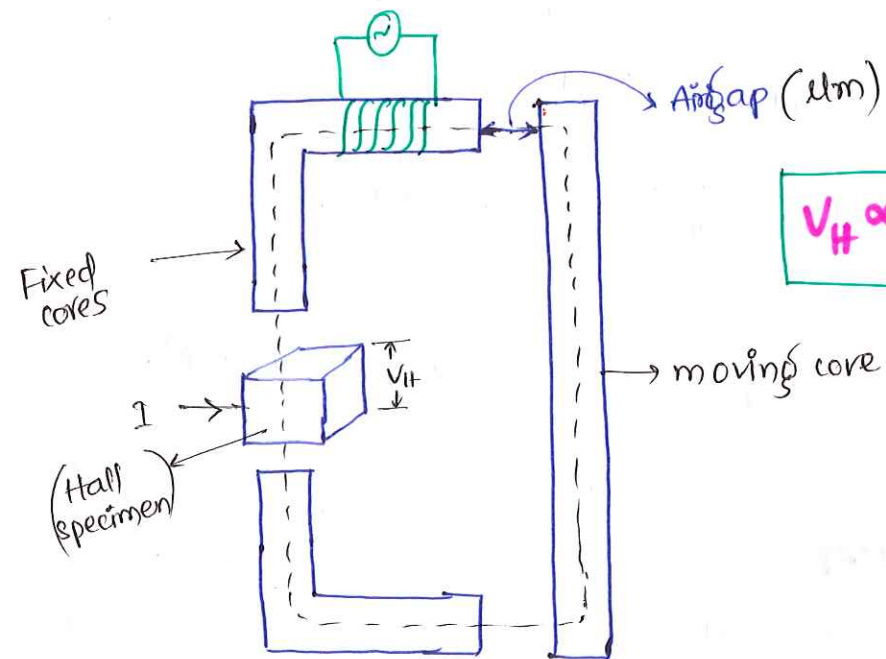


## application of Hall effect

$$V_H = \frac{BI}{ne\omega}$$

1.  $(V_H \propto B) \Rightarrow$  Gaussmeter.
2.  $V_H \propto (B \cdot I) \Rightarrow$  multiplies of two signals.
3.  $R_H = \frac{1}{nq}$ ;  $q = \pm \Rightarrow$  type of semiconductor;
4.  $V_H = \frac{BI}{j_v \omega}$ ;  $j_v, R_H \Rightarrow$  concentration of specimen.
5.  $\sigma = ne\mu = \beta eI \Rightarrow$  conductivity & mobility (if we know other).

with help of Hall effect .. we are going to measure displacement -- as follow.



$$V_H \propto B \propto \phi \propto \frac{l}{\text{Airgap}} \propto \frac{1}{x}$$

- cm  $\rightarrow$  potentiometer (centi)
- mm  $\rightarrow$  LVDT (milli)
- $\mu\text{m} \rightarrow$  Hall-pickup (micro)

### Hall effect

If a current carrying conductor placed in a magnetic field experiences electrical field which is perpendicular to both the directions of magnetic field & current.

This hall principle used in Hall-pickup transducer for the measurement of displacement in micrometer range.



## Measurement of Force/pressure/stress/strain

(03)

\*\*\* 1. piezo resistive transducer / strain-gauge / Load cell.

\*\* 2. piezo electric transducer.

### Piezo Resistive Transducer

Relation b/w  $G_f$  &  $\nu$  :-

$$R = \frac{\rho l}{A}$$

change in resistance ( $\Delta R$ ) due to stress ( $S$ ) applied

$$\frac{dR}{dS} = \frac{d}{dS} \left( \frac{\rho l}{A} \right)$$

$$\frac{dR}{dS} = \frac{\rho}{A} \left( \frac{dl}{dS} \right) - \frac{\rho l}{A^2} \left( \frac{dA}{dS} \right) + \frac{l}{A} \left( \frac{d\rho}{dS} \right)$$

multiplying both sides by  $\frac{1}{R} = \frac{A}{\rho l}$

$$\frac{1}{R} \left( \frac{dR}{dS} \right) = \frac{A}{\rho l} \left( \frac{\rho}{A} \frac{dl}{dS} - \frac{\rho l}{A^2} \frac{dA}{dS} + \frac{l}{A} \frac{d\rho}{dS} \right)$$

$$\frac{1}{R} \frac{dR}{dS} = \frac{1}{l} \frac{dl}{dS} - \frac{1}{A} \frac{dA}{dS} + \frac{l}{\rho} \frac{d\rho}{dS}$$

$$A = \frac{\pi D^2}{4} \Rightarrow \frac{1}{A} \frac{dA}{dS} = \frac{4}{\pi D^2} \frac{d}{dS} \left( \frac{\pi D^2}{4} \right) = \frac{2}{D} \frac{dD}{dS}$$

$$\therefore \frac{1}{R} \frac{dR}{dS} = \frac{1}{l} \frac{dl}{dS} - \frac{2}{D} \frac{dD}{dS} + \frac{l}{\rho} \frac{d\rho}{dS}$$

$$\frac{dR}{R} = \frac{dl}{l} - 2 \frac{dD}{D} + \frac{d\rho}{\rho}$$

$$G_f = \frac{dR/R}{d\epsilon/l} = 1 - \frac{2 \frac{dD}{D}}{d\epsilon/l} + \frac{d\rho/\rho}{d\epsilon/l} ; \text{poisson's Ratio } = \nu = - \frac{dD/D}{d\epsilon/l}$$

parameter

1. Longitudinal strain ( $\epsilon_l$ ) =  $\frac{\Delta l}{l}$   
l → length

2. Lateral strain ( $\epsilon_D$ ) =  $\frac{\Delta D}{D}$   
D → diamt

3. stress ( $S$ ) =  $\frac{F}{A}$

4. Young's modulus  $Y = E = \frac{\text{stress}}{\text{strain}} = \frac{S}{\epsilon_l}$

5. poisson's ratio =  $\nu = \frac{\epsilon_D}{\epsilon_l} = - \frac{\Delta D/D}{\Delta l/l}$

6. Gauge factor ( $G_f$ )

$$G_f = \frac{\Delta R/R}{\epsilon_l} = \frac{\Delta R/R}{\Delta l/l}$$

R → resist  
l → length  
D → diamt

$D = \text{dia}$ ,  $r = \text{radius}$

$\nu \rightarrow \text{poisson's ratio}$

$$G_f = \frac{dR/R}{d\epsilon/\epsilon} = 1 + 2 \cdot \left( \frac{d\nu/\nu}{d\epsilon/\epsilon} \right) + \frac{(d\rho/\rho)}{(d\epsilon/\epsilon)}$$

$$G_f = 1 + 2\gamma + \frac{d\rho/\rho}{d\epsilon/\epsilon}$$

Gauge factor  
↓

Material	$G_f$
Manganin	0.5
Nickel	10
Carbon	5
Cobalt	8.5
Semiconductor	$\pm 120$

1. Semiconductor

$$G_f = 1 + 2\gamma + \frac{d\rho/\rho}{d\epsilon/\epsilon}$$

2. Metals

$$\frac{d\rho/\rho}{d\epsilon/\epsilon} = \text{very small} \Rightarrow G_f = 1 + 2\gamma$$

In case of semiconductors by the applied stress the mobility of the carrier change which causes change in the resistivity and hence the gauge factor is very high for semiconductor.

$$(G_f)_{\text{semiconductor}} > (G_f)_{\text{metal}}$$

Q:- A carbon-gauge has  $G_f$  of 5 having a resistance of 120  $\Omega$ . If the applied strain is  $10^{-4}$ ; Find change in resistance, poisson ratio, Lateral strain

Sol:-

$$G_f = \frac{dR/R}{d\epsilon/\epsilon} = 5 \Rightarrow \frac{dR}{R} \left( \frac{1}{10^{-4}} \right) = 5$$

$$dR = 5 \times 10^{-4} \times 120 = 600 \times 10^{-4}$$

$$dR = 0.06 \Omega$$

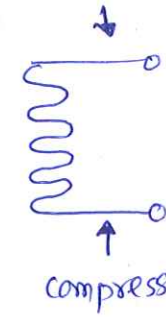
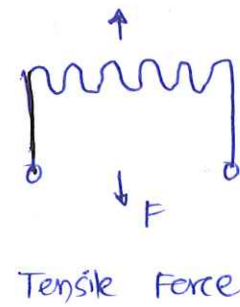
$$\text{poissons ration } (\gamma) ; G_f = 1 + 2\gamma = 5$$

$$2\gamma = 4 \Rightarrow \gamma = 2.$$

$$\text{lateral strain } \left( \frac{\Delta D}{D} \right) \Rightarrow \gamma = - \frac{\Delta D/D}{\Delta \epsilon/\epsilon} \Rightarrow \frac{\Delta D}{D} = \gamma \left( \frac{\Delta \epsilon}{\epsilon} \right) = 2 \times 10^{-4}$$

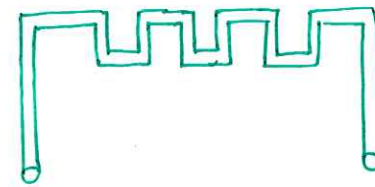
# Construction of strain gauges

## 1. Metal wire

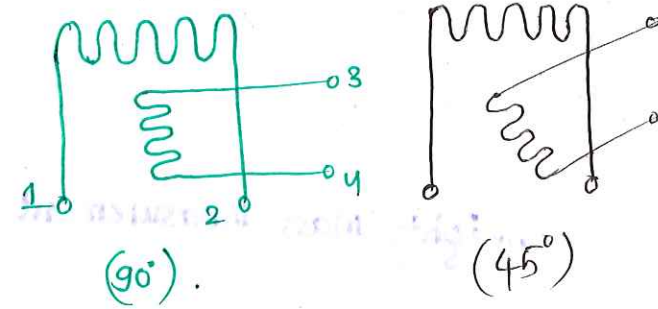


problem with metal wire is if we apply both one one wire... metal expands & breaks  
 ∴ go for metal foil.

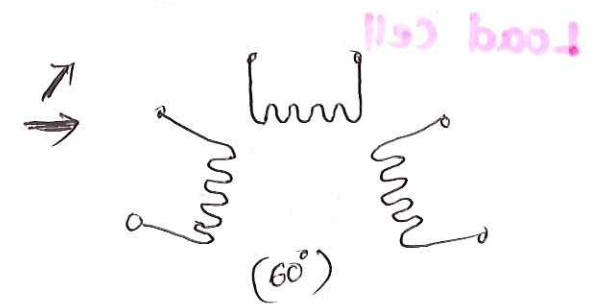
## 2. Metal Foil



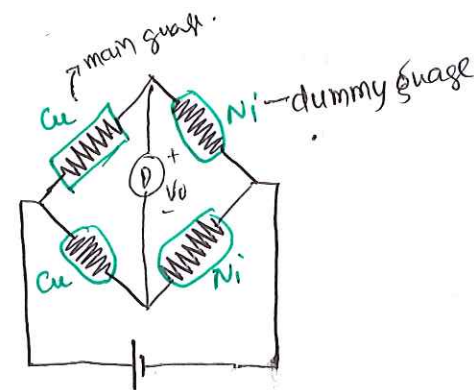
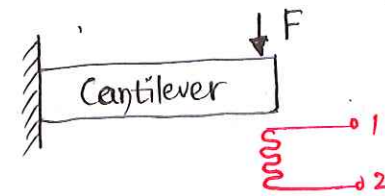
## 3. Rosetts (Rosettes)



Rosetts are preferred if direction of input we are not aware (or) unknown



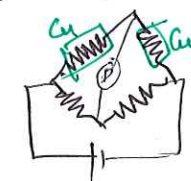
## Practical Arrangement



purpose of Dummy gauge (Ni) is for provision of temperature compensation.

### strain-gauges

1. Quarter Bridge
2. Half Bridge (no need of dummy)

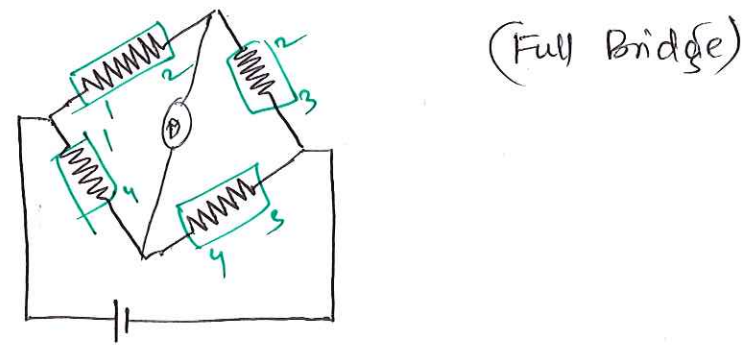
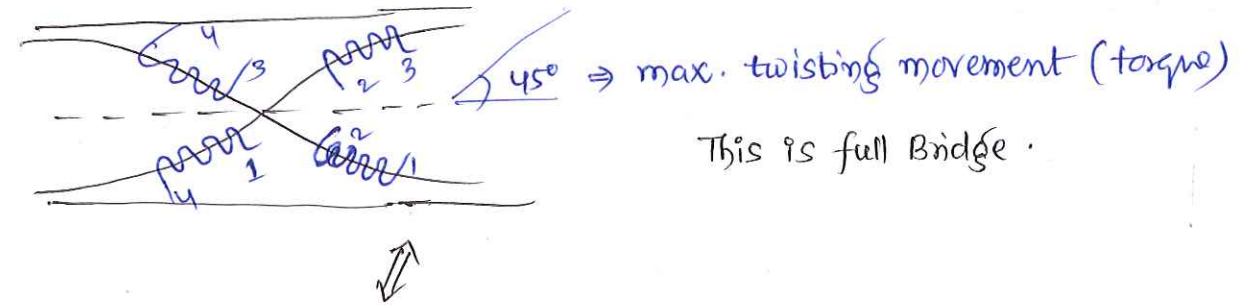


$\frac{2}{4} = \frac{1}{2}$  Bridge.

3. Full Bridge.

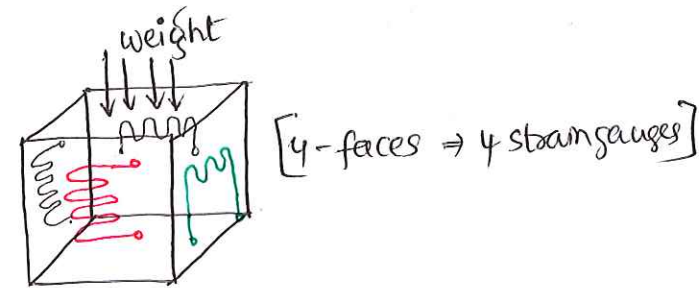


## Torque Measurement :-



## Weight/Mass Measurement :-

### Load Cell :-



In ... train signals  
Red, green,  
⇒ in station platform  
beneath platform ...  
Load cells are used  
⇒ after departure cells  
detect mass ... then  
corresponding colour  
on board.

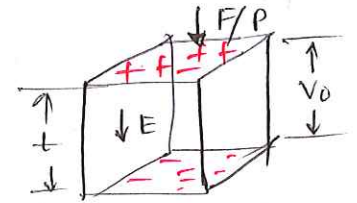
- \* Dummy gauge is used in the quarter bridge for compensation of temperature change.
- \* For measurement of torque strain gauges are kept at 45° position w.r.t. shaft axis and arranged in the full bridge format. Full bridges have 4 times more sensitivity than the quarter bridge.
- \* Load cell is used for measurement of weight by arranging in the full bridge format and the opp voltage converted into digital using ADC, which is displayed on 7-segment display unit.

# Piezo Electric Transducers

(Lithium Sulphate)

materials  $\rightarrow$  Quartz, Rochelle salt,  $BaTiO_3$ ,  $LiSO_4$ ,  $PbZrSO_4$  (Lead Zirconate)

$$\left\{ \begin{array}{l} \text{Force} \\ \text{pressure} \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \text{Electric field} \\ \text{voltage} \\ \text{produced} \end{array} \right\} \Rightarrow V_o \text{ (o/p)}$$



## parameters

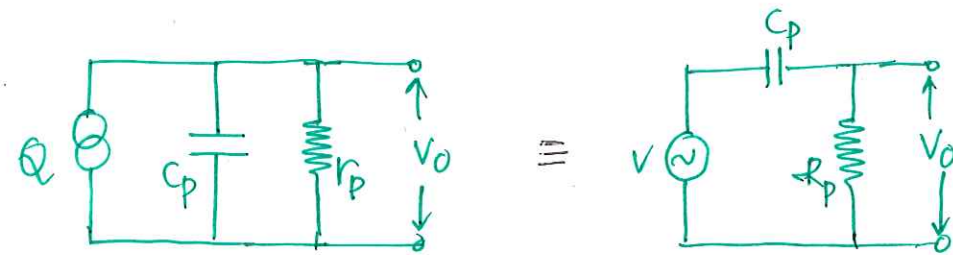
charge sensitivity =  $d = \frac{Q}{F}$  (C/N)

voltage sensitivity =  $\xi = \frac{E}{P} = \left( \frac{V \cdot m^2}{m \times N} \right) = \left( \frac{Vm}{N} \right)$

capacitance of crystal =  $C_p = \frac{\epsilon_0 \epsilon_r A}{t}$

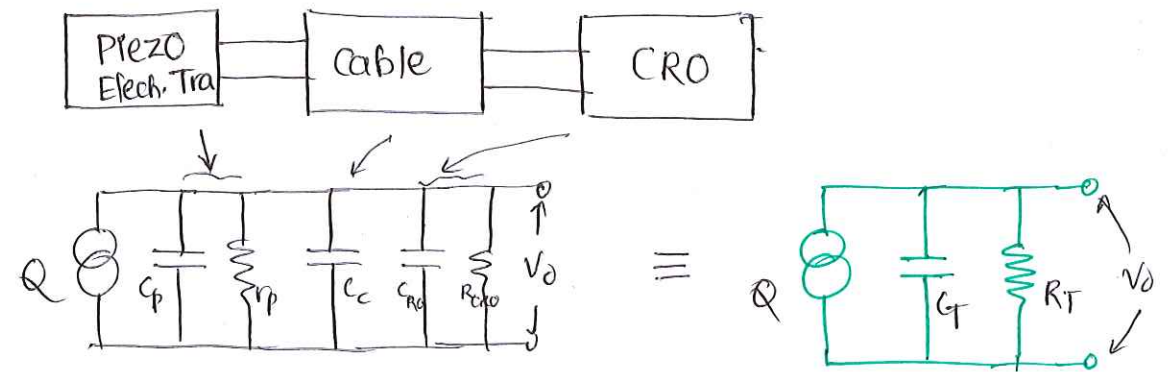
voltage =  $V_o = Et \Rightarrow \xi = \frac{E}{P} = \frac{V_o}{tP} \Rightarrow V_o = t\xi P \Rightarrow V_o \propto P$   
 $V_o = t\xi \left( \frac{F}{A} \right) \Rightarrow V_o \propto F$

## Electrical equivalent circuit :-



$$C_p = \frac{Q}{V} \Rightarrow V = \frac{Q}{C_p}$$

## practical arrangement :-



$$C_T = C_p + C_c + C_{CRO}$$

$$R_T = R_p \parallel R_{CRO}$$

Relation b/w  $g$  &  $d$  :-

$d = \text{charge sensitivity} = \frac{Q}{F}$

$g = \text{voltage sensitivity} = \frac{E}{P}$

$d = \frac{Q}{F} = \frac{C_p V_0}{F} = \frac{\left(\frac{\epsilon_0 \epsilon_r A}{t}\right) V_0}{P.A} = \frac{\epsilon_0 \epsilon_r V_0}{t P}$

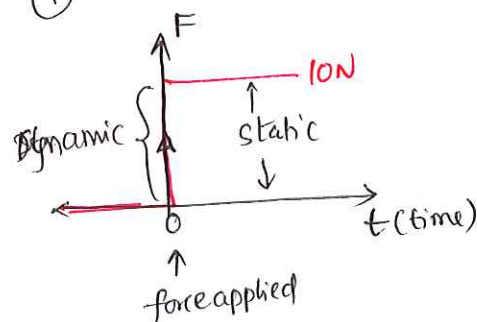
$d = \frac{\epsilon_r \epsilon_0}{t} \left(\frac{Et}{P}\right) ; \therefore V_0 = Et.$

$d = \epsilon_0 \epsilon_r \left(\frac{E}{P}\right)$

$d = \epsilon_0 \epsilon_r (g)$

$\frac{d}{g} = \epsilon = \text{permittivity of material} = \frac{\text{charge sensitivity}}{\text{voltage sensitivity}} = \epsilon_0 \epsilon_r$

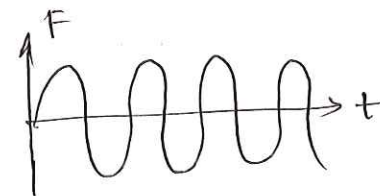
①



at;  $t=0 \Rightarrow V_0 \neq 0$

$t > 0$   
 $t < 0$  }  $\Rightarrow V_0 = 0$

②



at  $t \geq 0 \Rightarrow V_0 \neq 0$

( $\therefore F$  variable  $\Rightarrow C_p$  is not zero)

$\therefore$  From above ① & ② discussion point piezoelectrical transducer is suitable for any **Dynamic values of forces**, not for static forces.



Piezo-electric Transducers converts mechanical energy into electrical & vice-versa. By the application of electrical energy mechanical deformation occurs. Hence, it is used for the production of Ultrasonic waves.

It is used for measurement of Dynamic parameter variations  
 Eg... Acceleration  
Vibrations  
Variable force & pressure.

In case of static parameter, the charges on the surface of the crystal are discharged... & produces  $V_0 = 0$ .

Q:- A quartz piezoelectric transducer  $0.5 \text{ cm}^2$  area,  $1 \text{ mm}$  thickness has a charge sensitivity of  $2 \text{ pC/N}$ , is applied with a force of  $30 \times 10^{-3} \sin(150t) \text{ N}$ . Find peak-to-peak voltage swing at the o/p of the crystal.

Sol:-

$$d = 2 \text{ pC/N}; \quad A = 0.5 \text{ cm}^2; \quad t = 1 \text{ mm};$$

$$\frac{d}{\epsilon} = \epsilon_0 \epsilon_r; \quad C = \frac{\epsilon_0 \epsilon_r A}{t} = \epsilon_0 \epsilon_r \left( \frac{0.5 \times 10^{-4}}{10^{-3}} \right) = 0.05 \epsilon_0 \epsilon_r$$

$$F_{(\text{peak to peak})} = 2 \times 30 \times 10^{-3} = 60 \times 10^{-3} \text{ N};$$

$$\epsilon = \text{voltage sensitivity} = \frac{V}{P}; \quad V_0 = \epsilon t P = \frac{d}{\epsilon_0 \epsilon_r} t \cdot \frac{F}{A}$$

$$d \times F_{pp} = (\epsilon_0 \epsilon_r) \left( \frac{E}{P} \right) = \epsilon_0 \epsilon_r \frac{V_0/t}{F/A} = \frac{\epsilon_0 \epsilon_r A}{t} \left( \frac{V_0}{F} \right)$$

$$2 \times 60 \times 10^{-3} = (0.05 \epsilon_0 \epsilon_r) \cdot \frac{V_0}{30 \times 10^{-3} \sin(150t)}; \quad \epsilon_r = 1$$

$$V_0 = 0.1357 \sin(150t) \times 2 = 0.2714 \text{ V}$$

Q:- The o/p of an LVDT is connected to a 5V voltmeter to an amplifier having an amplification factor of 250, the o/p of 2m volts appears across the terminals of LVDT when the core moves to a distance of 0.5mm. Calculate

- (i) sensitivity of LVDT
- (ii) sensitivity of the whole setup
- (iii) The milli voltmeter scale has 100 divisions and can be read upto  $\frac{1}{2}$ th of the division. Calculate the resolution of the instrument in mm.

Sol:-

# Pressure Measurement

(i) pressure > atm. pressure

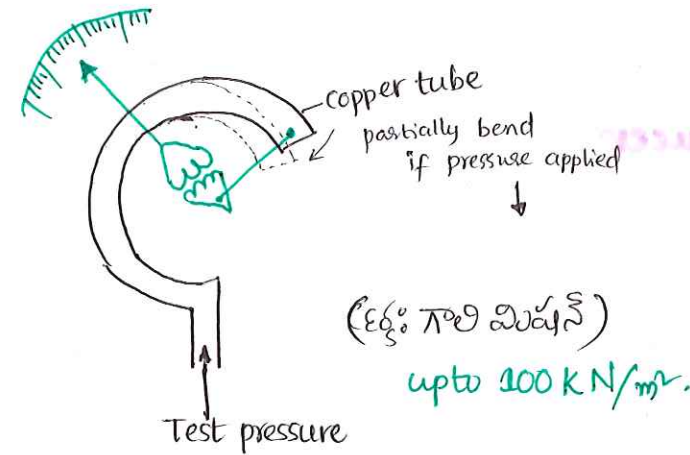
mechanical gauges :-

1. Bourdon Tube
2. Diaphragm
3. Bellows
4. Manometer

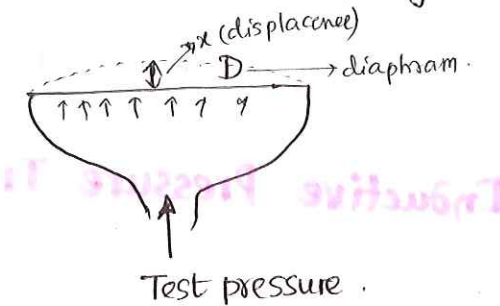
Electrical transducers :-

1. Inductive pr. Transducer
  2. Capacitive "
  3. photo electric "
  4. piezo resistive "
  5. piezo electric "
- } works on principle of diaphragm.

## Bourdon Tube / C-Tube

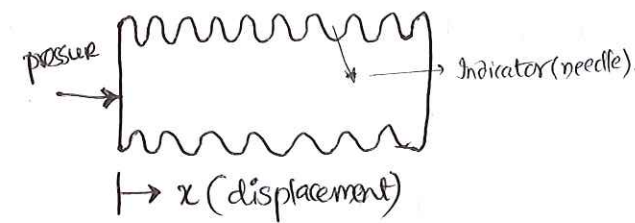


## Diaphragm (D) works as primary transducer.



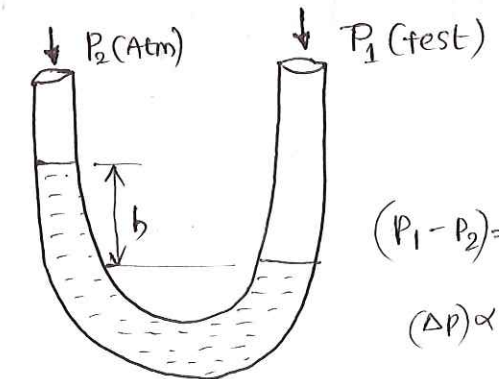
- > diaphragm should not react with chemicals
- > unwanted vibrations produces ... errors
- > high sensitive gauges.

## Bellows



Eg: kerosine sucking pump.

## Manometer



$$(P_1 - P_2) = \rho g b$$

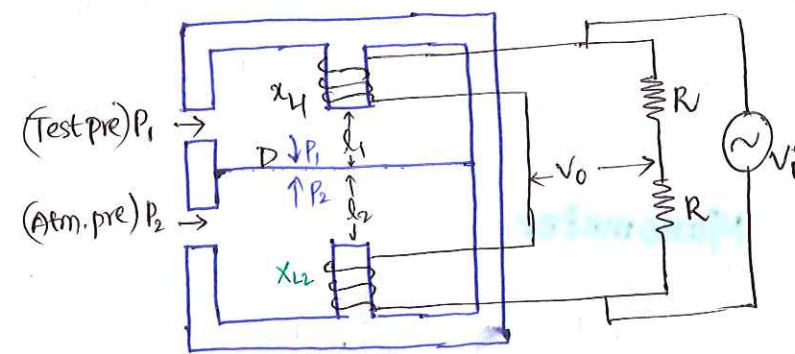
$$(\Delta P) \propto b$$

-> Used for calibration of other instruments.



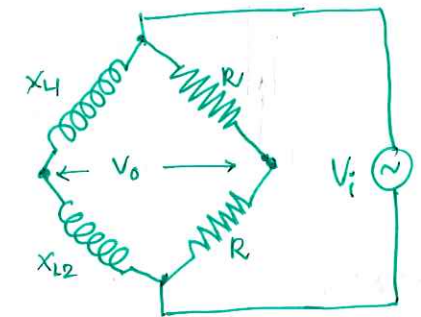
- \* Bourdon Tube is most practically used pressure sensing device. It measures upto  $100 \text{ kN/m}^2$
- \* Diaphragm & Bellow are used as a primary transducer (direct contact) which produces displacement for the applied pressure and this displacement is converted in terms of voltage using secondary transducers
- \* Diaphragm has more sensitivity & high accuracy. The material of the diaphragm should be selected such that it should not be interacted with the chemical whose pressure has to be measured.
- \* In the presence of vibrations the diaphragm produces errors.
- \* Manometer accuracy is very high & hence it is used for calibration of other pressure sensing devices.

### Inductive Pressure Transducer



D → diaphragm.

$X_L$  (coil inductance +  $l$  (air gap reactance)).



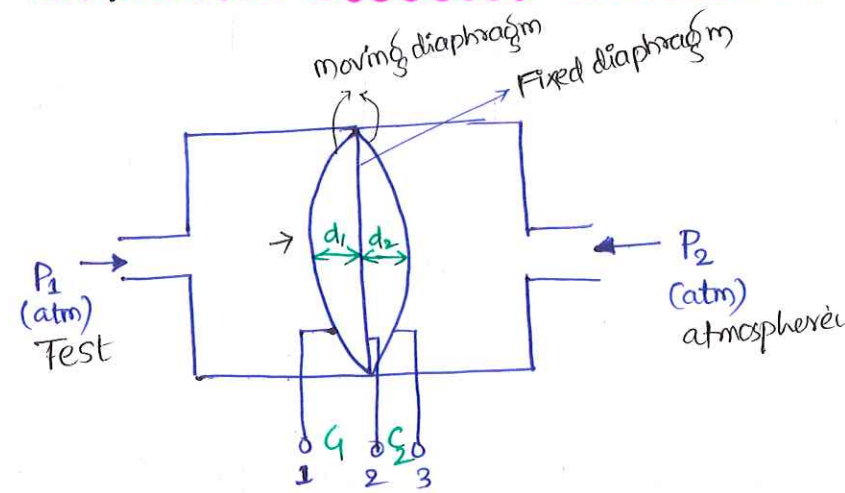
- If  $P_1 = P_2$ ;  $X_{L1} = X_{L2} \Rightarrow V_o = 0$   
( $l_1 = l_2$ )
- If  $P_1 \neq P_2$ ;  $X_{L1} \neq X_{L2} \Rightarrow V_o \neq 0$ .  
 $P_1 > P_2 \Rightarrow l_1 > l_2$

Bulky in size (due to inductance-coil)

By the difference of  $P_1$  &  $P_2$ , a diaphragm produces displacement so that inductance of the two coils ( $X_{L1} \neq X_{L2}$ ) not equal. causing the bridge-imbalance & output voltage is calibrated in terms of test pressure.

It measures both static & dynamic pressure, The size of the transducer is higher, It is affected by external magnetic fields (12)

## Capacitive Pressure Transducer :-



- ① If  $P_1 = P_2$ ,  $X_{c1} = X_{c2} \Rightarrow V_0 = 0$
- ② If  $P_1 \neq P_2$ ,  $X_{c1} \neq X_{c2} \Rightarrow V_0 \neq 0$

Electrostatic energy

$$T_d = \frac{V^2}{2} \frac{dc}{d\theta} \Rightarrow \theta \rightarrow \text{angle displacement}$$

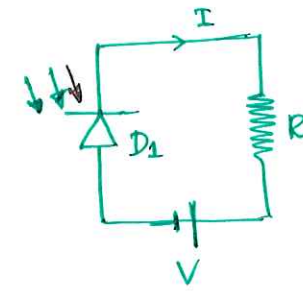
$$F = \frac{V^2}{2} \frac{dc}{dx} \Rightarrow \text{Linear displacement (x)}$$

If  $P_1 \neq P_2$  the diaphragms distances ( $d_1 \neq d_2$ ) not equal which causes unequal capacitances  $C_1$  &  $C_2$ , produces unbalance in the bridge ckt. The o/p voltage is calibrated in terms of test pressure.

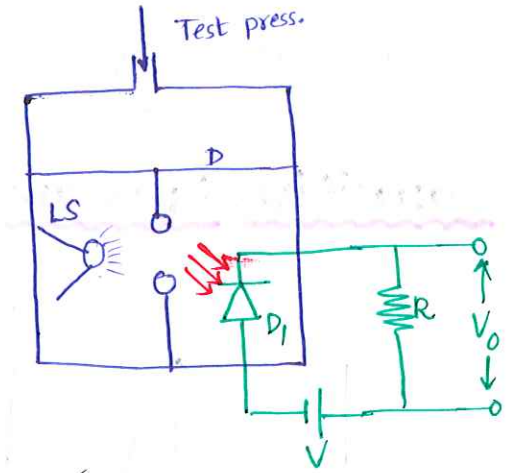
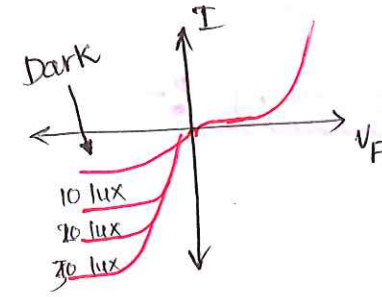
The size of the transducer is very small. It is most widely used in the industry for measurement of both static & dynamic pressures. Sensitivity is very high.

It is affected by chemicals & electrostatic fields. In the presence of vibrations produces error.

# Photo electric Pressure Transducer



$D_1 =$  photo diode (RB only)



LS  $\Rightarrow$  Light source

W  $\Rightarrow$  window

D  $\Rightarrow$  diaphragm,  $D_1 \Rightarrow$  diode.

By the application of test pressure, the diaphragm produces deflections so that window is partially closed depending on test pressure & hence light intensity on the photo diode is reduced so that output voltage across the resistor is changed, which is a function of the test pressure.

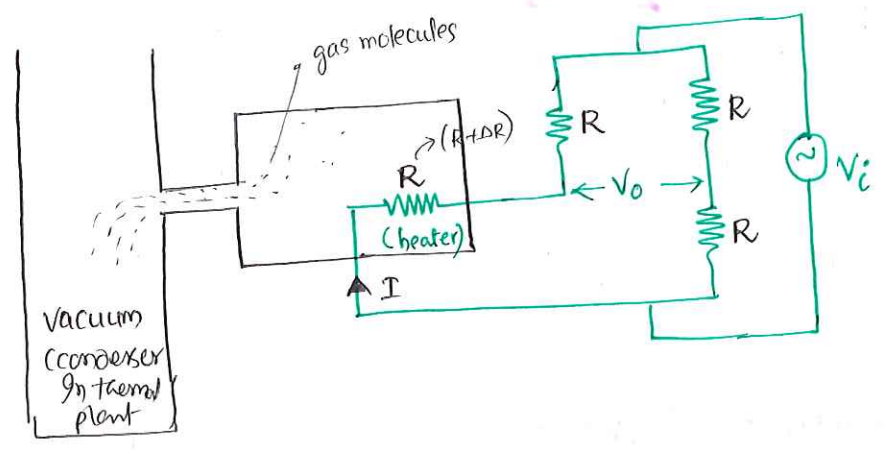
2.  $(P_{test}) < (\text{Atmospheric Pressure})_{(or)} (\text{vacuum})$ .

1. Pirani vacuum gauge.
2. Thermistor " "
3. Thermocouple " "
4. Ionization " "
5. McLeod " "
6. Knudsen " "
7. Alphatron " "

(In PS, plant, in condenser we need to maintain vacuum i.e. atm=1).



1. Pirani vacuum Gauge :- ( $\rightarrow$  upto  $+10^3$  mm of Hg)



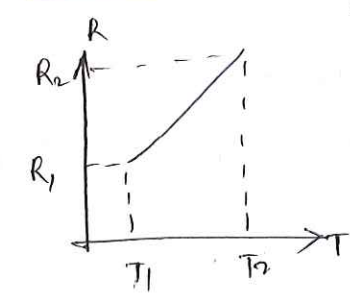
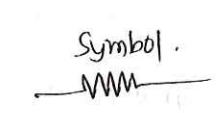
principle :- Thermal conductivity of gas molecules is a function of vacuum (or) low pressure. This principle is used in the pirani thermister, thermocouple & ionization vacuum gauges.

In the presence of vacuum the gas molecules inside the glass chamber are absorbed by the vacuum, so that the cooling medium is decreased which causes increase in temperature of the heater so that resistance of the heater increases causing the unbalance in the bridge ckt.

The output voltage of the bridge is calculated in terms of vacuum under measurement.

In Pirani gauge, the heater is made of **platinum/tungsten**. It measures upto  $10^3$  mm of Hg.

Resistor

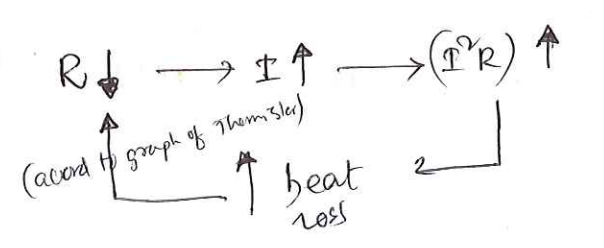


$$R_2 = R_1 (1 + \alpha(T_2 - T_1))$$

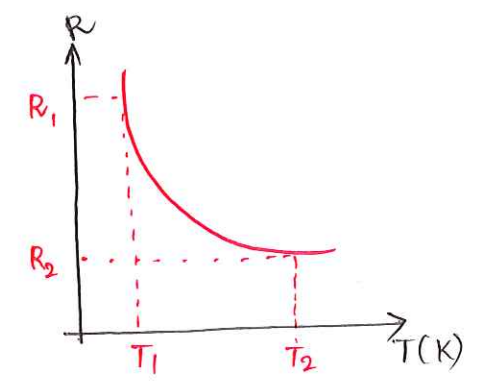
Thermister



$\rightarrow$  self-heating property



Thermister :- (all p.m. or - atq) :-



$$R_2 = R_1 e^{\beta \left( \frac{1}{T_2} - \frac{1}{T_1} \right)}$$

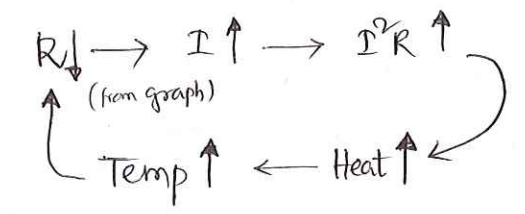
$\beta = \text{constant } (3000 - 4000)^\circ\text{K}$

$T_1, T_2 \Rightarrow \text{temperatures in } ^\circ\text{K}$

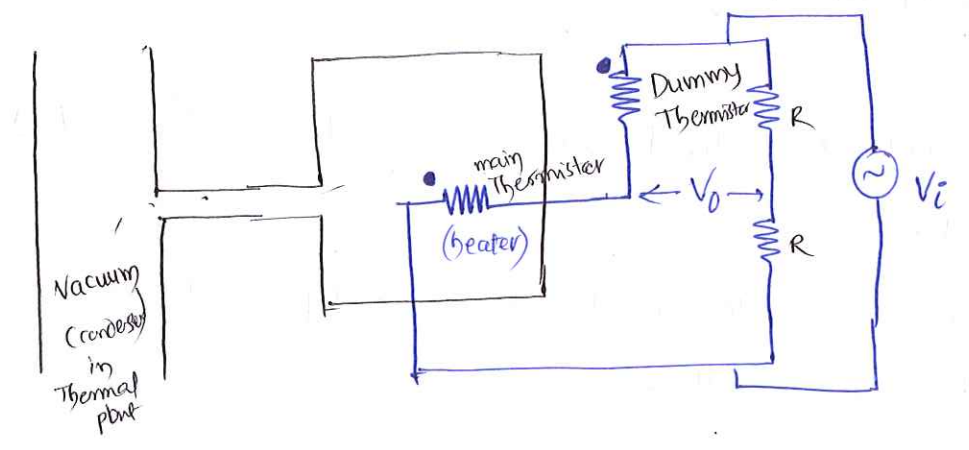
for  $\pm 100^\circ\text{C} \rightarrow (50\Omega \text{ to } 0.75\text{M}\Omega)$   
resistance change.

Self heating property of Thermister

— This is used in various measurement except temperature  
eg: flowrate, flow.



Thermister Vacuum Gauge :- ( $10^{-4}$  mm of Hg)



Thermister is made of metal oxides ( $\text{Mn, Ni, Co, Fe, Cu, W}$ )

The size of the thermister is very small in millimeter range.  
Hence, used in very small areas where other instruments are not suitable. It has negative temperature coefficient, the sensitivity of thermister is high. Hence used for measurement of small changes in the data.

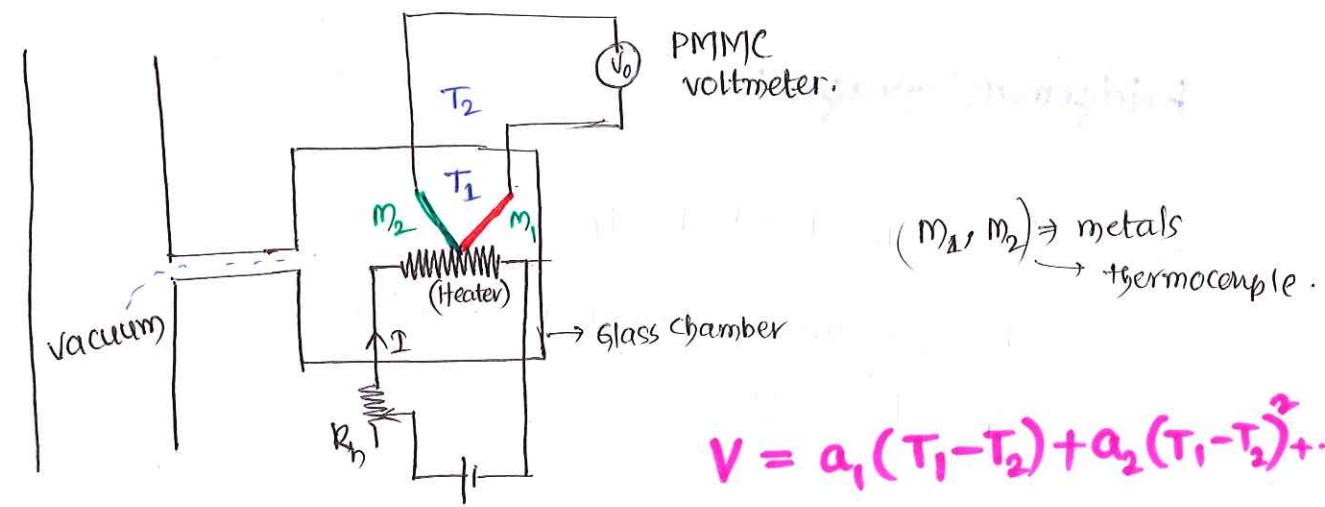
\* By using self-heating property non-electrical quantities other than temperature like vacuum liquid level, liquid flow (114)

liquid level  
liquid flow  
Thermal conductivity } measured by Thermister.

\* In the vacuum measurement because of higher sensitivity it measures upto  $10^{-4}$  mm of Hg.

\* In the measurement of temperature, self heating property is compensated by connecting a resistor across the thermister.

Thermocouple Vacuum Gauge :- ( $10^{-2}$  mm of Hg)



$$V = a_1(T_1 - T_2) + a_2(T_1 - T_2)^2 + \dots$$

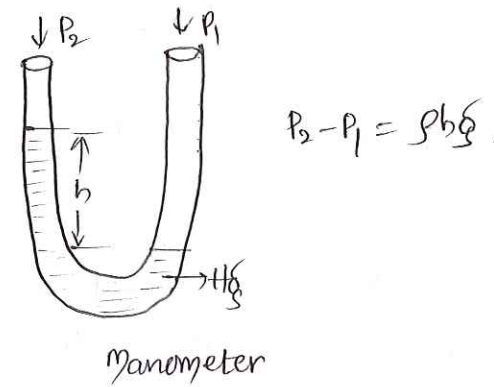
Thermocouple works on the principle of Seebeck Effect. In the presence of vacuum the gas molecules inside the gas chamber are absorbed by the vacuum causing increase in temp ( $T_1$ ) so that thermocouple output voltage is changed which is function of vacuum. This DC voltage is measured by PMMC voltmeter which is calibrated in terms of vacuum. It measures upto  $10^{-2}$  mm of Hg.



### \*\*\* McLeod Gauge :- ( $10^{-8}$ mm Hg)

Highest accurate  
Absolute meter.

\* McLeod  
Knudsen } gauges are working on  
Alphatron } the principle of Manometer



\* The accuracy of the instrument is very high measures pressure upto  $10^{-8}$  mm of Hg.

\* These instruments are used for calibrating pirani gauge, thermocouple, thermister & ionization vacuum gauges.

### ③ Very High pressure ( $> 1000$ kg/cm<sup>2</sup>)

#### Bridgeman Gauge :-

$$R_2 = R_1 (1 + b(P_2 - P_1))$$

$b \Rightarrow$  pressure coefficient of resistance.

$P_2, P_1 \Rightarrow$  pressures.

By the application of very high pressure, the materials like gold, manganin internal atomic structure changes causing change in the resistance. This principle is used in the Bridgeman gauge for measurement of very high pressure more than  $1000$  kg/cm<sup>2</sup>.

It can measure up to  $10^5$  kg/cm<sup>2</sup>,  $1 \text{ bar} = \text{kg/cm}^2$

# Flow measurement :-

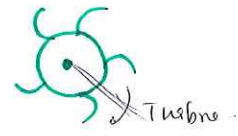
## Mechanical

1. Orifice plate
2. Venturi Tube
3. Rotameter
4. Vanes

## Electrical Transducers

1. Hot-wire Anemometer
2. Thermistor Flow meter
3. Turbine "
4. Electromagnetic "

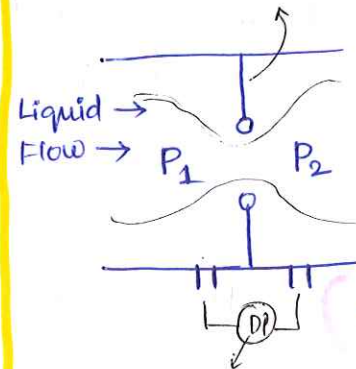
## Vanes



→ used for wind flow & rate of wind  
→ connected to counter

Wind flow

## Orifice plate :-

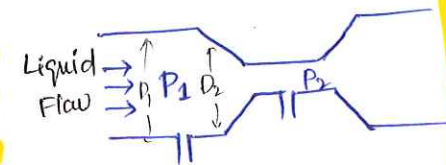


Differential Pressure meter.

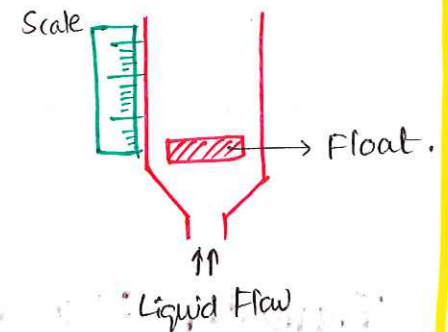
Large flow-rate measurement

Eg: canal, dams, river flow.

## Venturi Tube



## Rotameter



Laboratory purpose

## principle

If any obstruction is placed in the path of the liquid then the pressure difference occur before & after the object (obstruction); This pressure difference is a function of liquid flow-rate. This principle is used in the Orifice plate, venturi tube & rotameter.

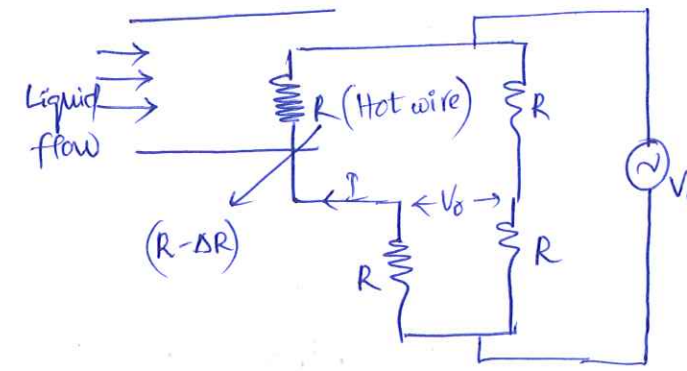
Orifice plate & venturi tube are used to measure liquid flow in open-canals, Large diameter pipes & open-channels, rivers etc.

Rotameter used in Labs for measurement of variable liquid flow.

Vanes are working similar to the turbine, used to measure wind flow.



## Hotwire Anemometer

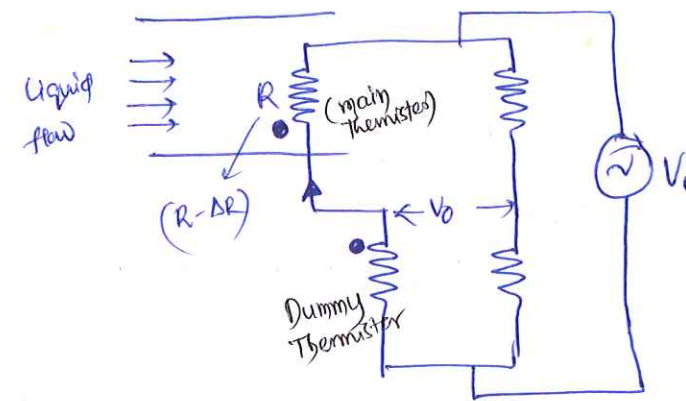


If liquid is flowing over a heated surface, the surface temperature is changed due to absorption of the heat by the liquid. This causes change in the resistance of the hot-wire.

The hot wire is made of **Tungsten/Platinum**.

In presence of liquid resistance will change & causing unbalance in Bridge ckt & o/p voltage is calibrated in terms of liquid flow. It is mostly used in Industry for measurement of **both static & dynamic liquid flow**.

## Thermistor Flow-meter (upto $10^{-3}$ m/sec) ∴ High sensitive.



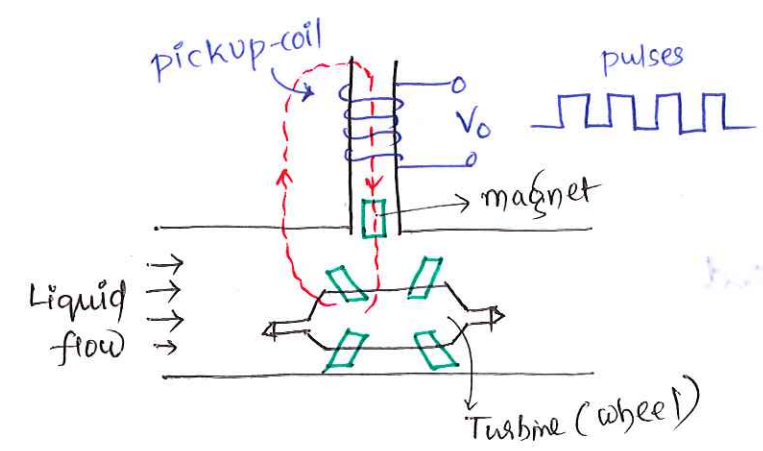
By using **Self heating** property the thermistor measures both **static & dynamic liquid flow**.

The sensitivity is very high so this measure very small liquid flow upto  $10^{-3}$  m/sec.

A dummy thermistor is used for compensation of atmospheric temperature changes.



## Turbine Flow-meter



- ① If Liquid flow = 0  
Turbine will not rotate  
no change of flux  
&  $V_o = 0$
- ② If Liquid flow  $\neq 0$   
turbine rotates through wheels.  
 $V_o \neq 0$  ( $\because$  change of flux)

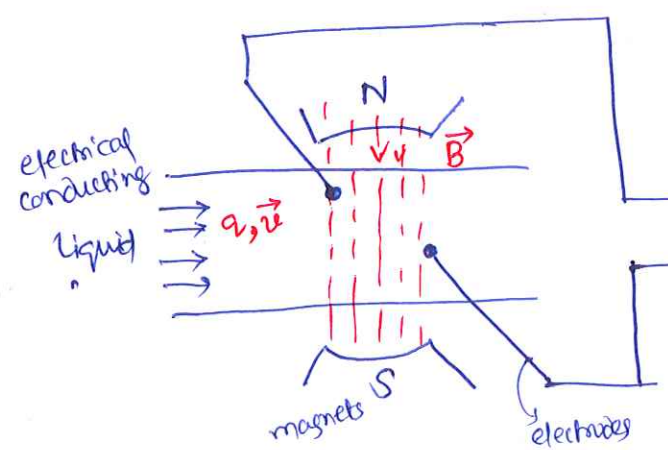
Ex: Petrol Bunk (gun - )

In the presence of liquid the turbine wheel rotates causing rate of change of flux of the permanent magnet. This will induce emf in the pickup coil in the form of pulses. These pulses are applied to a digital counter. Displayed in the 7 segment display unit.

**Application :-** Used to measure low-viscosity & high velocity liquid flow. Ex: Petro-chemical industries.

## Electromagnetic Flowmeter

- High viscos & Low-velocity liquids
- for only electrical conducting liquid



$$V_o \propto Bqv \Rightarrow V_o \propto v, V_o \propto (\text{flow-speed})$$

( $\because B, q = \text{constant}$ )

If an electrically conducting liquid having a charge of 'q' moving with velocity  $v$ , in an magnetic field an emf is induced across the electrodes which is the function of velocity of liquid.

This flow-meter is used for measurement of high viscosity & low velocity liquid flow & It is depending on density of the liquid.

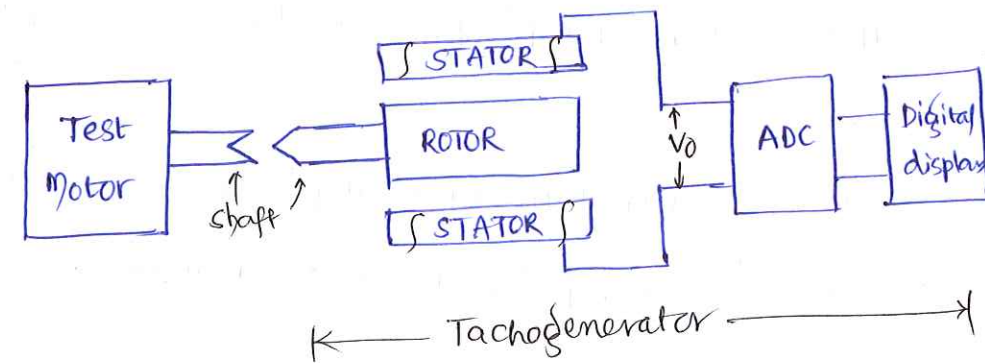
Ex:- paint, slurry, sludges liquid flow measurements.

### Measurement of Angular Speed

1. Tacho generator
2. Inductive Reluctance Tachometer.
3. Photo Electric tachometer
4. Stroboscope.

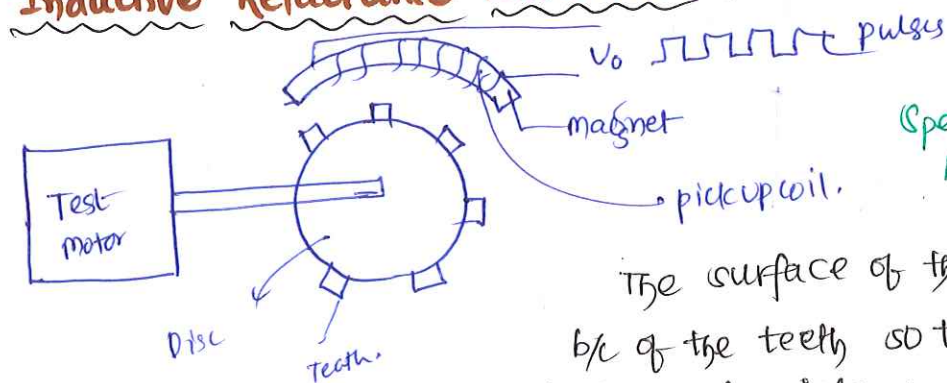
(1,4) - Most practically used  
(2,3) - out dated.

### Tachogenerator :-



Tachogenerator works on the principle of generator. It is widely used in the laboratory for the measurement of both static & dynamic speed measurement of electrical motors and generators.

### Inductive Reluctance Tachometer :-

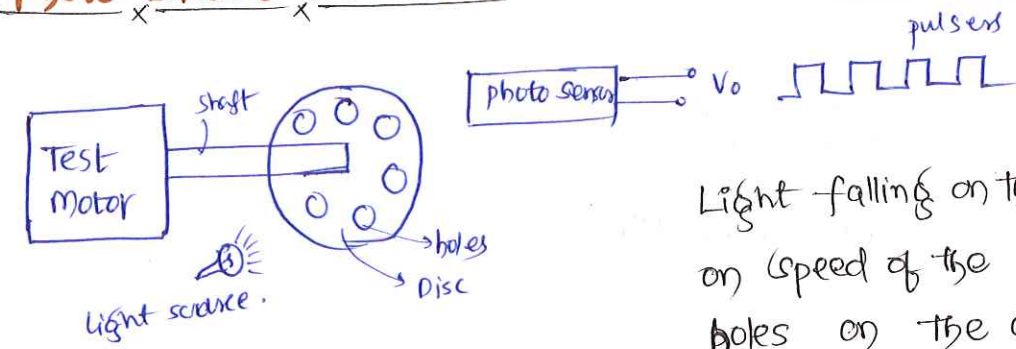


$$\text{Speed in RPM} = \frac{\text{no. of pulses/sec} \times 60}{\text{no. of Teeth}}$$

The surface of the disc is not uniform b/c of the teeth, so that reluctance is different at different points of the surface. This produces rate of change of flux which induces an emf in the pickup coil in the form of pulses. These pulses are applied to the digital counter which is calibrated in terms of RPM.

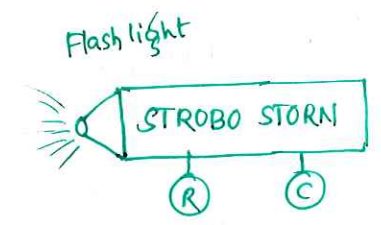
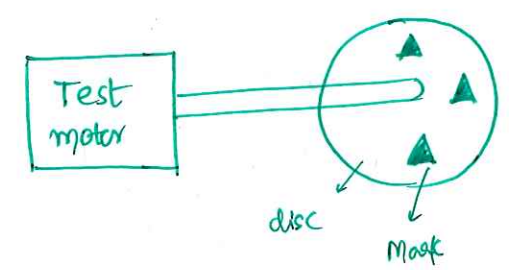


## Photo Electric Tachometer :-



Light falling on the photo sensor depends on speed of the test motor, and no. of holes on the disc. These holes are fixed so that light intensity is depends on the speed of the motor it produces pulses of o/p of photosensor; These pulses are applied to a digital counter, it is calibrated in terms of RPM. This type of arrangement is used in research laboratories for measuring static & dynamic speeds of test motor which are controlled by different controllers.

## Strobo storn :-



$$F = \text{no. of flashes / sec}$$

$$n = \text{no. of marks on disc}$$

R = No. of Flashes reactor.  
C = Flash light controller.

$$\text{speed in RPM} = N = \frac{F}{n}$$

strobostorn is working on the principle of stroboscopic effect. This device produce flash light and if flashing rate is equal to the speed of the test motor then the no. of markings on the disc looks stationary at this time, reader (R) records no. of flashes from this speed is calculated. It is a **non-contacting type** speed measuring device so that accuracy is very high it measures speed upto **20,000 rpm**. It is most widely used in the industrial application for measurement of both **static & dynamic speeds**



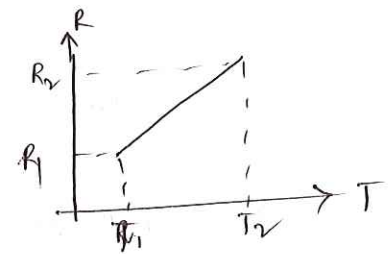
# Temperature Measurement



- (i) RTD
  - (ii) Thermister
  - (iii) Thermocouple
  - (iv) Bimetallic strip
- (i) pyrometer

1. RTD / Thermometer ↙ Liquid (Hg) - upto 400°C  
↘ solid - upto 900°C

(Resistance Temperature Detector)



$$R_2 = R_1 (1 + \alpha (T_2 - T_1))$$

Material	$\alpha$ ( $^{\circ}\text{C}^{-1}$ )	Temp. range	Relative cost
platinum	0.0039	-250 to 900	150
Nickel	0.0066	-100 to 150	10
Copper	0.0045	-125 to 250	1

motors, T<sub>s</sub>-line, T<sub>s</sub>-F  $\Rightarrow$  2nd order system.

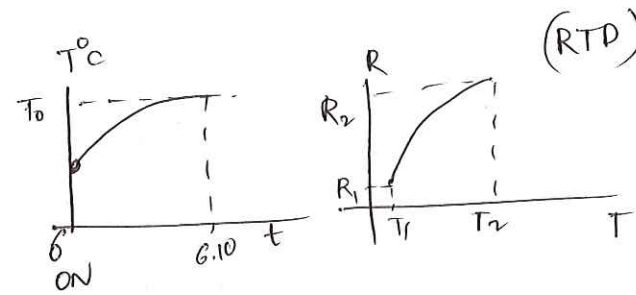
ORDER of system  $\rightarrow$  should be matched for measurement.

(a) order of instrument

For measuring certain unknown quantities like current, temp, power, voltage .. etc the measuring equipment must behave similar to the quantity to be measured .. this is called order of the instrument.

Eg:- The electrical equipments like motors, generators, TFs and T<sub>s</sub>-lines containing RLC parameters ... so that behaving as a second order system for measuring current, voltage, power etc from these devices, second order instruments has to be used. i.e. (A), (V), (W) - all second order devices.

Variation of temperature w.r.t. time is of 1st order behaviour for measuring this change of temperature RTD has to be used which is of first order instrument. (118)

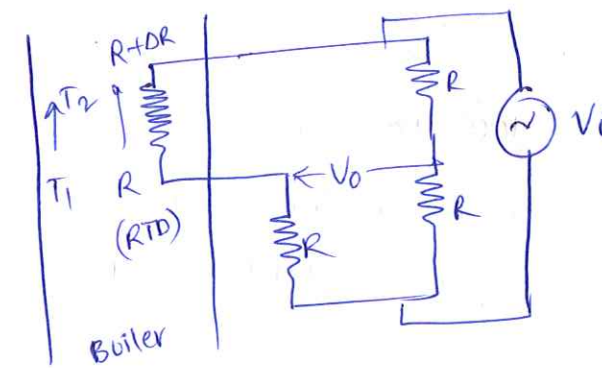


RTD  $\Rightarrow$  1st Order instrument  
 $\rightarrow$  temperature measurement  
 ( $\because$  temp. variable is 1st order).

### Application of RTD

- Used to measure temperature of electrical heaters, turbines, micro-ovens, small-size boilers.

### practical arrangement:-



Q:- A platinum RTD has a resistance of  $100\ \Omega$  at  $20^\circ\text{C}$  is arranged in the bridge format for measurement of temperature of the boiler. If the temperature is changed to  $200^\circ\text{C}$  assume  $\alpha = 0.004^\circ\text{C}$  find the o/p voltage.

Sol:-

above ckt ;  $R = 100\ \Omega$  ;  $V_i = 5\text{V}$  ;  $\alpha = 0.004^\circ\text{C}$

$$R_2 = 100 (1 + 0.004 \times (200 - 20)) = 100 (1 + 4 \times 10^{-3} \times 180)$$

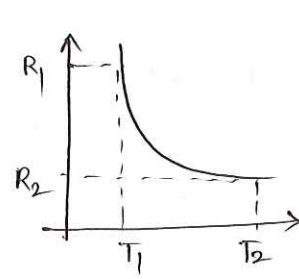
$$= 100 (1.72)$$

$$= 172\ \Omega$$

$$V_0 = V_i \left( \frac{100}{100 + 172} - \frac{100}{100 + 100} \right) = -0.66\text{V}$$

## Thermister :- (-100°C to 300°C)

- \* has negative temperature coefficient of resistance



$$R_2 = R_1 e^{\beta \left( \frac{1}{T_1} - \frac{1}{T_2} \right)}$$

$$\beta = 3000 - 4500^\circ \text{K}$$

$$T_1, T_2 \Rightarrow ^\circ \text{K}$$

sensitivity (S)

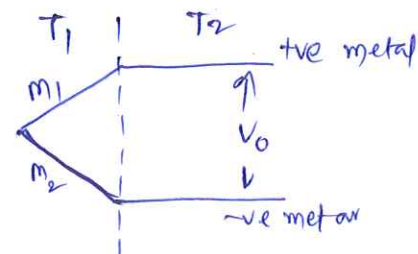
$$S = \frac{\Delta R/R_2}{\Delta T} = \frac{-\beta}{T_2^2}$$

$$\Delta R = R_1 - R_2$$

$$\Delta T = T_2 - T_1$$

- \* The size of the thermister is very small in millimeter range and hence it is used for measurement of temperature where thermocouple & RTD are not suitable.
- \* The variation of resistance is very large in the mega ohm range and hence the sensitivity is high.
- \* It measures temperature b/w (-100°C to 300°C)
- \* It is used as a time delay unit in the electronic circuits.
- \* Used as Temperature compensation in the BJT so that <sup>useful</sup> for stabilization of Q-point.
- \* Used in electrical heaters & electronic equipment for temperature control.
- \* By using self-heating property non-electrical quantities like liquid flow, liquid level, thermal conductivity, vacuum pressure, etc. measured
- \* In the temperature measurement the self heating property is compensated by connecting a resistor across the thermister.

## Thermo couple :-



$$V_0 = a_1(T_1 - T_2) + a_2(T_1 - T_2)^2 + \dots \text{upto } 9^{\text{th}} \text{ degree}$$

$T_1$  = hot junction temperature

$T_2$  = cold/ref. junction.



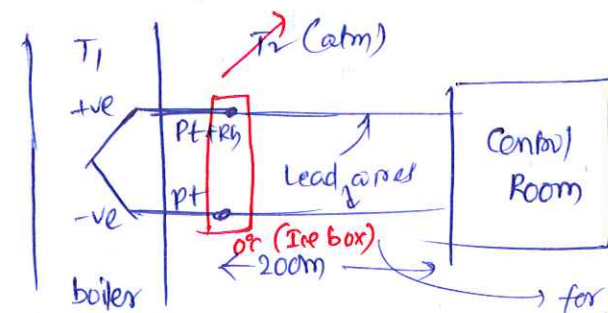
Type	+ve metal	-ve metal	temperature Range (°C)	voltage swing (mV)	Lead wires	
					+ve	-ve
K	chromel	Alumel	-200 to 1300°C	55	Iron	Cu-Ni
T	Cu	Constantan	-200 to 300°C	25	Cu	Constantan
J	Iron	Constantan	-150 to 1000	50	Iron	Constantan
S	Pt + 10% Rh	Pt	0 to 1500	15	Cu	Cu-Ni
R	Pt + 13% Rh	Pt	0 to 1600	20	Cu	Cu-Ni
B	Pt + 30% Rh	Pt + 6% Rh	30 to 1800*	13.5*	Cu	Cu-Ni
E	chromel	constantan	0 to 1000	75	Iron	Constantan

constantan → 55% Cu + 45% Ni  
 chromel → 90% Ni + 10% Chromium  
 Alumel → 94% Ni + 3% Mn + 2% Al + 1% Si  
 platinum (Pt) → Pt  
 Rh → Rhodium

high sensitive - E type  
 temp. range - B type

- \* Thermocouple works on the principle of **Seebeck effect**.
- \* Type B thermocouple measures highest temperature upto 1800°C
- \* It produces very low output voltages so that sensitivity is very small and stability is very high over wide range of temperature.
- \* Type 'E' has higher sensitivity b/c it produces max. o/p voltage
- \* For reducing the cost of the measuring system the thermocouple metals are terminated with the lead-wires b/c the cost of lead-wires is less.

Application :- Thermocouples are used for measurement of higher temperatures of electrical boilers of higher capacity and in nuclear power stations for measurement of reactor temperature and used to measure turbine temperature. For measuring the temperature at a specific point in an area.



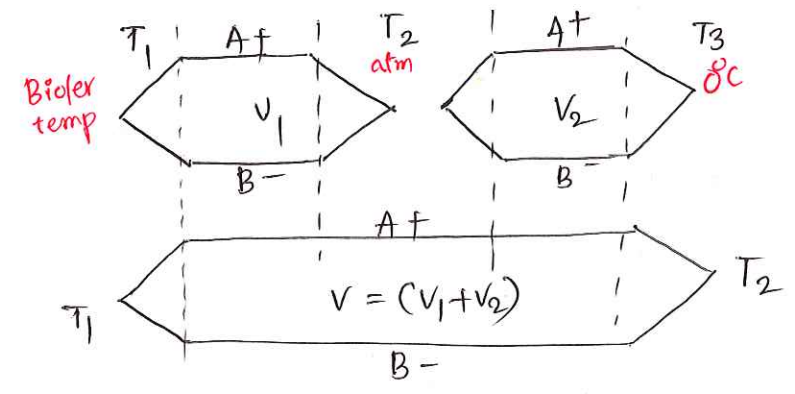
for one thermo couple (for 100s of thermocouples Ice box maintenance is difficult).

$$V_0 = a_1 (T_1 - T_2)^0 + a_2 (T_1 - T_2)^1 + \dots$$

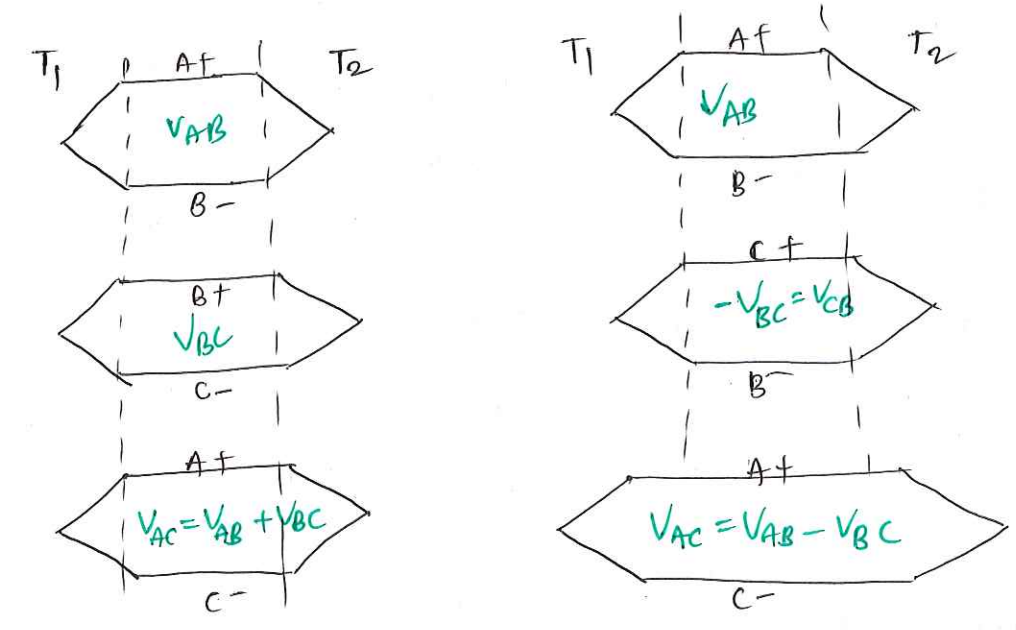
$$V_0 = a_1 (T_1)^0 + a_2 T_1^2$$

### Thermo Electric Laws

#### 1. Law of Intermediate Temperature :



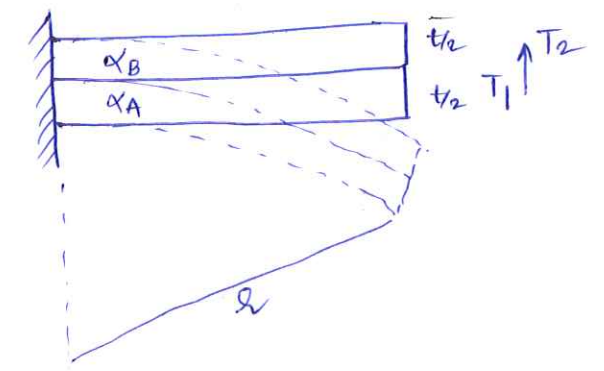
#### 2. Law of Intermediate metals :



### Cold Junction Compensation

Due to atmospheric temperature change the thermocouple output voltage produces error in the measurement of hot junction temperature so that for compensation of this temperature ~~the~~ cold junction compensation is used. In this method an electronic ckt is used which will simulate an output voltage similar to the thermocouple so that, the thermocouple cold junction is operating at 0°C.

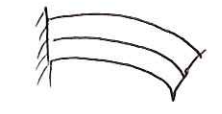
### Bimetallic strips :-



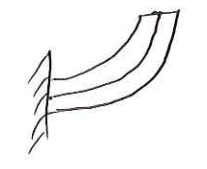
$$\text{Radius of curvature } (r) = \frac{2t}{(\alpha_A - \alpha_B)(T_2 - T_1)}$$

assume :-  $\alpha_A > \alpha_B$

(i)  $T_2 > T_1 \Rightarrow r = +ve$



(ii)  $T_2 < T_1 \Rightarrow r = -ve$



Applications :- MCB (Main Circuit Breaker), Iron Box.

principle :- The thermal expansion of metals are different depending on the value of 'alpha'. This property is used in the bimetallic strip. It is made of Ni-Iron Alloy with manganese & chromium.

\* Used for temperature control of air condition refrigerators, electric heaters, oil burners ... etc.



