Chapter 4 performance of short and medium lines

Classification of Overhead Transmission Lines:

A transmission line has *three constants *R*, *L* and *C* distributed uniformly along the whole length of the line. The resistance and inductance form the series impedance. The capacitance existing between conductors for 1-phase line or from a conductor to neutral for a 3-phase line forms a shunt path throughout the length of the line. Therefore, capacitance effects introduce complications in transmission line calculations. Depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as:

Short transmission lines. When the length of an overhead transmission line is upto about 50 km and the line voltage is comparatively low (< 20 kV), it is usually considered as a short transmission line. Due to smaller length and lower voltage, the capacitance effects are small and hence can be neglected. Therefore, while studying the performance of a short transmission line, only resistance and inductance of the line are taken into account.

Medium transmission lines. When the length of an overhead transmission line is about 50- 150 km and the line voltage is moderately high (>20 kV < 100 kV), it is considered as a medium transmission line. Due to sufficient length and voltage of the line, the capacitance effects are taken into account. For purposes of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more points.

Long transmission lines. When the length of an overhead transmission line is more than 150 km and line voltage is very high (> 100 kV), it is considered as a long transmission line. For the treatment of such a line, the line constants are considered uniformly distributed over the whole length of the line and rigorous methods are employed for solution.

Important Terms:

While studying the performance of a transmission line, it is desirable to determine its voltage regula- tion and transmission efficiency. We shall explain these two terms in turn.

1.**Voltage regulation.** When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage (V_R) of the line is generally less than the sending end voltage (V_S). This voltage drop ($V_S - V_R$) in the line is expressed as a percentage of receiving end voltage V_R and is called voltage regulation.

*The difference in voltage at the receiving end of a transmission line **between conditions of no load and full load is called* **voltage regulation** and is expressed as a percentage of the receiving end voltage.

Mathematically %age of voltage regulation = V_s -V_R / V_R X100

Obviously, it is desirable that the voltage regulation of a transmission line should be low *i.e.*, the increase in load current should make very little difference in the receiving end voltage.

2.Transmission efficiency. The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.

The ratio of receiving end power to the sending end power of a transmission line is known as the

transmission efficiency *of the line i.e.* % age Transmission efficiency, η_T

$$= \frac{\text{Receiving end power}}{\text{Sending end power}} \times 100$$

$= \left(V_R I_R \cos \phi_R \; / \; V_S I_S \cos \phi_S \right) \times 100$

where V_R, I_R and $\cos \phi_R$ are the receiving end voltage, current and power factor while V_S, I_S and $\cos \phi_S$ are the corresponding values at the sending end.

Performance of Single Phase Short Transmission Lines

As stated earlier, the effects of line capacitance are neglected for a short transmission line. Therefore, while studying the performance of such a line, only resistance and inductance of the line are taken into account. The equivalent circuit of a single phase short transmission line is shown in Fig. 10.1 (*i*). Here, the total line resistance and inductance are shown as concentrated or lumped instead of being distributed. The circuit is a simple a.c. series circuit.

Let
I = load current
R = loop resistance *i.e.*, resistance of both conductors
X_L = loop reactance
V_R = receiving end voltage
$\cos \phi_R$ = receiving end power factor (lagging)
V_S = sending end voltage
$\cos \phi_S$ = sending end power factor

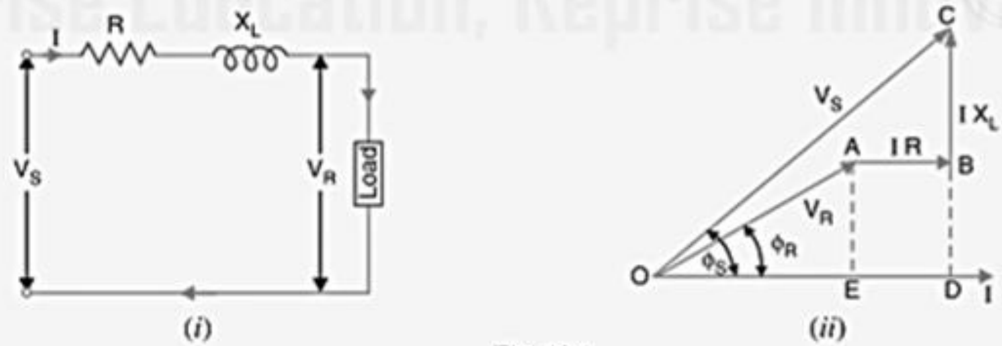


Fig. 10.1

The *phasor diagram of the line for lagging load power factor is shown in Fig. 10.1 (*ii*). From the right angled traingle *ODC*, we get,

$$(OC)^2 = (OD)^2 + (DC)^2$$

or
$$V_S^2 = (OE + ED)^2 + (DB + BC)^2$$
$$= (V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2$$

$\therefore$
$$V_S = \sqrt{(V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2}$$

(*i*) %age Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100$

(*ii*) Sending end *p.f.*, $\cos \phi_S = \dfrac{OD}{OC} = \dfrac{V_R \cos \phi_R + IR}{V_S}$

(*iii*) Power delivered $= V_R I_R \cos \phi_R$

Line losses $= I^2 R$

Power sent out $= V_R I_R \cos \phi_R + I^2 R$

%age Transmission efficiency $= \dfrac{\text{Power delivered}}{\text{Power sent out}} \times 100$

$$= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I^2 R} \times 100$$

An approximate expression for the sending end voltage V_S can be obtained as follows. Draw perpendicular from *B* and *C* on *OA* produced as shown in Fig. 10.2. Then *OC* is *nearly* equal to *OF* *i.e.*,

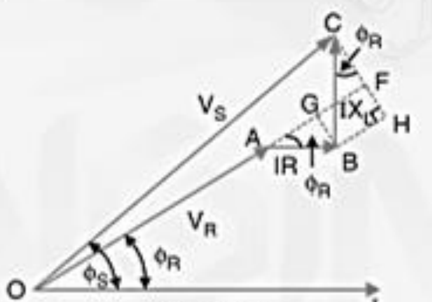


Fig. 10.2

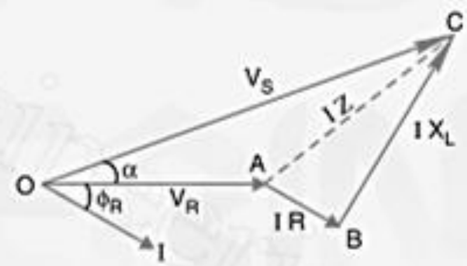


Fig. 10.3

$$OC = OF = OA + AF = OA + AG + GF$$
$$= OA + AG + BH$$

$\therefore$
$$V_S = V_R + I\,R \cos \phi_R + I\,X_L \sin \phi_R$$

Solution in complex notation. It is often convenient and profitable to make the line calculations in complex notation.

Taking $\overrightarrow{V_R}$ as the reference phasor, draw the phasor diagram as shown in Fig 10.3. It is clear that $\overrightarrow{V_S}$ is the phasor sum of $\overrightarrow{V_R}$ and $\vec{I}\,\vec{Z}$.

$$^*\overrightarrow{V_R} = V_R + j\,0$$
$$\vec{I} = \vec{I}\ \angle -\phi_R = I\,(\cos \phi_R - j \sin \phi_R)$$
$$\vec{Z} = R + jX_L$$

$\therefore$
$$\overrightarrow{V_S} = \overrightarrow{V_R} + \vec{I}\,\vec{Z}$$
$$= (V_R + j\,0) + I\,(\cos \phi_R - j \sin \phi_R)\,(R + j\,X_L)$$

$$= (V_R + IR\cos\phi_R + IX_L\sin\phi_R) + j\,(IX_L\cos\phi_R - IR\sin\phi_R)$$

$$\therefore \quad V_S = \sqrt{(V_R + IR\cos\phi_R + IX_L\sin\phi_R)^2 + (IX_L\cos\phi_R - IR\sin\phi_R)^2}$$

The second term under the root is quite small and can be neglected with reasonable accuracy. Therefore, approximate expression for V_S becomes :

$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$$

The following poins may be noted :

(*i*) The approximate formula for V_S $(= V_R + IR\cos\phi_R + IX_L\sin\phi_R)$ gives fairly correct results for lagging power factors. However, appreciable error is caused for leading power factors. Therefore, approximate expression for V_S should be used for lagging p.f. only.

(*ii*) The solution in complex notation is in more presentable form.

Three-Phase Short Transmission Lines

For reasons associated with economy, transmission of electric power is done by 3-phase system. This system may be regarded as consisting of three single phase units, each wire transmitting one-third of the total power. As a matter of convenience, we generally analyse 3-phase system by considering

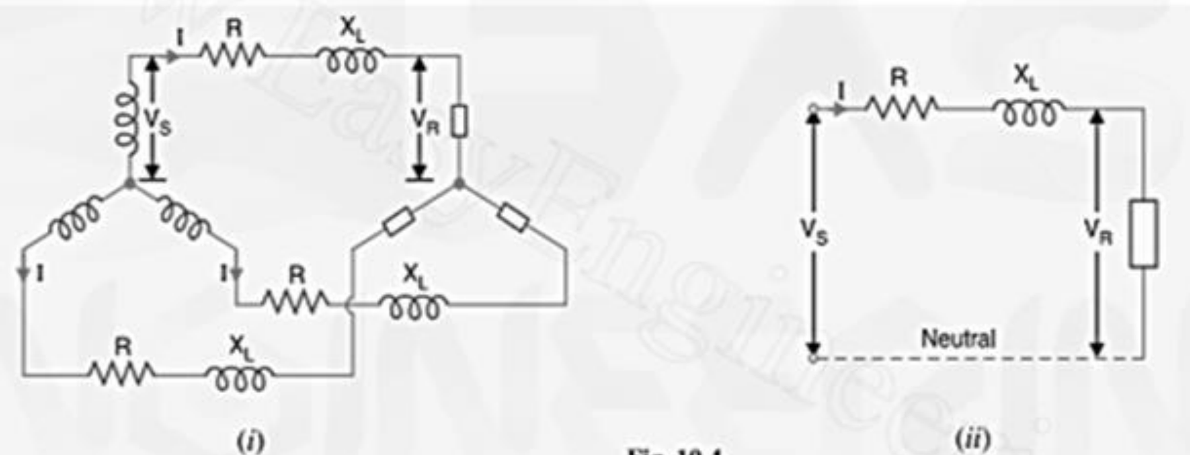


Fig. 10.4

*one phase only. Therefore, expression for regulation, efficiency etc. derived for a single phase line can also be applied to a 3-phase system. Since only one phase is considered, phase values of 3-phase system should be taken. Thus, V_S and V_R are the phase voltages, whereas R and X_L are the resistance and inductive reactance per phase respectively.

Fig. 10.4 (*i*) shows a *Y*-connected generator supplying a balanced *Y*-connected load through a transmission line. Each conductor has a resistance of $R\,\Omega$ and inductive reactance of $X_L\,\Omega$. Fig. 10.4 (*ii*) shows one phase separately. The calculations can now be made in the same way as for a single phase line.

Effect of Load p.f. on Regulation and Efficiency

The regulation and efficiency of a transmission line depend to a considerable extent upon the power factor of the load.

1. Effect on regulation. The expression for voltage regulation of a short transmission line is given by :

$$\%\text{age Voltage regulation} = \frac{IR\cos\phi_R + IX_L\sin\phi_R}{V_R} \times 100 \qquad (\textit{for lagging p.f.})$$

$$\%\text{age Voltage regulation} = \frac{I R \cos \phi_R - I X_L \sin \phi_R}{V_R} \times 100 \qquad \textit{(for leading p.f.)}$$

The following conclusions can be drawn from the above expressions :

(*i*) When the load p.f. is lagging or unity or such leading that $I R \cos \phi_R > I X_L \sin \phi_R$, then voltage regulation is positive *i.e.*, receiving end voltage V_R will be less than the sending end voltage V_S.

(*ii*) For a given V_R and I, the voltage regulation of the line increases with the decrease in p.f. for lagging loads.

(*iii*) When the load p.f. is leading to this extent that $I X_L \sin \phi_R > I R \cos \phi_R$, then voltage regulation is negative *i.e.* the receiving end voltage V_R is more than the sending end voltage V_S.

(*iv*) For a given V_R and I, the voltage regulation of the line decreases with the decrease in p.f. for leading loads.

2. Effect on transmission efficiency. The power delivered to the load depends upon the power factor.

$$P = V_R * I \cos \phi_R \quad \text{(For 1-phase line)}$$

$$\therefore \quad I = \frac{P}{V_R \cos \phi_R}$$

$$P = 3\, V_R I \cos \phi_R \quad \text{(For 3-phase line)}$$

$$\therefore \quad I = \frac{P}{3 V_R \cos \phi_R}$$

Power Factor Meter


Power Factor Regulator

It is clear that in each case, for a given amount of power to be transmitted (P) and receiving end voltage (V_R), the load current I is inversely proportional to the load p.f. $\cos \phi_R$. Consequently, with the decrease in load p.f., the load current and hence the line losses are increased. This leads to the conclusion that transmission efficiency of a line decreases with the decrease in load p.f. and *vice-versa*,

Example *A single phase overhead transmission line delivers 1100 kW at 33 kV at 0·8 p.f. lagging. The total resistance and inductive reactance of the line are 10 Ω and 15 Ω respectively. Determine :* (*i*) *sending end voltage* (*ii*) *sending end power factor and* (*iii*) *transmission efficiency.*

Solution.

Load power factor, $\cos \phi_R = 0{\cdot}8$ lagging

Total line impedance, $\vec{Z} = R + j X_L = 10 + j\,15$

Receiving end voltage, $V_R = 33$ kV $= 33{,}000$ V

$$\therefore \quad \text{Line current, } I = \frac{kW \times 10^3}{V_R \cos\phi_R} = \frac{1100 \times 10^3}{33{,}000 \times 0{\cdot}8} = 41{\cdot}67 \text{ A}$$

$$\text{As } \cos\phi_R = 0{\cdot}8 \quad \therefore \quad \sin\phi_R = 0{\cdot}6$$

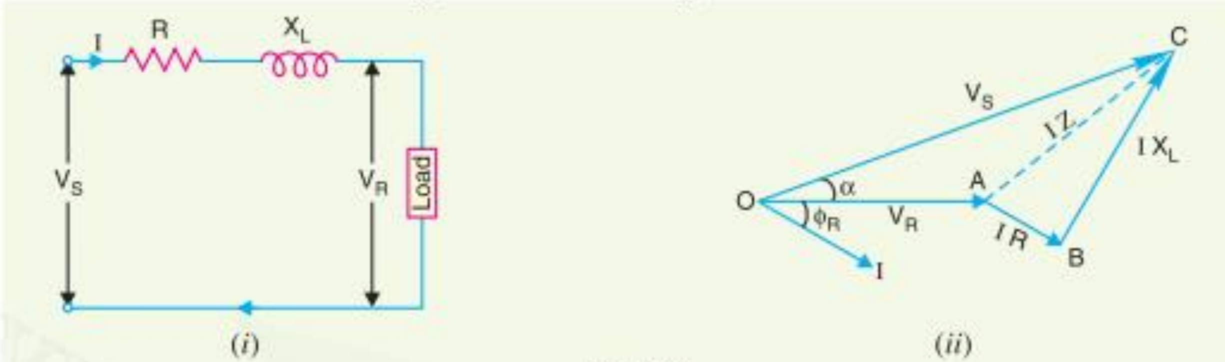


Fig. 10.5

The equivalent circuit and phasor diagram of the line are shown in Figs. 10.5 (*i*) and 10.5 (*ii*) respectively. Taking receiving end voltage $\overrightarrow{V_R}$ as the reference phasor,

$$\overrightarrow{V_R} = V_R + j\,0 = 33000 \text{ V}$$

$$\vec{I} = I(\cos\phi_R - j\sin\phi_R)$$

$$= 41{\cdot}67\,(0{\cdot}8 - j\,0{\cdot}6) = 33{\cdot}33 - j\,25$$

(*i*) Sending end voltage, $\overrightarrow{V_S} = \overrightarrow{V_R} + \vec{I}\,\vec{Z}$

$$= 33{,}000 + (33{\cdot}33 - j\,25{\cdot}0)\,(10 + j\,15)$$

$$= 33{,}000 + 333{\cdot}3 - j250 + j500 + 375$$

$$= 33{,}708{\cdot}3 + j\,250$$

$$\therefore \quad \text{Magnitude of } V_S = \sqrt{(33{,}708{\cdot}3)^2 + (250)^2} = \textbf{33,709 V}$$

(*ii*) Angle between $\overrightarrow{V_S}$ and $\overrightarrow{V_R}$ is

$$\alpha = \tan^{-1}\frac{250}{33{,}708{\cdot}3} = \tan^{-1} 0{\cdot}0074 = 0{\cdot}42^\circ$$

∴ Sending end power factor angle is

$$\phi_S = \phi_R + \alpha = 36{\cdot}87^\circ + 0{\cdot}42^\circ = 37{\cdot}29^\circ$$

∴ Sending end p.f., $\cos\phi_S = \cos 37{\cdot}29^\circ =$ **0·7956 lagging**

(*iii*) Line losses $= I^2 R = (41{\cdot}67)^2 \times 10 = 17{,}364 \text{ W} = 17{\cdot}364 \text{ kW}$

Output delivered $= 1100$ kW

Power sent $= 1100 + 17{\cdot}364 = 1117{\cdot}364$ kW

$$\therefore \quad \text{Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power sent}} \times 100 = \frac{1100}{1117{\cdot}364} \times 100 = \textbf{98·44\%}$$

Note. V_S and ϕ_S can also be calculated as follows :

$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R \text{ (approximately)}$$

$$= 33{,}000 + 41{\cdot}67 \times 10 \times 0{\cdot}8 + 41{\cdot}67 \times 15 \times 0{\cdot}6$$

$$= 33{,}000 + 333{\cdot}36 + 375{\cdot}03$$

$$= 33708{\cdot}39 \text{ V which is approximately the same as above}$$

$$\cos\phi_S = \frac{V_R\cos\phi_R + IR}{V_S} = \frac{33{,}000 \times 0{\cdot}8 + 41{\cdot}67 \times 10}{33{,}708{\cdot}39} = \frac{26{,}816{\cdot}7}{33{,}708{\cdot}39}$$

$$= 0{\cdot}7958$$

As stated earlier, this method gives fairly correct results for lagging p.f. The reader will find that this method is used in the solution of some numericals.

Example *What is the maximum length in km for a 1-phase transmission line having copper conductor of 0·775 cm² cross-section over which 200 kW at unity power factor and at 3300V are to be delivered ? The efficiecny of transmission is 90%. Take specific resistance as 1.725 μ Ω cm.*

Solution.

$$\text{Receiving end power} = 200 \text{ kW} = 2{,}00{,}000 \text{ W}$$

$$\text{Transmission efficiency} = 0{\cdot}9$$

$$\therefore \quad \text{Sending end power} = \frac{2{,}00{,}000}{0\cdot 9} = 2{,}22{,}222 \text{ W}$$

$$\therefore \quad \text{Line losses} = 2{,}22{,}222 - 2{,}00{,}000 = 22{,}222 \text{ W}$$

$$\text{Line current, } I = \frac{200 \times 10^3}{3{,}300 \times 1} = 60{\cdot}6 \text{ A}$$

Let R Ω be the resistance of one conductor.

$$\text{Line losses} = 2 I^2 R$$

or

$$22{,}222 = 2\,(60{\cdot}6)^2 \times R$$

$$\therefore \quad R = \frac{22{,}222}{2 \times (60\cdot 6)^2} = 3{\cdot}025\,\Omega$$

Now,

$$R = \rho\, l/a$$

$$\therefore \quad l = \frac{Ra}{\rho} = \frac{3\cdot 025 \times 0\cdot 775}{1\cdot 725 \times 10^{-6}} = 1{\cdot}36{\times}10^6 \text{ cm} = \textbf{13·6 km}$$

Example *An overhead 3-phase transmission line delivers 5000 kW at 22 kV at 0·8 p.f. lagging. The resistance and reactance of each conductor is 4 Ω and 6 Ω respectively. Determine : (i) sending end voltage (ii) percentage regulation (iii) transmission efficiency.*

Solution.

$$\text{Load power factor, } \cos\phi_R = 0{\cdot}8 \text{ lagging}$$

$$\text{Receiving end voltage/phase, }{}^*V_R = 22{,}000/\sqrt{3} = 12{,}700 \text{ V}$$

$$\text{Impedance/phase, } \quad \vec{Z} = 4 + j\,6$$

$$\text{Line current, } \quad I = \frac{5000 \times 10^3}{3 \times 12700 \times 0\cdot 8} = 164 \text{ A}$$

$$\text{As } \cos\phi_R = 0{\cdot}8 \quad \therefore \sin\phi_R = 0{\cdot}6$$

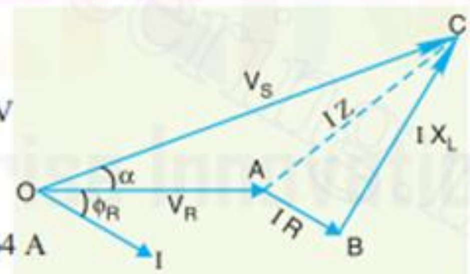


Fig. 10.6

Taking $\overrightarrow{V_R}$ as the reference phasor (see Fig. 10.6),

$$\overrightarrow{V_R} = V_R + j\,0 = 12700 \text{ V}$$

$$\vec{I} = I(\cos\phi_R - j\sin\phi_R) = 164\,(0{\cdot}8 - j\,0{\cdot}6) = 131{\cdot}2 - j\,98{\cdot}4$$

(*i*) Sending end voltage per phase is

$$\overrightarrow{V_S} = \overrightarrow{V_R} + \vec{I}\,\vec{Z} = 12700 + (131{\cdot}2 - j\,98{\cdot}4)\,(4 + j\,6)$$

$$= 12700 + 524{\cdot}8 + j\,787{\cdot}2 - j\,393{\cdot}6 + 590{\cdot}4$$

$$= 13815{\cdot}2 + j\,393{\cdot}6$$

$$\text{Magnitude of } V_S = \sqrt{(13815\cdot 2)^2 + (393\cdot 6)^2} = 13820{\cdot}8 \text{ V}$$

$$\text{Line value of } V_S = \sqrt{3} \times 13820{\cdot}8 = 23938 \text{ V} = \mathbf{23{\cdot}938 \text{ kV}}$$

(*ii*)
$$\% \text{age Regulation} = \frac{V_S - V_R}{V_R} \times 100 = \frac{13820{\cdot}8 - 12700}{12700} \times 100 = \mathbf{8{\cdot}825\%}$$

(*iii*)
$$\text{Line losses} = 3I^2R = 3 \times (164)^2 \times 4 = 3{,}22{,}752 \text{ W} = 322{\cdot}752 \text{ kW}$$

$$\therefore \quad \text{Transmission efficiency} = \frac{5000}{5000 + 322{\cdot}752} \times 100 = \mathbf{93{\cdot}94\%}$$

Example *Estimate the distance over which a load of 15000 kW at a p.f. 0·8 lagging can be delivered by a 3-phase transmission line having conductors each of resistance 1 Ω per kilometre. The voltage at the receiving end is to be 132 kV and the loss in the transmission is to be 5%.*

Solution.

Line current,
$$I = \frac{\text{Power delivered}}{\sqrt{3} \times \text{line voltage} \times \text{power factor}} = \frac{15000 \times 10^3}{\sqrt{3} \times 132 \times 10^3 \times 0{\cdot}8} = 82 \text{ A}$$

$$\text{Line losses} = 5\% \text{ of power delivered} = 0{\cdot}05 \times 15000 = 750 \text{ kW}$$

Let R Ω be the resistance of one conductor.

$$\text{Line losses} = 3\,I^2R$$

or
$$750 \times 10^3 = 3 \times (82)^2 \times R$$

$$\therefore \quad R = \frac{750 \times 10^3}{3 \times (82)^2} = 37{\cdot}18\,\Omega$$

Resistance of each conductor per km is 1 Ω (*given*).

$$\therefore \quad \text{Length of line} = \mathbf{37{\cdot}18 \text{ km}}$$

Example *A 3-phase line delivers 3600 kW at a p.f. 0·8 lagging to a load. If the sending end voltage is 33 kV, determine (i) the receiving end voltage (ii) line current (iii) transmission efficiency. The resistance and reactance of each conductor are 5·31 Ω and 5·54 Ω respectively.*

Solution.

Resistance of each conductor, $R = 5{\cdot}31\,\Omega$

Reactance of each conductor, $X_L = 5{\cdot}54\,\Omega$

Load power factor, $\cos\phi_R = 0{\cdot}8$ (*lagging*)

Sending end voltage/phase, $V_S = 33{,}000/\sqrt{3} = 19{,}052$ V

Let V_R be the phase voltage at the receiving end.

Line current,
$$I = \frac{\text{Power delivered / phase}}{V_R \times \cos\phi_R} = \frac{1200 \times 10^3}{V_R \times 0{\cdot}8}$$

$$= \frac{150 \times 10^5}{V_R} \qquad ...(i)$$

(*i*) Using approximate expression for V_S, we get,

$$V_S = V_R + I\,R\cos\phi_R + I\,X_L\sin\phi_R$$

or
$$19{,}052 = V_R + \frac{15 \times 10^5}{V_R} \times 5{\cdot}31 \times 0{\cdot}8 + \frac{15 \times 10^5}{V_R} \times 5{\cdot}54 \times 0{\cdot}6$$

or
$$V_R^2 - 19{,}052\,V_R + 1{,}13{,}58{,}000 = 0$$

Solving this equation, we get, $V_R = 18{,}435$ V

$$\therefore \quad \text{Line voltage at the receiving end} = \sqrt{3} \times 18{,}435 = 31{,}930 \text{ V} = \mathbf{31{\cdot}93 \text{ kV}}$$

(*ii*) Line current, $I = \frac{15 \times 10^5}{V_R} = \frac{15 \times 10^5}{18{,}435} =$ **81·36 A**

(*iii*) Line losses, $= 3 I^2 R = 3 \times (81 \cdot 36)^2 \times 5 \cdot 31 = 1{,}05{,}447 \text{ W} = 105 \cdot 447 \text{ kW}$

$\therefore$ Transmission efficiency $= \frac{3600}{3600 + 105 \cdot 447} \times 100 =$ **97·15%**

Example *A short 3-ϕ transmission line with an impedance of (6 + j 8) Ω per phase has sending and receiving end voltages of 120 kV and 110 kV respectively for some receiving end load at a p.f. of 0·9 lagging. Determine* (*i*) *power output and* (*ii*) *sending end power factor.*

Solution.

Resistance of each conductor, $R = 6\ \Omega$

Reactance of each conductor, $X_L = 8\ \Omega$

Load power factor, $\cos \phi_R = 0 \cdot 9$ lagging

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63508 \text{ V}$

Sending end voltage/phase, $V_S = 120 \times 10^3/\sqrt{3} = 69282 \text{ V}$

Let I be the load current. Using approximate expression for V_S, we get,

$$V_S = V_R + I R \cos \phi_R + I X_L \sin \phi_R$$

or $$69282 = 63508 + I \times 6 \times 0 \cdot 9 + I \times 8 \times 0 \cdot 435$$

or $$8 \cdot 88 I = 5774$$

or $$I = 5774/8 \cdot 88 = 650 \cdot 2 \text{ A}$$

(*i*) Power output $= \frac{3 V_R I \cos \phi_R}{1000} \text{ kW} = \frac{3 \times 63508 \times 650 \cdot 2 \times 0 \cdot 9}{1000}$

$=$ **1,11,490 kW**

(*ii*) Sending end p.f., $\cos \phi_S = \frac{V_R \cos \phi_R + I R}{V_S} = \frac{63508 \times 0 \cdot 9 + 650 \cdot 2 \times 6}{69282} =$ **0·88 lag**

Example *An 11 kV, 3-phase transmission line has a resistance of 1·5 Ω and reactance of 4 Ω per phase. Calculate the percentage regulation and efficiency of the line when a total load of 5000 kVA at 0.8 lagging power factor is supplied at 11 kV at the distant end.*

Solution.

Resistance of each conductor, $R = 1 \cdot 5\ \Omega$

Reactance of each conductor, $X_L = 4\ \Omega$

Receiving end voltage/phase, $V_R = \frac{11 \times 10^3}{\sqrt{3}} = 6351 \text{ V}$

Load power factor, $\cos \phi_R = 0 \cdot 8$ lagging

Load current, $I = \frac{\text{Power delivered in kVA} \times 1000}{3 \times V_R}$

$= \frac{5000 \times 1000}{3 \times 6351} = 262 \cdot 43 \text{A}$

Using the approximate expression for V_S (sending end voltage per phase), we get,

$$V_S = V_R + I R \cos \phi_R + I X_L \sin \phi_R$$

$$= 6351 + 262 \cdot 43 \times 1 \cdot 5 \times 0 \cdot 8 + 262 \cdot 43 \times 4 \times 0 \cdot 6 = 7295 \cdot 8 \text{ V}$$

% regulation $= \frac{V_S - V_R}{V_R} \times 100 = \frac{7295 \cdot 8 - 6351}{6351} \times 100 =$ **14·88%**

Line losses $= 3 I^2 R = 3 \times (262 \cdot 43)^2 \times 1 \cdot 5 = 310 \times 10^3 \text{ W} = 310 \text{ kW}$

$$\text{Output power} = 5000 \times 0{\cdot}8 = 4000 \text{ kW}$$

$$\text{Input power} = \text{Ouput power} + \text{line losses} = 4000 + 310 = 4310 \text{ kW}$$

$$\text{Transmission efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{4000}{4310} \times 100 = \mathbf{92{\cdot}8\%}$$

Example *A 3-phase, 50 Hz, 16 km long overhead line supplies 1000 kW at 11 kV, 0·8 p.f. lagging. The line resistance is 0·03 Ω per phase per km and line inductance is 0·7 mH per phase per km. Calculate the sending end voltage, voltage regulation and efficiency of transmission.*

Solution.

Resistance of each conductor, $R = 0{\cdot}03 \times 16 = 0{\cdot}48\,\Omega$

Reactance of each conductor, $X_L = 2\pi f L \times 16 = 2\pi \times 50 \times 0{\cdot}7 \times 10^{-3} \times 16 = 3{\cdot}52\,\Omega$

Receiving end voltage/phase, $V_R = \dfrac{11 \times 10^3}{\sqrt{3}} = 6351 \text{ V}$

Load power factor, $\cos\phi_R = 0{\cdot}8$ lagging

$$\text{Line current, } I = \frac{1000 \times 10^3}{3 \times V_R \times \cos\phi} = \frac{1000 \times 10^3}{3 \times 6351 \times 0{\cdot}8} = 65{\cdot}6\text{A}$$

$$\text{Sending end voltage/phase, } V_S = V_R + I R \cos\phi_R + I X_L \sin\phi_R$$

$$= 6351 + 65{\cdot}6 \times 0{\cdot}48 \times 0{\cdot}8 + 65{\cdot}6 \times 3{\cdot}52 \times 0{\cdot}6 = 6515 \text{ V}$$

$$\therefore \quad \text{\%age Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100 = \frac{6515 - 6351}{6351} \times 100 = \mathbf{2{\cdot}58\%}$$

$$\text{Line losses} = 3 I^2 R = 3 \times (65{\cdot}6)^2 \times 0{\cdot}48 = 6{\cdot}2 \times 10^3 \text{ W} = 6{\cdot}2 \text{ kW}$$

$$\text{Input power} = \text{Output power} + \text{Line losses} = 1000 + 6{\cdot}2 = 1006{\cdot}2 \text{ kW}$$

$$\therefore \quad \text{Transmission efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{1000}{1006{\cdot}2} \times 100 = \mathbf{99{\cdot}38\%}$$

Medium Transmission Lines

In short transmission line calculations, the effects of the line capacitance are neglected because such lines have smaller lengths and transmit power at relatively low voltages (< 20 kV). However, as the length and voltage of the line increase, the capacitance gradually becomes of greater importance. Since medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, the effects of capacitance cannot be neglected. Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.

The capacitance is uniformly distributed over the entire length of the line. However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points. Such a treatment of localising the line capacitance gives reasonably accurate results. The most commonly used methods (known as *localised capacitance methods*) for the solution of medium transmissions lines are :

(*i*) End condenser method (*ii*) Nominal *T* method (*iii*) Nominal π method.

Although the above methods are used for obtaining the performance calculations of medium lines, they can also be used for short lines if their line capacitance is given in a particular problem.

End Condenser Method

In this method, the capacitance of the line is lumped or concentrated at the receiving or load end as shown in Fig. 10.8. This method of localising the line capacitance at the load end overestimates the effects of capacitance. In Fig. 10.8, one phase of the 3-phase transmission line is shown as it is more convenient to work in phase instead of line-to-line values.

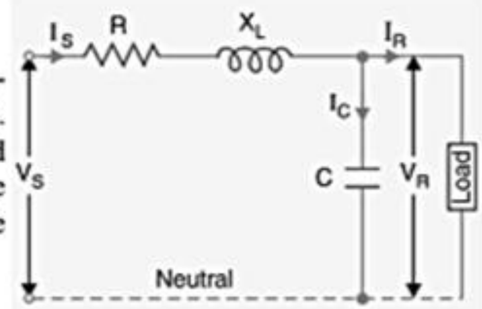


Fig. 10.8

Let I_R = load current per phase

R = resistance per phase

X_L = inductive reactance per phase

C = capacitance per phase

$\cos \phi_R$ = receiving end power factor (*lagging*)

V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig 10.9. Taking the receiving end voltage $\overrightarrow{V_R}$ as the reference phasor, we have, $\overrightarrow{V_R} = V_R + j\,0$

Load current, $\overrightarrow{I_R} = I_R\,(\cos\phi_R - j\sin\phi_R)$

Capacitive current, $\overrightarrow{I_C} = j\,\overrightarrow{V_R}\,\omega C = j\,2\pi f C\,\overrightarrow{V_R}$

The sending end current $\overrightarrow{I_S}$ is the phasor sum of load current $\overrightarrow{I_R}$ and capacitive current $\overrightarrow{I_C}$ *i.e.*,

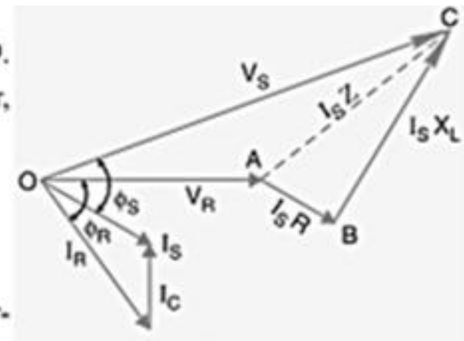


Fig. 10.9

$$\overrightarrow{I_S} = \overrightarrow{I_R} + \overrightarrow{I_C}$$
$$= I_R\,(\cos\phi_R - j\sin\phi_R) + j\,2\pi f C\,V_R$$
$$= I_R\cos\phi_R + j\,(-I_R\sin\phi_R + 2\pi f C V_R)$$

Voltage drop/phase $= \overrightarrow{I_S}\,\overrightarrow{Z} = \overrightarrow{I_S}\,(R + jX_L)$

Sending end voltage, $\overrightarrow{V_S} = \overrightarrow{V_R} + \overrightarrow{I_S}\,\overrightarrow{Z} = \overrightarrow{V_R} + \overrightarrow{I_S}\,(R + j\,X_L)$

Thus, the magnitude of sending end voltage V_S can be calculated.

$$\%\text{ Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\%\text{ Voltage transmission efficiency} = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} \times 100$$

$$= \frac{V_R\,I_R\cos\phi_R}{V_R\,I_R\cos\phi_R + I_S^2\,R} \times 100$$

Limitations. Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks :

(*i*) There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.

(*ii*) This method overestimates the effects of line capacitance.

Example *A (medium) single phase transmission line 100 km long has the following constants :*

Resistance/km = 0·25 Ω ; *Reactance/km = 0·8 Ω*

Susceptance/km = 14 × 10⁻⁶ siemen ; *Receiving end line voltage = 66,000 V*

Assuming that the total capacitance of the line is localised at the receiving end alone, determine (i) the sending end current (ii) the sending end voltage (iii) regulation and (iv) supply power factor. The line is delivering 15,000 kW at 0.8 power factor lagging. Draw the phasor diagram to illustrate your calculations.

Solution. Figs. 10.10 (*i*) and (*ii*) show the circuit diagram and phasor diagram of the line respectively.

Total resistance, $\quad R = 0{\cdot}25 \times 100 = 25\,\Omega$

Total reactance, $\quad X_L = 0{\cdot}8 \times 100 = 80\,\Omega$

Total susceptance, $\quad Y = 14 \times 10^{-6} \times 100 = 14 \times 10^{-4}\,S$

Receiving end voltage, $\quad V_R = 66{,}000$ V

$\therefore$ Load current, $\quad I_R = \dfrac{15{,}000 \times 10^3}{66{,}000 \times 0 \cdot 8} = 284$ A

$$\cos\phi_R = 0{\cdot}8\,; \qquad \sin\phi_R = 0{\cdot}6$$

Taking receiving end voltage as the reference phasor [see Fig.10.10 (*ii*)], we have,

$$\overrightarrow{V_R} = V_R + j\,0 = 66{,}000 V$$

Load current, $\quad \overrightarrow{I_R} = I_R\,(\cos\phi_R - j\sin\phi_R) = 284\,(0{\cdot}8 - j\,0{\cdot}6) = 227 - j\,170$

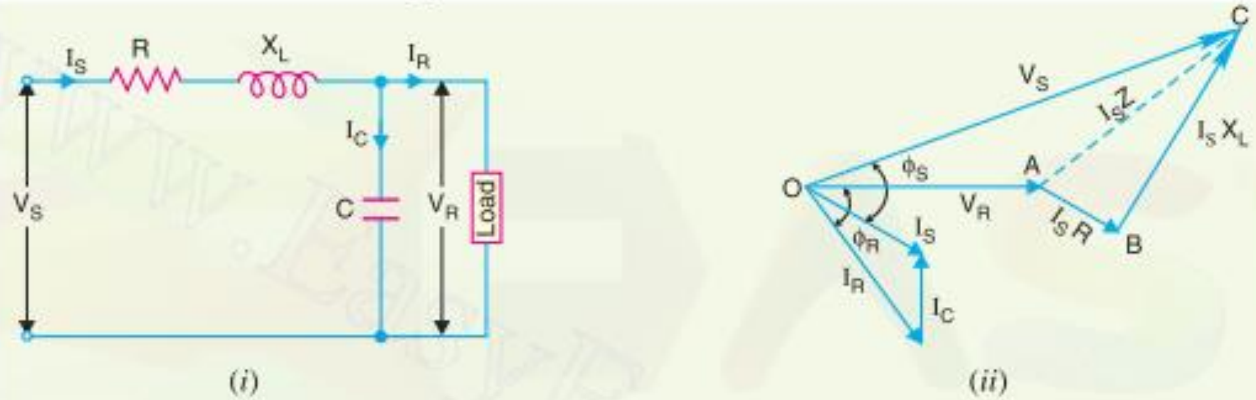


Fig. 10.10

Capacitive current, $\quad \overrightarrow{I_C} = j\,Y \times V_R = j\,14 \times 10^{-4} \times 66000 = j\,92$

(*i*) Sending end current, $\quad \overrightarrow{I_S} = \overrightarrow{I_R} + \overrightarrow{I_C} = (227 - j\,170) + j\,92$

$$= 227 - j\,78 \qquad \text{... }(i)$$

Magnitude of $I_S = \sqrt{(227)^2 + (78)^2} =$ **240 A**

(*ii*) Voltage drop $= \overrightarrow{I_S}\,\overrightarrow{Z} = \overrightarrow{I_S}(R + j\,X_L) = (227 - j\,78)\,(\,25 + j\,80)$

$$= 5{,}675 + j\,18{,}160 - j\,1950 + 6240$$

$$= 11{,}915 + j\,16{,}210$$

Sending end voltage, $\quad \overrightarrow{V_S} = \overrightarrow{V_R} + \overrightarrow{I_S}\,\overrightarrow{Z} = 66{,}000 + 11{,}915 + j\,16{,}210$

$$= 77{,}915 + j\,16{,}210 \qquad \text{...}(ii)$$

Magnitude of $V_S = \sqrt{(77915)^2 + (16210)^2} =$ **79583V**

(*iii*) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{79{,}583 - 66{,}000}{66{,}000} \times 100 =$ **20·58%**

(*iv*) Referring to exp. (*i*), phase angle between $\overrightarrow{V_R}$ and $\overrightarrow{I_R}$ is :

$$\theta_1 = \tan^{-1} - 78/227 = \tan^{-1}(-\,0{\cdot}3436) = -\,18{\cdot}96°$$

Referring to exp. (*ii*), phase angle between $\overrightarrow{V_R}$ and $\overrightarrow{V_S}$ is :

$$\theta_2 = \tan^{-1}\frac{16210}{77915} = \tan^{-1}(0 \cdot 2036) = 11 \cdot 50°$$

$\therefore$ Supply power factor angle, $\quad \phi_S = 18{\cdot}96° + 11{\cdot}50° = 30{\cdot}46°$

$\therefore$ Supply p.f. $= \cos\phi_S = \cos 30{\cdot}46° =$ **0·86 lag**

Nominal T Method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. 10.11. Therefore, in this arrangement, full charging current flows over half the line. In Fig. 10.11, one phase of 3-phase transmission line is shown as it is advantageous to work in phase instead of line-to-line values.

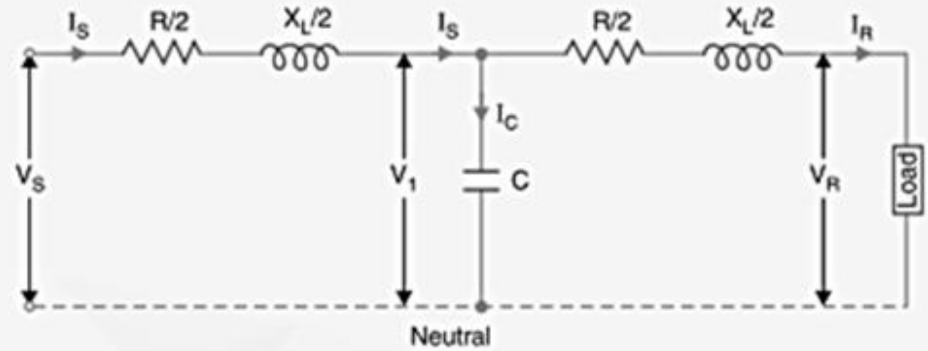


Fig. 10.11

Let I_R = load current per phase ; R = resistance per phase

X_L = inductive reactance per phase ; C = capacitance per phase

$\cos\phi_R$ = receiving end power factor (*lagging*) ; V_S = sending end voltage/phase

V_1 = voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig. 10.12. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have,

Receiving end voltage, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

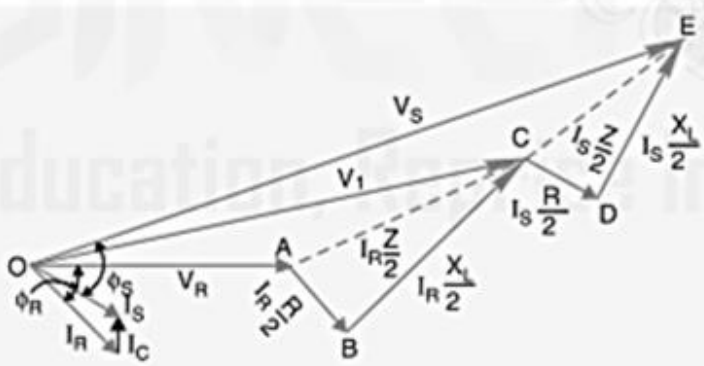


Fig. 10.12

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$

$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

Capacitive current, $\vec{I_C} = j\,\omega\,C\,\vec{V_1} = j\,2\pi f\,C\,\vec{V_1}$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\frac{\vec{Z}}{2} = \vec{V_1} + \vec{I_S}\left(\frac{R}{2} + j\frac{X_L}{2}\right)$

Example *A 3-phase, 50-Hz overhead transmission line 100 km long has the following constants :*

Resistance/km/phase $= 0.1\ \Omega$

Inductive reactance/km/phase $= 0{\cdot}2\ \Omega$

Capacitive susceptance/km/phase $= 0{\cdot}04 \times 10^{-4}$ *siemen*

Determine (i) the sending end current (ii) sending end voltage (iii) sending end power factor and (iv) transmission efficiency when supplying a balanced load of 10,000 kW at 66 kV, p.f. 0·8 lagging. Use nominal T method.

Solution. Figs. 10.13 (*i*) and 10.13 (*ii*) show the circuit diagram and phasor diagram of the line respectively.

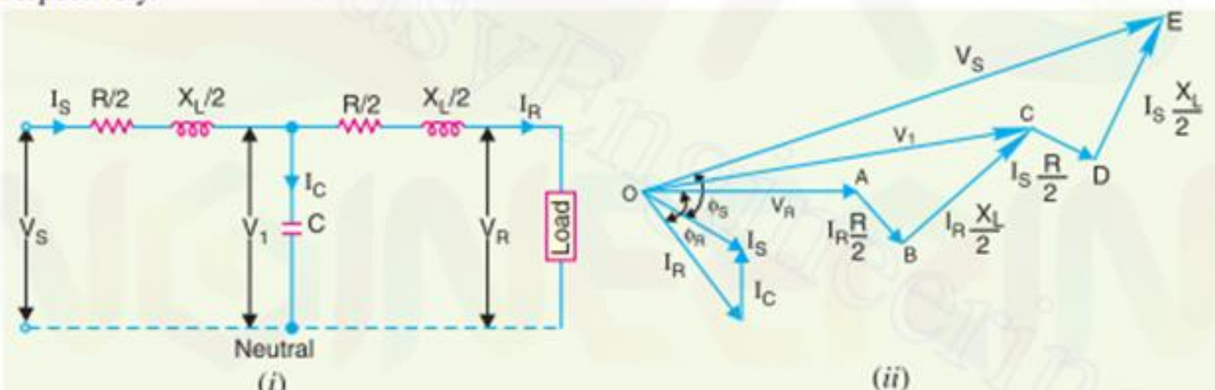


Fig. 10.13

Total resistance/phase, $R = 0{\cdot}1 \times 100 = 10\ \Omega$

Total reactance/phase. $X_L = 0{\cdot}2 \times 100 = 20\ \Omega$

Capacitive susceptance, $Y = 0{\cdot}04 \times 10^{-4} \times 100 = 4 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66{,}000/\sqrt{3} = 38105\ V$

Load current, $I_R = \dfrac{10{,}000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}8} = 109$ A

$$\cos\phi_R = 0{\cdot}8\,;\ \sin\phi_R = 0{\cdot}6$$

Impedance per phase, $\vec{Z} = R + j\,X_L = 10 + j\,20$

(*i*) Taking receiving end voltage as the reference phasor [see Fig. 10.13 (*ii*)], we have,

Receiving end voltage, $\vec{V_R} = V_R + j\,0 = 38{,}105\ V$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 109\,(0{\cdot}8 - j\,0{\cdot}6) = 87{\cdot}2 - j\,65{\cdot}4$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2 = 38,105 + (87{\cdot}2 - j\,65{\cdot}4)\,(5 + j\,10)$

$$= 38{,}105 + 436 + j\,872 - j\,327 + 654 = 39{,}195 + j\,545$$

Charging current, $\vec{I_C} = j\,Y\,\vec{V_1} = j\,4\times10^{-4}(39{,}195 + j\,545) = -0\cdot218 + j\,15\cdot6$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (87\cdot2 - j\,65\cdot4) + (-0\cdot218 + j\,15\cdot6)$

$= 87\cdot0 - j\,49\cdot8 = 100\angle -29°47'$ A

∴ Sending end current $=$ **100 A**

(ii) Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 = (39{,}195 + j\,545) + (87\cdot0 - j\,49\cdot8)\,(5 + j\,10)$

$= 39{,}195 + j\,545 + 434\cdot9 + j\,870 - j\,249 + 498$

$= 40128 + j\,1170 = 40145\angle 1°40'$ V

∴ Line value of sending end voltage

$= 40145 \times \sqrt{3} = 69\,533$ V $=$ **69·533 kV**

(iii) Referring to phasor diagram in Fig. 10.14,

θ_1 = angle between $\vec{V_R}$ and $\vec{V_S}$ = 1°40′

θ_2 = angle between $\vec{V_R}$ and $\vec{I_S}$ = 29° 47′

∴ ϕ_S = angle between $\vec{V_S}$ and $\vec{I_S}$

$= \theta_1 + \theta_2 = 1°40' + 29°47' = 31°27'$

∴ Sending end power factor, $\cos\phi_S = \cos 31°27' =$ **0·853 lag**

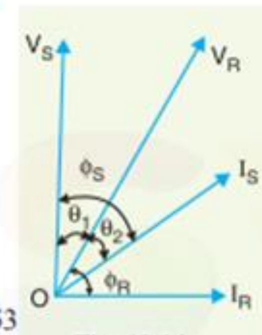


Fig. 10.14

(iv) Sending end power $= 3\,V_S I_S \cos\phi_S = 3 \times 40{,}145 \times 100 \times 0\cdot853$

$= 10273105$ W $= 10273\cdot105$ kW

Power delivered $= 10{,}000$ kW

∴ Transmission efficiency $= \dfrac{10{,}000}{10273\cdot105} \times 100 =$ **97·34%**

Example *A 3-phase, 50 Hz transmission line 100 km long delivers 20 MW at 0·9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0·2 Ω and 0·4 Ω respectively, while capacitance admittance is 2·5 × 10⁻⁶ siemen/km/phase. Calculate :* (i) *the current and voltage at the sending end* (ii) *efficiency of transmission. Use nominal T method.*

Solution. Figs. 10.15 (*i*) and 10.15 (*ii*) show the circuit diagram and phasor diagram respectively.

Total resistance/phase, $R = 0\cdot2 \times 100 = 20\ \Omega$

Total reactance/phase, $X_L = 0\cdot4 \times 100 = 40\ \Omega$

Total capacitance admittance/phase, $Y = 2\cdot5 \times 10^{-6} \times 100 = 2\cdot5 \times 10^{-4}$ S

Phase impedance, $\vec{Z} = 20 + j40$

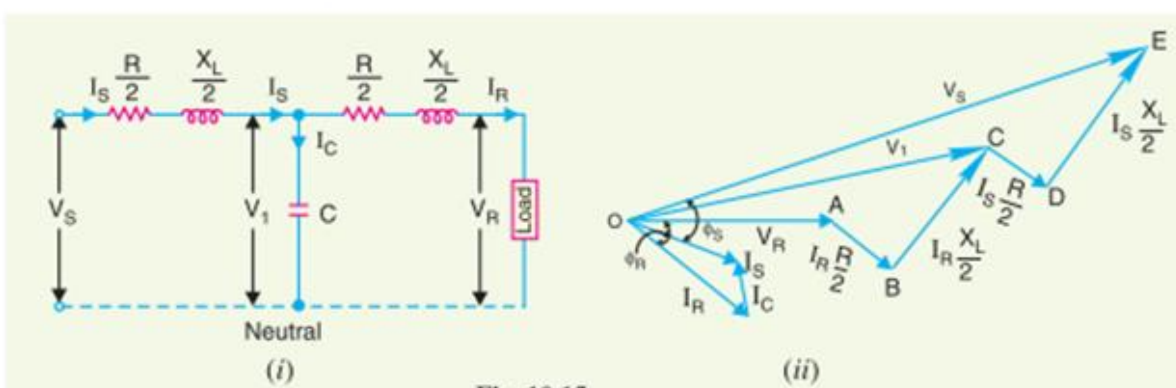


Fig. 10.15

$$\text{Receiving end voltage/phase, } V_R = 110 \times 10^3/\sqrt{3} = 63508 \text{ V}$$

$$\text{Load current, } I_R = \frac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}9} = 116.6 \text{ A}$$

$$\cos\phi_R = 0{\cdot}9 \text{ ; } \sin\phi_R = 0{\cdot}435$$

(*i*) Taking receiving end voltage as the reference phasor [see phasor diagram 10.15 (*ii*)], we have,

$$\overrightarrow{V_R} = V_R + j0 = 63508 \text{ V}$$

Load current, $\overrightarrow{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 116{\cdot}6\,(0{\cdot}9 - j\,0{\cdot}435) = 105 - j50{\cdot}7$

Voltage across *C*, $\overrightarrow{V_1} = \overrightarrow{V_R} + \overrightarrow{I_R}\,\overrightarrow{Z}/2 = 63508 + (105 - j\,50{\cdot}7)\,(10 + j\,20)$

$$= 63508 + (2064 + j1593) = 65572 + j1593$$

Charging current, $\overrightarrow{I_C} = j\,Y\overrightarrow{V_1} = j\,2{\cdot}5 \times 10^{-4}\,(65572 + j1593) = -0{\cdot}4 + j\,16{\cdot}4$

Sending end current, $\overrightarrow{I_S} = \overrightarrow{I_R} + \overrightarrow{I_C} = (105 - j\,50{\cdot}7) + (-0{\cdot}4 + j16{\cdot}4)$

$$= (104{\cdot}6 - j\,34{\cdot}3) = 110\,\angle -18°9'\text{ A}$$

$\therefore$ Sending end current $= 110$ A

Sending end voltage, $\overrightarrow{V_S} = \overrightarrow{V_1} + \overrightarrow{I_S}\,\overrightarrow{Z}/2$

$$= (65572 + j1593) + (104{\cdot}6 - j34{\cdot}3)\,(10 + j20)$$

$$= 67304 + j\,3342$$

$\therefore$ Magnitude of $V_S = \sqrt{(67304)^2 + (3342)^2} = 67387\ V$

$\therefore$ Line value of sending end voltage

$$= 67387 \times \sqrt{3} = 116717 \text{ V} = 116{\cdot}717 \text{ kV}$$

(*ii*) Total line losses for the three phases

$$= 3\,I_S^{\,2}\,R/2 + 3I_R^{\,2}\,R/2$$

$$= 3 \times (110)^2 \times 10 + 3 \times (116{\cdot}6)^2 \times 10$$

$$= 0{\cdot}770 \times 10^6 \text{ W} = 0{\cdot}770 \text{ MW}$$

$\therefore$ Transmission efficiency $= \dfrac{20}{20 + 0{\cdot}770} \times 100 = 96{\cdot}29\%$

Nominal π Method

In this method, capacitance of each conductor (*i.e.*, line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in Fig. 10.16. It is obvious that capacitance at the sending end has no effect on the line drop. However, its charging current must be added to line current in order to obtain the total sending end current.

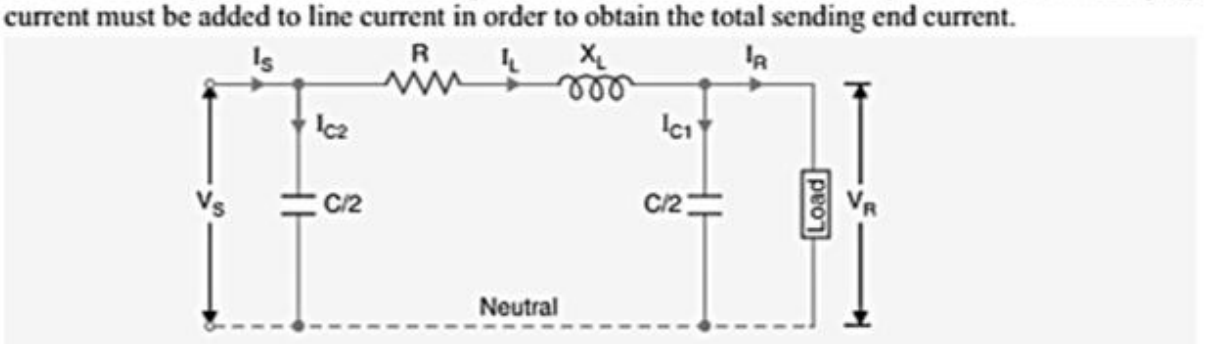


Fig. 10.16

Let I_R = load current per phase

R = resistance per phase

X_L = inductive reactance per phase

C = capacitance per phase

$\cos \phi_R$ = receiving end power factor (*lagging*)

V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig. 10.17. Taking the receiving end voltage as the reference phasor, we have,

$$\overrightarrow{V_R} = V_R + j\,0$$

Load current, $\overrightarrow{I_R} = I_R (\cos \phi_R - j \sin \phi_R)$

Charging current at load end is

$$\overrightarrow{I_{C1}} = j\,\omega\,(C/2)\,\overrightarrow{V_R} = j\,\pi\,f\,C\,\overrightarrow{V_R}$$

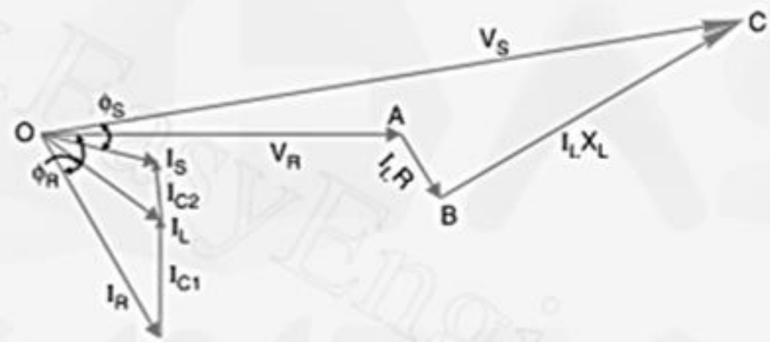


Fig. 10.17

Line current, $\overrightarrow{I_L} = \overrightarrow{I_R} + \overrightarrow{I_{C1}}$

Sending end voltage, $\overrightarrow{V_S} = \overrightarrow{V_R} + \overrightarrow{I_L}\,\overrightarrow{Z} = \overrightarrow{V_R} + \overrightarrow{I_L}\,(R + jX_L)$

Charging current at the sending end is

$$\overrightarrow{I_{C2}} = j\,\omega\,(C/2)\,\overrightarrow{V_S} = j\,\pi\,f\,C\,\overrightarrow{V_S}$$

∴ Sending end current, $\overrightarrow{I_S} = \overrightarrow{I_L} + \overrightarrow{I_{C2}}$

Example *A 3-phase, 50Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0·1 Ω, 0·5 Ω and 3×10^{-6} S per km per phase. If the line delivers 50 MW at 110 kV and 0·8 p.f. lagging, determine the sending end voltage and current. Assume a nominal π circuit for the line.*

Solution. Fig. 10.18 shows the circuit diagram for the line.

Total resistance/phase, $R = 0{\cdot}1 \times 150 = 15\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}5 \times 150 = 75\ \Omega$

Capacitive admittance/phase, $Y = 3 \times 10^{-6} \times 150 = 45 \times 10^{-5}$ S

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63{,}508$ V

Load current, $$I_R = \frac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}8} = 328 \text{ A}$$

$$\cos\phi_R = 0{\cdot}8\ ;\ \sin\phi_R = 0{\cdot}6$$

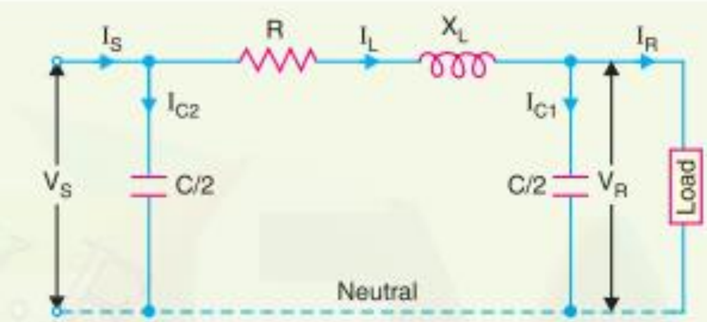


Fig. 10.18

Taking receiving end voltage as the reference phasor, we have,

$$\overrightarrow{V_R} = V_R + j\,0 = 63{,}508 \text{ V}$$

Load current, $\overrightarrow{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 328\,(0{\cdot}8 - j0{\cdot}6) = 262{\cdot}4 - j196{\cdot}8$

Charging current at the load end is

$$\overrightarrow{I_{C1}} = \overrightarrow{V_R}\, j\,\frac{Y}{2} = 63{,}508 \times j\,\frac{45 \times 10^{-5}}{2} = j\,14{\cdot}3$$

Line current, $\overrightarrow{I_L} = \overrightarrow{I_R} + \overrightarrow{I_{C1}} = (262{\cdot}4 - j\,196{\cdot}8) + j\,14{\cdot}3 = 262{\cdot}4 - j\,182{\cdot}5$

Sending end voltage, $\overrightarrow{V_S} = \overrightarrow{V_R} + \overrightarrow{I_L}\,\overrightarrow{Z} = \overrightarrow{V_R} + \overrightarrow{I_L}\,(R + j\,X_L)$

$= 63,\,508 + (262{\cdot}4 - j\,182{\cdot}5)\,(15 + j\,75)$

$= 63{,}508 + 3936 + j\,19{,}680 - j\,2737{\cdot}5 + 13{,}687$

$= 81{,}131 + j\,16{,}942{\cdot}5 = 82{,}881 \angle 11°47'$ V

$\therefore$ Line to line sending end voltage $= 82{,}881 \times \sqrt{3} = 1{,}43{,}550$ V $=$ **143·55 kV**

Charging current at the sending end is

$$I_{C2} = j\overrightarrow{V_S}\,Y/2 = (81{,}131 + j\,16{,}942\cdot5)\,j\,\frac{45 \times 10^{-5}}{2}$$

$= -3{\cdot}81 + j\,18{\cdot}25$

Sending end current, $\overrightarrow{I_S} = \overrightarrow{I_L} + \overrightarrow{I_{C_2}} = (262{\cdot}4 - j\,182{\cdot}5) + (-3{\cdot}81 + j\,18{\cdot}25)$

$= 258{\cdot}6 - j\,164{\cdot}25 = 306{\cdot}4 \angle -32{\cdot}4°$ A

$\therefore$ Sending end current $=$ **306·4 A**

Chapter5 EHV transmission

EHV AC Transmission:

NECESSITY OF EHVAC TRANSMISSION:

1.With the increase in transmission voltage, for same amount of power to be transmitted current in the line decreases which reduces I2R losses. This will lead to increase in transmission efficiency.

2.With decrease in transmission current, size of conductor required reduces which decreases the volume of conductor.

3.The transmission capacity is proportional to square of operating voltages. Thus the transmission capacity of line increases with increase in voltage.

4.With increase in level of transmission voltage, the installation cost of the transmission line per km decreases.

5.It is economical with EHV transmission to interconnect the power systems on a large scale.

6.The no. of circuits and the land requirement for transmission decreases with the use of higher transmission voltages.

ADVANTAGES :

- Reduction in the current.
- Reduction in the losses.
- Reduction in volume of conductor material required.
- Decrease in voltage drop & improvement of voltage regulation.
- Increase in Transmission Efficiency.
- Increased power handling capacity.
- The no. of circuits & the land requirement reduces as transmission voltage increases.
- The total line cost per MW per km decreases considerably with the increase in line voltage.

PROBLEMS INVOLVED IN EHV TRANSMISSION:

1. Corona loss and radio interference
2. Heavy supporting structure and erection difficulties
3. Insulation requirement
4. Suitability considerations
5. Current carrying capacity
6. Ferranti effect
7. Environmental and biological aspects
8. Equipment cost

HVDC Transmission System

We know that AC power is generated in the generating station. This should first be converted into DC. The conversion is done with the help of rectifier. The DC power will flow through the overhead lines. At the user end, this DC has to be converted into AC. For that purpose, an inverter is placed at the receiving end.

Thus, there will be a rectifier terminal in one end of HVDC substation and an inverter terminal in the other end. The power of the sending end and user end will be always equal (Input Power = Output Power).

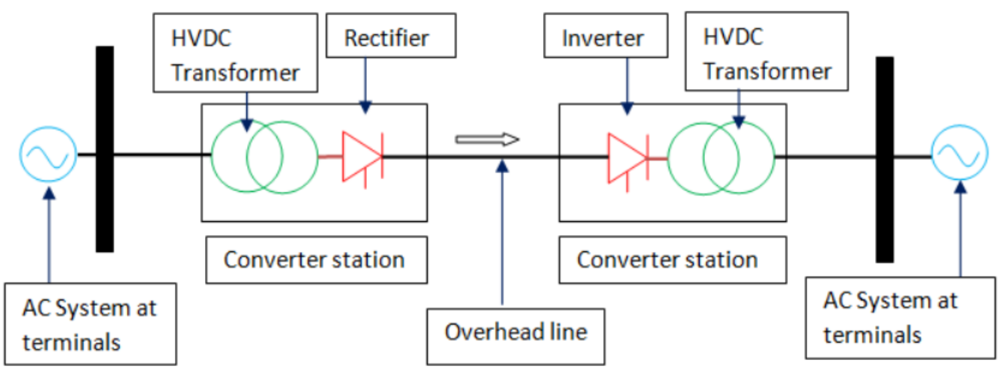


Figure 1: HVDC Substation Layout

When there are two converter stations at both ends and a single transmission line is termed as two terminal DC systems. When there are two or more converter stations and DC transmission lines is termed as multi-terminal DC substation.

Comparison of both HVAC and HVDC Transmission System

HVDC Transmission System	HVAC Transmission System
Low losses.	Losses are high due to the skin effect and corona discharge

Better Voltage regulation and Control ability.	Voltage regulation and Control ability is low.
Transmit more power over a longer distance.	Transmit less power compared to a HVDC system.
Less insulation is needed.	More insulation is required.
Reliability is high.	Low Reliability.
Asynchronous interconnection is possible.	Asynchronous interconnection is not possible.
Reduced line cost due to fewer conductors.	Line cost is high.
Towers are cheaper, simple and narrow.	Towers are bigger compared to HVDC.

Disadvantages of HVDC Transmission

- Converters with small overload capacity are used.
- Circuit Breakers, Converters and AC filters are expensive especially for small distance transmission.
- No transformers for altering the voltage level.
- HVDC link is extremely complicated.
- Uncontrollable power flow.

Application of HVDC Transmission

- Undersea and underground cables
- AC network interconnections
- Interconnecting Asynchronous system

Chapter 6 Distribution systems

Types of D.C. Distributors

The most general method of classifying d.c. distributors is the way they are fed by the feeders. On this basis, d.c. distributors are classified as:

(*i*) Distributor fed at one end

(*ii*) Distributor fed at both ends

(*iii*) Distributor fed at the centre

(*iv*) Ring distributor.

(*i*) **Distributor fed at one end.** In this type of feeding, the distributor is connected to the supply at one end and loads are taken at different points along the length of the distributor. Fig. 13.1 shows the single line diagram of a d.c. distributor *A B* fed at the end *A* (also known as *singly fed distributor*) and loads I_1, I_2 and I_3 tapped off at points *C*, *D* and *E* respectively.

Fig. 13.1

The following points are worth noting in a singly fed distributor :

(*a*) The current in the various sections of the distributor away from feeding point goes on decreasing. Thus current in section *AC* is more than the current in section *CD* and current in section *CD* is more than the current in section *DE*.

(*b*) The voltage across the loads away from the feeding point goes on decreasing. Thus in Fig. 13.1, the minimum voltage occurs at the load point *E*.

(*c*) In case a fault occurs on any section of the distributor, the whole distributor will have to be disconnected from the supply mains. Therefore, continuity of supply is interrupted.

(*ii*) **Distributor fed at both ends.** In this type of feeding, the distributor is connected to the supply mains at both ends and loads are tapped off at different points along the length of the distributor. The voltage at the feeding points may or may not be equal. Fig. 13.2 shows a distributor *A B* fed at the ends *A* and *B* and loads of I_1, I_2 and I_3 tapped off at points *C*, *D* and *E* respectively. Here, the load voltage goes on decreasing as we move away from one feeding point *say A*, reaches minimum value and then again starts rising and reaches maximum value when we reach the other feeding point *B*. The minimum voltage occurs at some load point and is never fixed. It is shifted with the variation of load on different sections of the distributor.

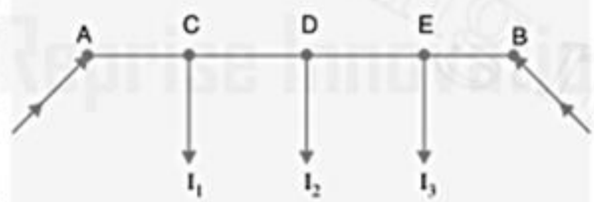


Fig. 13.2

Advantages

(*a*) If a fault occurs on any feeding point of the distributor, the continuity of supply is maintained from the other feeding point.

(*b*) In case of fault on any section of the distributor, the continuity of supply is maintained from the other feeding point.

(c) The area of X-section required for a doubly fed distributor is much less than that of a singly fed distributor.

(*iii*) **Distributor fed at the centre.** In this type of feeding, the centre of the distributor is connected to the supply mains as shown in Fig. 13.3. It is equivalent to two singly fed distributors, each distributor having a common feeding point and length equal to half of the total length.

Fig. 13.3

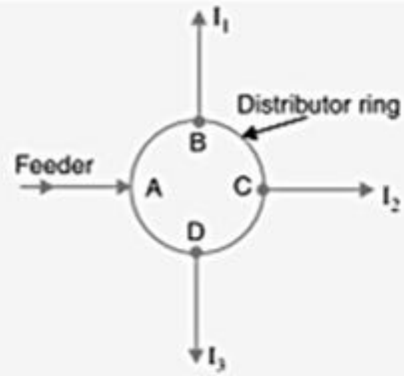
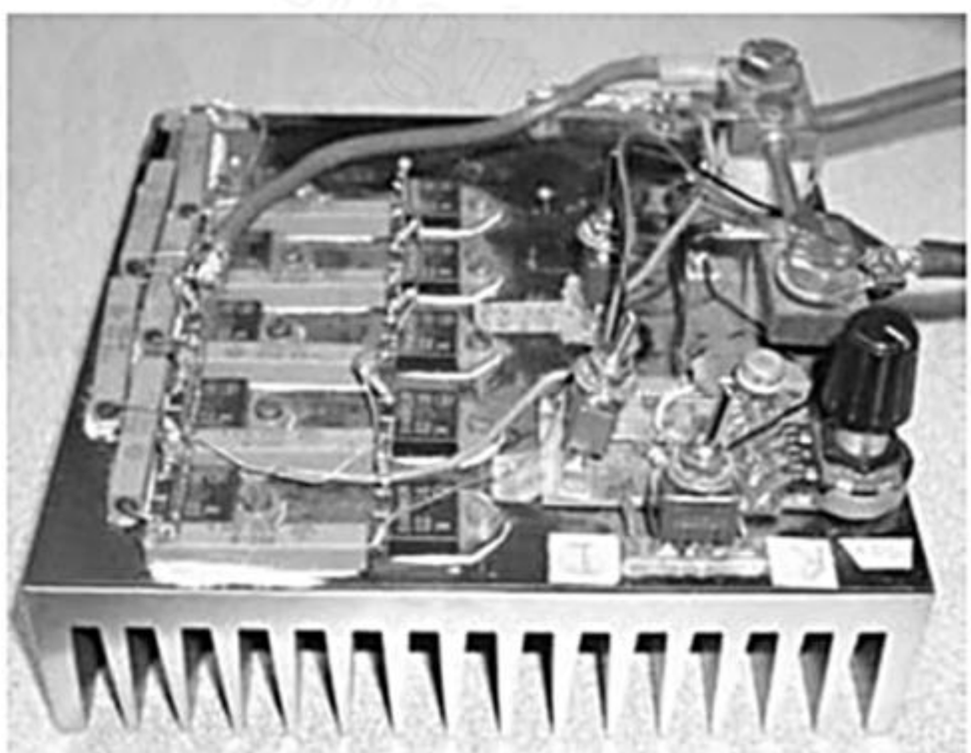


Fig. 13.4

(*iv*) **Ring mains.** In this type, the distributor is in the form of a closed ring as shown in Fig.13.4. It is equivalent to a straight distributor fed at both ends with equal voltages, the two ends being brought together to form a closed ring. The distributor ring may be fed at one or more than one point.

D.C. Distribution Calculations

In addition to the methods of feeding discussed above, a distributor may have (*i*) concentrated loading (*ii*) uniform loading (*iii*) both concentrated and uniform loading. The concentrated loads are those which act on particular points of the distributor. A common example of such loads is that tapped off for domestic use. On the other hand, distributed loads are those which act uniformly on all points of the distributor.



D.C. Load

Ideally, there are no distributed loads. However, a nearest example of distributed load is a large number of loads of same wattage connected to the distributor at equal distances.

In d.c. distribution calculations, one important point of interest is the determination of point of minimum potential on the distributor. The point where it occurs depends upon the loading conditions and the method of feeding the distributor. The distributor is so designed that the minimum potential on it is not less than 6% of rated voltage at the consumer's terminals. In the next sections, we shall discuss some important cases of d.c. distributors separately.

D.C. Distributor Fed at one End—Concentrated Loading

Fig. 13.5 shows the single line diagram of a 2-wire d.c. distributor AB fed at one end A and having concentrated loads I_1, I_2, I_3 and I_4 tapped off at points C, D, E and F respectively.

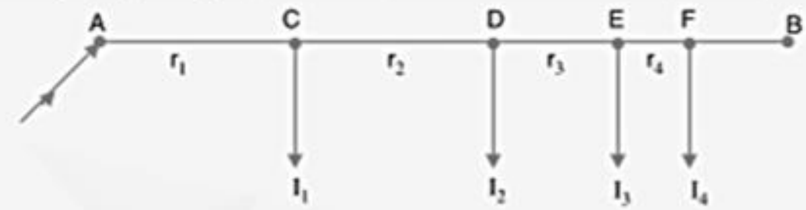


Fig. 13.5

Let r_1, r_2, r_3 and r_4 be the resistances of both wires (go and return) of the sections AC, CD, DE and EF of the distributor respectively.

Current fed from point A $= I_1 + I_2 + I_3 + I_4$

Current in section AC $= I_1 + I_2 + I_3 + I_4$

Current in section CD $= I_2 + I_3 + I_4$

Current in section DE $= I_3 + I_4$

Current in section EF $= I_4$

Voltage drop in section AC $= r_1 (I_1 + I_2 + I_3 + I_4)$

Voltage drop in section CD $= r_2 (I_2 + I_3 + I_4)$

Voltage drop in section DE $= r_3 (I_3 + I_4)$

Voltage drop in section EF $= r_4 I_4$

∴ Total voltage drop in the distributor

$$= r_1 (I_1 + I_2 + I_3 + I_4) + r_2 (I_2 + I_3 + I_4) + r_3 (I_3 + I_4) + r_4 I_4$$

It is easy to see that the minimum potential will occur at point F which is farthest from the feeding point A.

Example . *A 2-wire d.c. distributor cable AB is 2 km long and supplies loads of 100A, 150A, 200A and 50A situated 500 m, 1000 m, 1600 m and 2000 m from the feeding point A. Each conductor has a resistance of 0·01 Ω per 1000 m. Calculate the p.d. at each load point if a p.d. of 300 V is maintained at point A.*

Solution. Fig. 13.6 shows the single line diagram of the distributor with its tapped currents.

Resistance per 1000 m of distributor $= 2 \times 0{\cdot}01 = 0{\cdot}02\,\Omega$

Resistance of section AC, R_{AC} $= 0{\cdot}02 \times 500/1000 = 0{\cdot}01\,\Omega$

Resistance of section CD, R_{CD} $= 0{\cdot}02 \times 500/1000 = 0{\cdot}01\,\Omega$

Resistance of section DE, R_{DE} $= 0{\cdot}02 \times 600/1000 = 0{\cdot}012\,\Omega$

Resistance of section EB, R_{EB} $= 0{\cdot}02 \times 400/1000 = 0{\cdot}008\,\Omega$

Referring to Fig. 13.6, the currents in the various sections of the distributor are :

$I_{EB} = 50$ A ; $I_{DE} = 50 + 200 = 250$ A

$I_{CD} = 250 + 150 = 400$ A ; $I_{AC} = 400 + 100 = 500$ A

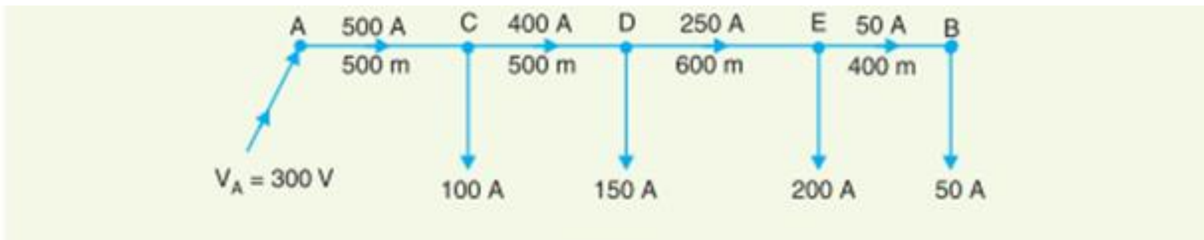


Fig. 13.6

P.D. at load point C,

$$V_C = \text{Voltage at } A - \text{Voltage drop in } AC$$
$$= V_A - I_{AC} R_{AC}$$
$$= 300 - 500 \times 0{\cdot}01 = \mathbf{295\ V}$$

P.D. at load point D,

$$V_D = V_C - I_{CD} R_{CD}$$
$$= 295 - 400 \times 0{\cdot}01 = \mathbf{291\ V}$$

P.D. at load point E,

$$V_E = V_D - I_{DE} R_{DE}$$
$$= 291 - 250 \times 0{\cdot}012 = \mathbf{288\ V}$$

P.D. at load point B,

$$V_B = V_E - I_{EB} R_{EB}$$
$$= 288 - 50 \times 0{\cdot}008 = \mathbf{287{\cdot}6\ V}$$

Example *A 2-wire d.c. distributor AB is 300 metres long. It is fed at point A. The various loads and their positions are given below :*

At point	*distance from A in metres*	*concentrated load in amperes*
C	*40*	*30*
D	*100*	*40*
E	*150*	*100*
F	*250*	*50*

If the maximum permissible voltage drop is not to exceed 10 V, find the cross-sectional area of the distributor. Take $\rho = 1{\cdot}78 \times 10^{-8}\ \Omega m$.

Solution. The single line diagram of the distributor along with its tapped currents is shown in Fig. 13.7. Suppose that resistance of 100 m length of the distributor is r ohms. Then resistance of various sections of the distributor is :

$$R_{AC} = 0{\cdot}4\, r\, \Omega \;;\; R_{CD} = 0{\cdot}6\, r\, \Omega \;;\; R_{DE} = 0{\cdot}5\, r\, \Omega \;;\; R_{EF} = r\, \Omega$$

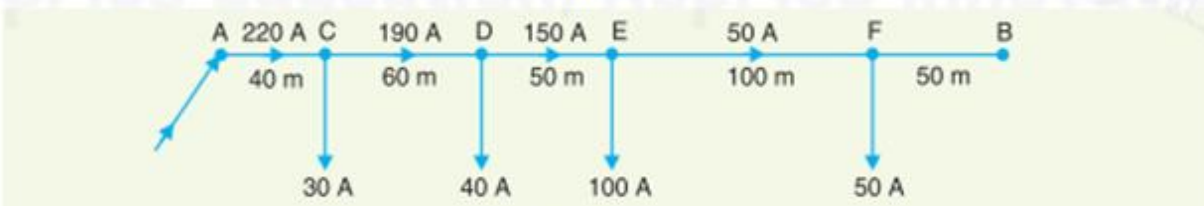


Fig. 13.7

Referring to Fig. 13.7, the currents in the various sections of the distributor are :

$$I_{AC} = 220\text{ A} \;;\; I_{CD} = 190\text{ A} \;;\; I_{DE} = 150\text{ A} \;;\; I_{EF} = 50\text{ A}$$

Total voltage drop over the distributor

$$= I_{AC} R_{AC} + I_{CD} R_{CD} + I_{DE} R_{DE} + I_{EF} R_{EF}$$
$$= 220 \times 0{\cdot}4r + 190 \times 0{\cdot}6r + 150 \times 0{\cdot}5r + 50 \times r$$
$$= 327\, r$$

As the maximum permissible drop in the distributor is 10 V,

$$\therefore \qquad 10 = 327\, r$$

or $r = 10/327 = 0{\cdot}03058\ \Omega$

X-sectional area of conductor $= \frac{*\rho l}{r/2} = \frac{1{\cdot}78\times10^{-8}\times100}{\frac{0{\cdot}03058}{2}} = 116{\cdot}4\times10^{-6}\ \text{m}^2 =$ **$1{\cdot}164\ \text{cm}^2$**

Example *Two tram cars (A & B) 2 km and 6 km away from a sub-station return 40 A and 20 A respectively to the rails. The sub-station voltage is 600 V d.c. The resistance of trolley wire is 0·25 Ω/km and that of track is 0·03 Ω/km. Calculate the voltage across each tram car.*

Solution. The tram car operates on d.c. supply. The positive wire is placed overhead while the rail track acts as the negative wire. Fig. 13.8 shows the single line diagram of the arrangement.

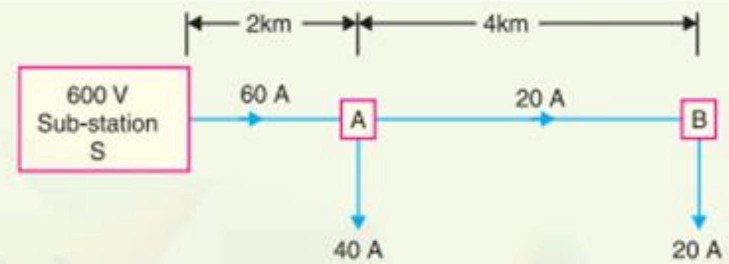


Fig. 13.8

Resistance of trolley wire and track/km

$= 0{\cdot}25 + 0{\cdot}03 = 0{\cdot}28\Omega$

Current in section $SA = 40 + 20 = 60$ A

Current in section $AB = 20$ A

Voltage drop in section $SA = 60\times0{\cdot}28\times2 = 33{\cdot}6$ V

Voltage drop in section $AB = 20\times0{\cdot}28\times4 = 22{\cdot}4$ V

∴ Voltage across tram $A = 600 - 33{\cdot}6 =$ **566·4 V**

Voltage across tram $B = 566{\cdot}4 - 22{\cdot}4 =$ **544 V**

Uniformly Loaded Distributor Fed at One End

Fig 13.11 shows the single line diagram of a 2-wire d.c. distributor AB fed at one end A and loaded uniformly with i amperes per metre length. It means that at every 1 m length of the distributor, the load tapped is i amperes. Let l metres be the length of the distributor and r ohm be the resistance per metre run.

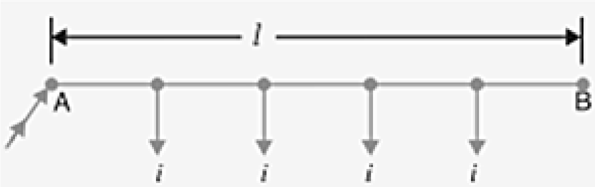


Fig. 13.11

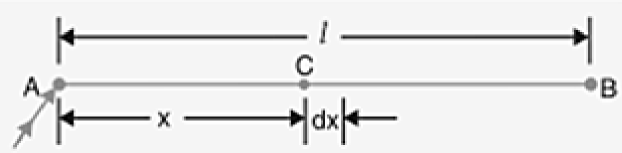


Fig. 13.12

Consider a point C on the distributor at a distance x metres from the feeding point A as shown in Fig. 13.12. Then current at point C is

$$= i\,l - i\,x \text{ amperes} = i\,(l - x) \text{ amperes}$$

Now, consider a small length dx near point C. Its resistance is $r\ dx$ and the voltage drop over length dx is

$$dv = i\,(l - x)\,r\,dx = i\,r\,(l - x)\,dx$$

Total voltage drop in the distributor upto point C is

$$v = \int_0^x i\,r\,(l - x)\,dx = i\,r\left(l\,x - \frac{x^2}{2}\right)$$

The voltage drop upto point B (*i.e.* over the whole distributor) can be obtained by putting $x = l$ in the above expression.

∴ Voltage drop over the distributor AB

$$= i\,r\left(l \times l - \frac{l^2}{2}\right)$$

$$= \tfrac{1}{2}\,i\,r\,l^2 = \tfrac{1}{2}\,(i\,l)\,(r\,l)$$

$$= \tfrac{1}{2}\,I\,R$$

where $i\,l = I$, the total current entering at point A

$r\,l = R$, the total resistance of the distributor

Thus, in a uniformly loaded distributor fed at one end, the total voltage drop is equal to that produced by the whole of the load assumed to be concentrated at the middle point.

Example *A 2-wire d.c. distributor 200 metres long is uniformly loaded with 2A/metre. Resistance of single wire is 0·3 Ω/km. If the distributor is fed at one end, calculate :*

(*i*) *the voltage drop upto a distance of 150 m from the feeding point*

(*ii*) *the maximum voltage drop*

Solution.

Current loading, $i = 2$ A/m

Resistance of distributor per metre run,

$$r = 2 \times 0{\cdot}3/1000 = 0{\cdot}0006\,\Omega$$

Length of distributor, $l = 200$ m

(*i*) Voltage drop upto a distance x metres from feeding point

$$= i\,r\left(l\,x - \frac{x^2}{2}\right) \quad \text{[See Art. 13·4]}$$

Here, $x = 150$ m

∴ Desired voltage drop $= 2 \times 0{\cdot}0006\left(200 \times 150 - \dfrac{150 \times 150}{2}\right) = \mathbf{22{\cdot}5\ V}$

(*ii*) Total current entering the distributor,

$$I = i \times l = 2 \times 200 = 400 \text{ A}$$

Total resistance of the distributor,

$$R = r \times l = 0{\cdot}0006 \times 200 = 0{\cdot}12\,\Omega$$

∴ Total drop over the distributor

$$= \frac{1}{2}\,I\,R = \frac{1}{2} \times 400 \times 0{\cdot}12 = \mathbf{24\ V}$$

Example *Calculate the voltage at a distance of 200 m of a 300 m long distributor uniformly loaded at the rate of 0.75 A per metre. The distributor is fed at one end at 250 V. The resistance of the distributor (go and return) per metre is 0·00018 Ω. Also find the power loss in the distributor.*

Solution.

Voltage drop at a distance x from supply end

$$= i\,r\left(l\,x - \frac{x^2}{2}\right)$$

Here $i = 0{\cdot}75$ A/m; $l = 300$ m ; $x = 200$ m ; $r = 0{\cdot}00018\ \Omega$/m

$$\therefore \quad \text{Voltage drop} = 0{\cdot}75 \times 0{\cdot}00018\left[300 \times 200 - \frac{(200)^2}{2}\right] = 5.4\text{ V}$$

Voltage at a distance of 200 m from supply end

$$= 250 - 5{\cdot}4 = 244{\cdot}6\text{ V}$$

Power loss in the distributor is

$$P = \frac{i^2\,r\,l^3}{3} = \frac{(0\cdot75)^2 \times 0\cdot00018 \times (300)^3}{3} = 911{\cdot}25\text{ W}$$

TUTORIAL PROBLEMS

1. A 2-wire d.c. distributor 500 m long is loaded uniformly at the rate of 0·4A/m. If the voltage drop in the distributor is not to exceed 5V, calculate the area of X-section of each conductor required when the distributor is fed at one end. Take resistivity of conductor material as $1{\cdot}7 \times 10^{-8}\ \Omega$m. [3·4 cm^2]
2. A uniformly distributed load on a distributor of length 500 m is rated at 1 A per metre length. The distributor is fed from one end at 220V. Determine the voltage drop at a distance of 400 m from the feeding point. Assume a loop resistance of $2 \times 10^{-5}\ \Omega$ per metre. [2·4 V]
3. A 250 m, 2-wire d.c. distributor fed from one end is loaded uniformly at the rate of 0·8 A per metre. The resistance of each conductor is 0·0002 Ω per metre. Find the necessary voltage at the feeding point to maintain 250 V at the far end of the distributor. [260 V]

Distributor Fed at Both Ends — Concentrated Loading

Whenever possible, it is desirable that a long distributor should be fed at both ends instead of at one end only, since total voltage drop can be considerably reduced without increasing the cross-section of the conductor. The two ends of the distributor may be supplied with (*i*) equal voltages (*ii*) unequal voltages.

(*i*) **Two ends fed with equal voltages.** Consider a distributor AB fed at both ends with equal voltages V volts and having concentrated loads I_1, I_2, I_3, I_4 and I_5 at points C, D, E, F and G respectively as shown in Fig. 13.14. As we move away from one of the feeding points, *say* A, p.d. goes on decreasing till it reaches the minimum value at some load point, *say* E, and then again starts rising and becomes V volts as we reach the other feeding point B.

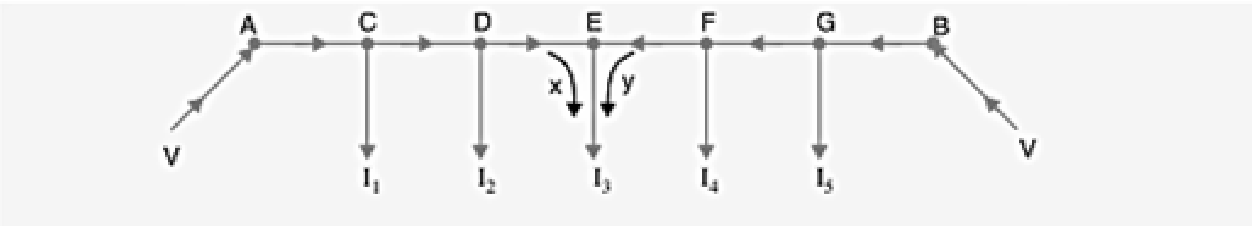


Fig. 13.14

All the currents tapped off between points A and E (minimum p.d. point) will be supplied from the feeding point A while those tapped off between B and E will be supplied from the feeding point B. The current tapped off at point E itself will be partly supplied from A and partly from B. If these currents are x and y respectively, then,

$$I_3 = x + y$$

Therefore, we arrive at a very important conclusion that at the point of minimum potential, current comes from both ends of the distributor.

Point of minimum potential. It is generally desired to locate the point of minimum potential. There is a simple method for it. Consider a distributor AB having three concentrated loads I_1, I_2 and I_3 at points C, D and E respectively. Suppose that current supplied by feeding end A is I_A. Then current distribution in the various sections of the distributor can be worked out as shown in Fig. 13.15 (*i*). Thus

$$I_{AC} = I_A\,; \qquad I_{CD} = I_A - I_1$$

$$I_{DE} = I_A - I_1 - I_2\,; \qquad I_{EB} = I_A - I_1 - I_2 - I_3$$

Fig. 13.15

Voltage drop between A and B = Voltage drop over AB

or $\quad V - V = I_A R_{AC} + (I_A - I_1) R_{CD} + (I_A - I_1 - I_2) R_{DE} + (I_A - I_1 - I_2 - I_3) R_{EB}$

From this equation, the unknown I_A can be calculated as the values of other quantities are generally given. Suppose *actual* directions of currents in the various sections of the distributor are indicated as shown in Fig. 13.15 (*ii*). The load point where the currents are coming from both sides of the distributor is the point of minimum potential *i.e.* point E in this case

(*ii*) **Two ends fed with unequal voltages.** Fig. 13.16 shows the distributor AB fed with unequal voltages ; end A being fed at V_1 volts and end B at V_2 volts. The point of minimum potential can be found by following the same procedure as discussed above. Thus in this case,

Voltage drop between A and B = Voltage drop over AB

or $\quad V_1 - V_2$ = Voltage drop over AB

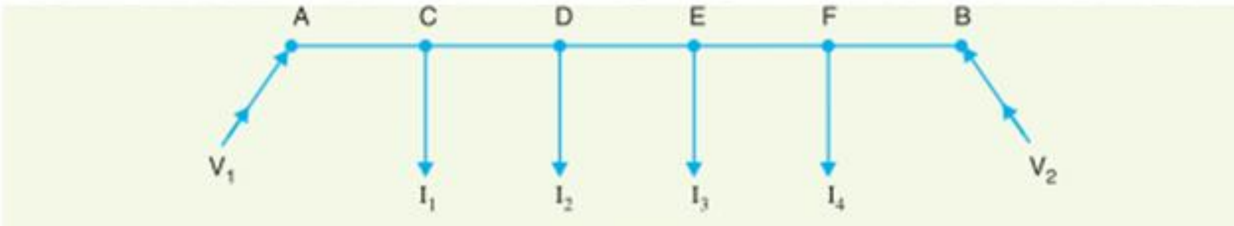


Fig. 13.16

Example *A 2-wire d.c. street mains AB, 600 m long is fed from both ends at 220 V. Loads of 20 A, 40 A, 50 A and 30 A are tapped at distances of 100m, 250m, 400m and 500 m from the end A respectively. If the area of X-section of distributor conductor is 1cm², find the minimum consumer voltage. Take* $\rho = 1{\cdot}7 \times 10^{-6}\ \Omega\,cm$.

Solution. Fig. 13.17 shows the distributor with its tapped currents. Let I_A amperes be the current supplied from the feeding end A. Then currents in the various sections of the distributor are as shown in Fig. 13.17.

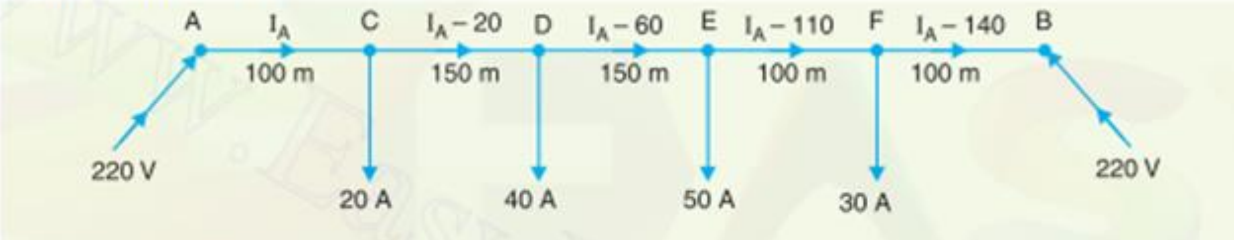


Fig. 13.17

Resistance of 1 m length of distributor

$$= 2 \times \frac{1{\cdot}7 \times 10^{-6} \times 100}{1} = 3{\cdot}4 \times 10^{-4}\ \Omega$$

Resistance of section AC, $R_{AC} = (3{\cdot}4 \times 10^{-4}) \times 100 = 0{\cdot}034\ \Omega$

Resistance of section CD, $R_{CD} = (3{\cdot}4 \times 10^{-4}) \times 150 = 0{\cdot}051\ \Omega$

Resistance of section DE, $R_{DE} = (3{\cdot}4 \times 10^{-4}) \times 150 = 0{\cdot}051\ \Omega$

Resistance of section EF, $R_{EF} = (3{\cdot}4 \times 10^{-4}) \times 100 = 0{\cdot}034\ \Omega$

Resistance of section FB, $R_{FB} = (3{\cdot}4 \times 10^{-4}) \times 100 = 0{\cdot}034\ \Omega$

Voltage at B = Voltage at A − Drop over length AB

or
$$V_B = V_A - [I_A R_{AC} + (I_A - 20) R_{CD} + (I_A - 60) R_{DE} + (I_A - 110) R_{EF} + (I_A - 140) R_{FB}]$$

or
$$220 = 220 - [0{\cdot}034\, I_A + 0{\cdot}051\,(I_A - 20) + 0{\cdot}051\,(I_A - 60) + 0{\cdot}034\,(I_A - 110) + 0{\cdot}034\,(I_A - 140)]$$

$$= 220 - [0{\cdot}204\, I_A - 12{\cdot}58]$$

or
$$0{\cdot}204 I_A = 12{\cdot}58$$

∴
$$I_A = 12{\cdot}58/0{\cdot}204 = 61{\cdot}7\text{ A}$$

The *actual distribution of currents in the various sections of the distributor is shown in Fig. 13.18. It is clear that currents are coming to load point E from both sides *i.e.* from point D and point F. Hence, E is the point of minimum potential.

∴ Minimum consumer voltage,

$$V_E = V_A - [I_{AC} R_{AC} + I_{CD} R_{CD} + I_{DE} R_{DE}]$$

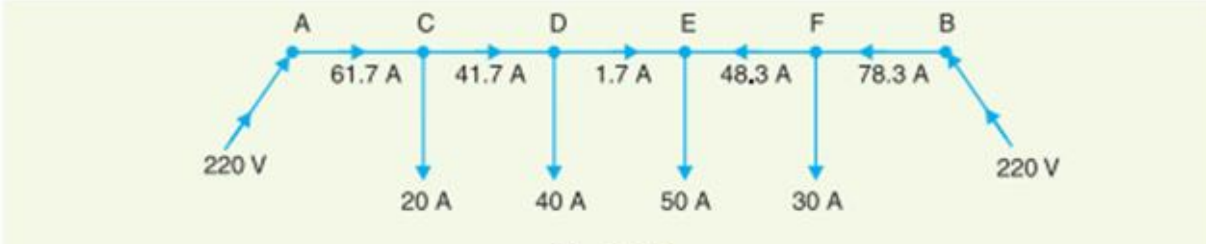


Fig. 13.18

$$= 220 - [61{\cdot}7 \times 0{\cdot}034 + 41{\cdot}7 \times 0{\cdot}051 + 1{\cdot}7 \times 0{\cdot}051]$$
$$= 220 - 4{\cdot}31 = \mathbf{215{\cdot}69\ V}$$

Example *A 2-wire d.c. distributor AB is fed from both ends. At feeding point A, the voltage is maintained as at 230 V and at B 235 V. The total length of the distributor is 200 metres and loads are tapped off as under :*

25 A at 50 metres from A ; 50 A at 75 metres from A

30 A at 100 metres from A ; 40 A at 150 metres from A

The resistance per kilometre of one conductor is 0·3 Ω. Calculate :

(i) currents in various sections of the distributor

(ii) minimum voltage and the point at which it occurs

Solution. Fig. 13.19 shows the distributor with its tapped currents. Let I_A amperes be the current supplied from the feeding point A. Then currents in the various sections of the distributor are as shown in Fig 13.19.

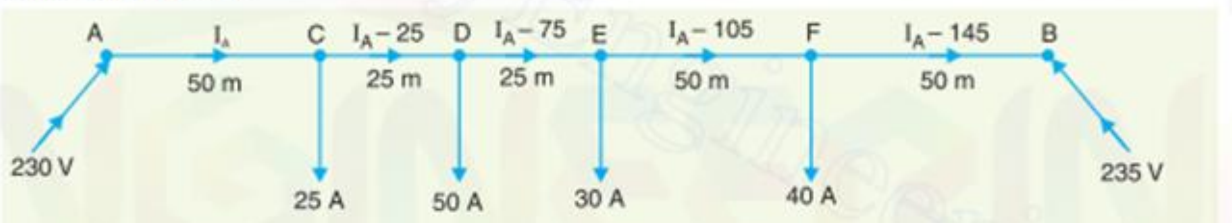


Fig. 13.19

Resistance of 1000 m length of distributor (both wires)

$$= 2 \times 0{\cdot}3 = 0{\cdot}6\,\Omega$$

Resistance of section AC, $R_{AC} = 0{\cdot}6 \times 50/1000 = 0{\cdot}03\,\Omega$

Resistance of section CD, $R_{CD} = 0{\cdot}6 \times 25/1000 = 0{\cdot}015\,\Omega$

Resistance of section DE, $R_{DE} = 0{\cdot}6 \times 25/1000 = 0{\cdot}015\,\Omega$

Resistance of section EF, $R_{EF} = 0{\cdot}6 \times 50/1000 = 0{\cdot}03\,\Omega$

Resistance of section FB, $R_{FB} = 0{\cdot}6 \times 50/1000 = 0{\cdot}03\,\Omega$

Voltage at B = Voltage at A − Drop over AB

or
$$V_B = V_A - [I_A R_{AC} + (I_A - 25) R_{CD} + (I_A - 75) R_{DE} + (I_A - 105) R_{EF} + (I_A - 145) R_{FB}]$$

or
$$235 = 230 - [0{\cdot}03\, I_A + 0{\cdot}015\,(I_A - 25) + 0{\cdot}015\,(I_A - 75) + 0{\cdot}03\,(I_A - 105) + 0{\cdot}03\,(I_A - 145)]$$

or
$$235 = 230 - [0{\cdot}12\, I_A - 9]$$

∴
$$I_A = \frac{239 - 235}{0{\cdot}12} = 33{\cdot}34\ \text{A}$$

(i) ∴ Current in section AC, $I_{AC} = I_A = \mathbf{33{\cdot}34\ A}$

Current in section CD, $I_{CD} = I_A - 25 = 33{\cdot}34 - 25 = \mathbf{8{\cdot}34\ A}$

Current in section DE, $I_{DE} = I_A - 75 = 33{\cdot}34 - 75 = -41{\cdot}66$ A *from D to E*
$= $ **41·66 A** *from E to D*

Current in section EF, $I_{EF} = I_A - 105 = 33{\cdot}34 - 105 = -71{\cdot}66$ A *from E to F*
$= $ **71·66 A** *from F to E*

Current in section FB, $I_{FB} = I_A - 145 = 33{\cdot}34 - 145 = -111{\cdot}66$ A *from F to B*
$= $ **111·66 A** *from B to F*

(*ii*) The actual distribution of currents in the various sections of the distributor is shown in Fig. 13.20. The currents are coming to load point D from both sides of the distributor. Therefore, load point D is the point of minimum potential.

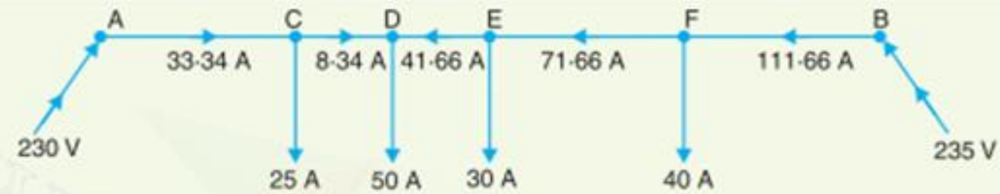


Fig. 13.20

$$\text{Voltage at } D,\ V_D = V_A - [I_{AC} R_{AC} + I_{CD} R_{CD}]$$
$$= 230 - [33{\cdot}34 \times 0{\cdot}03 + 8{\cdot}34 \times 0{\cdot}015]$$
$$= 230 - 1{\cdot}125 = \mathbf{228{\cdot}875\ V}$$

Example *A two-wire d.c. distributor AB, 600 metres long is loaded as under :*

Distance from A (metres) :	*150*	*300*	*350*	*450*
Loads in Amperes :	*100*	*200*	*250*	*300*

The feeding point A is maintained at 440 V and that of B at 430 V. If each conductor has a resistance of 0·01 Ω per 100 metres, calculate :

(*i*) *the currents supplied from A to B,* (*ii*) *the power dissipated in the distributor.*

Solution. Fig. 13.21 shows the distributor with its tapped currents. Let I_A amperes be the current supplied from the feeding point A. Then currents in the various sections of the distributor are as shown in Fig.13.21.

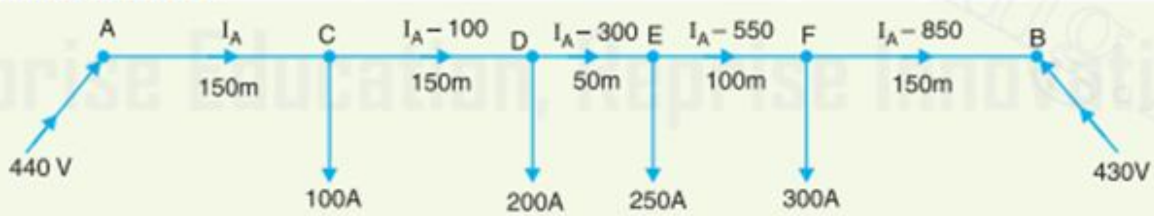


Fig. 13.21

Resistance of 100 m length of distributor (both wires)
$$= 2 \times 0{\cdot}01 = 0{\cdot}02\,\Omega$$

Resistance of section AC, $R_{AC} = 0{\cdot}02 \times 150/100 = 0{\cdot}03\,\Omega$
Resistance of section CD, $R_{CD} = 0{\cdot}02 \times 150/100 = 0{\cdot}03\,\Omega$
Resistance of section DE, $R_{DE} = 0{\cdot}02 \times 50/100 = 0{\cdot}01\,\Omega$
Resistance of section EF, $R_{EF} = 0{\cdot}02 \times 100/100 = 0{\cdot}02\,\Omega$
Resistance of section FB, $R_{FB} = 0{\cdot}02 \times 150/100 = 0{\cdot}03\,\Omega$

Voltage at B = Voltage at A — Drop over AB

or
$$V_B = V_A - [I_A R_{AC} + (I_A - 100) R_{CD} + (I_A - 300) R_{DE} + (I_A - 550) R_{EF} + (I_A - 850) R_{FB}]$$

or $$430 = 440 - [0{\cdot}03\, I_A + 0{\cdot}03\,(I_A - 100) + 0{\cdot}01\,(I_A - 300) + 0{\cdot}02\,(I_A - 550) + 0{\cdot}03\,(I_A - 850)]$$

or $$430 = 440 - [0{\cdot}12\, I_A - 42{\cdot}5]$$

$$\therefore \quad I_A = \frac{482{\cdot}5 - 430}{0{\cdot}12} = 437{\cdot}5 \text{ A}$$

The actual distribution of currents in the various sections of the distributor is shown in Fig.13.22. Incidentally, E is the point of minimum potential.

(*i*) Referring to Fig. 13.22, it is clear that

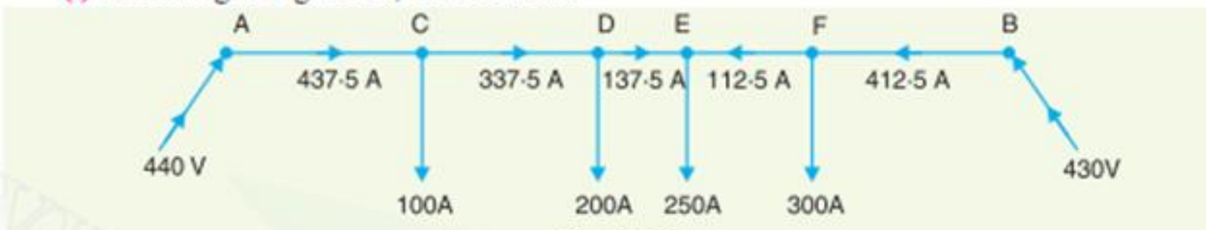


Fig. 13.22

Current supplied from end A, I_A = **437·5 A**

Current supplied from end B, I_B = **412·5 A**

(*ii*) Power loss in the distributor

$$= I_{AC}^2 R_{AC} + I_{CD}^2 R_{CD} + I_{DE}^2 R_{DE} + I_{EF}^2 R_{EF} + I_{FB}^2 R_{FB}$$

$$= (437{\cdot}5)^2 \times 0{\cdot}03 + (337{\cdot}5)^2 \times 0{\cdot}03 + (137{\cdot}5)^2 \times 0{\cdot}01 + (112{\cdot}5)^2 \times 0{\cdot}02 + (412{\cdot}5)^2 \times 0{\cdot}03$$

= 5742 + 3417 + 189 + 253 + 5104 = 14,705 watts = **14·705 kW**

Example *An electric train runs between two sub-stations 6 km apart maintained at voltages 600 V and 590 V respectively and draws a constant current of 300 A while in motion. The track resistance of go and return path is 0·04 Ω/km. Calculate :*

(*i*) *the point along the track where minimum potential occurs*

(*ii*) *the current supplied by the two sub-stations when the train is at the point of minimum potential*

Solution. The single line diagram is shown in Fig. 13.23 where substation A is at 600 V and sub-station B at 590 V. Suppose that minimum potential occurs at point M at a distance x km from the substation A. Let I_A amperes be the current supplied by the sub-station A. Then current supplied by sub-station B is $300 - I_A$ as shown in Fig 13.23.

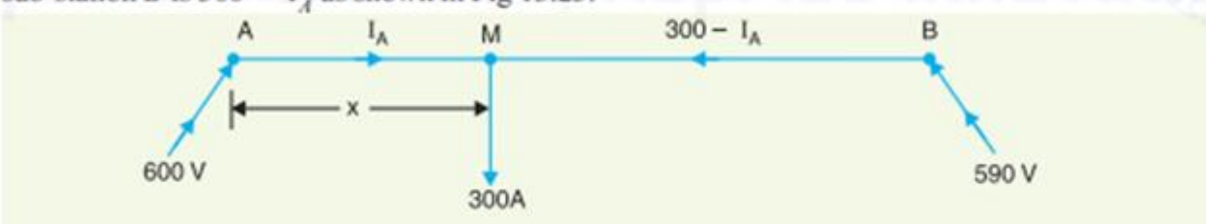


Fig. 13.23

Resistance of track (go and return path) per km

$$= 0{\cdot}04\,\Omega$$

Track resistance for section AM, $R_{AM} = 0{\cdot}04\,x\ \Omega$

Track resistance for section MB, $R_{MB} = 0{\cdot}04\,(6 - x)\Omega$

Potential at M, $V_M = V_A - I_A R_{AM}$ ... (*i*)

Also, Potential at M, $V_M = V_B - (300 - I_A) R_{MB}$ ... (*ii*)

From equations (*i*) and (*ii*), we get,

$$V_A - I_A R_{AM} = V_B - (300 - I_A) R_{MB}$$

or $$600 - 0{\cdot}04x\, I_A = 590 - (300 - I_A) \times 0{\cdot}04\,(6 - x)$$

or $$600 - 0{\cdot}04x\, I_A = 590 - 0{\cdot}04\,(1800 - 300\,x - 6\,I_A + I_A \times x)$$

or $$600 - 0{\cdot}04x\, I_A = 590 - 72 + 12\,x + 0{\cdot}24\,I_A - 0{\cdot}04xI_A$$

or $$0{\cdot}24 I_A = 82 - 12\,x$$

or $$I_A = 341{\cdot}7 - 50\,x$$

Substituting the value of I_A in eq. (*i*), we get,

$$V_M = V_A - (341{\cdot}7 - 50\,x) \times 0{\cdot}04x$$

∴ $$V_M = 600 - 13{\cdot}7x + 2x^2 \qquad ...(iii)$$

(*i*) For V_M to be minimum, its differential coefficient *w.r.t.* x must be zero *i.e.*

$$\frac{d}{dx}(600 - 13{\cdot}7x + 2x^2) = 0$$

or $$0 - 13{\cdot}7 + 4x = 0$$

∴ $$x = 13{\cdot}7/4 = \mathbf{3{\cdot}425\ km}$$

i.e. minimum potential occurs at a distance of 3·425 km from the sub-station A.

(*ii*) ∴ Current supplied by sub-station A

$$= 341{\cdot}7 - 50 \times 3{\cdot}425 = 341{\cdot}7 - 171{\cdot}25 = \mathbf{170{\cdot}45\ A}$$

Current supplied by sub-station B $= 300 - I_A = 300 - 170{\cdot}45 = \mathbf{129{\cdot}55\ A}$

TUTORIAL PROBLEMS

1. A 2-wire d.c. distributor AB is fed at both ends at the same voltage of 230 V. The length of the distributor is 500 metres and the loads are tapped off from the end A as shown below :

Load :	100 A	60 A	40 A	100 A
Distance :	50 m	150 m	250 m	400 m

If the maximum voltage drop of 5·5 V is to be allowed, find the X-sectional area of each conductor and point of minimum potential. Specific resistance of conductor material may be taken as $1{\cdot}73 \times 10^{-8}\ \Omega$ m. **[1·06 cm² ; 250 m from A]**

2. A d.c. distributor AB is fed at both ends. At feeding point A, the voltage is maintained at 235 V and at B at 236 V. The total length of the distributor is 200 metres and loads are tapped off as under :

20 A at 50 m from A
40 A at 75 m from A
25 A at 100 m from A
30 A at 150 m from A

The resistance per kilometre of one conductor is 0·4 Ω. Calculate the minimum voltage and the point at which it occurs. **[232·175 V ; 75 m from point A]**

3. A two conductor main AB, 500 m in length is fed from both ends at 250 volts. Loads of 50 A, 60 A, 40 A and 30 A are tapped at distance of 100 m, 250 m, 350 m and 400 m from end A respectively. If the X-section of conductor be 1 cm² and specific resistance of the material of the conductor is 1·7 μ Ω cm, determine the minimum consumer voltage. **[245·07 V]**

Ring Distributor

A distributor arranged to form a closed loop and fed at one or more points is called a *ring distributor*. Such a distributor starts from one point, makes a loop through the area to be served, and returns to the original point. For the purpose of calculating voltage distribution, the distributor can be considered as consisting of a series of open distributors fed at both ends. The principal advantage of ring distributor is that by proper choice in the number of feeding points, great economy in copper can be affected.

The most simple case of a ring distributor is the one having only one feeding point as shown in Fig. 13.36(*ii*). Here A is the feeding point and tappings are taken from points B and C. For the purpose of calculations, it is equivalent to a straight distributor fed at both ends with equal voltages.

Example *A 2-wire d.c. ring distributor is 300 m long and is fed at 240 V at point A. At point B, 150 m from A, a load of 120 A is taken and at C, 100 m in the opposite direction, a load of 80 A is taken. If the resistance per 100 m of single conductor is 0·03 Ω, find :*

(*i*) *current in each section of distributor*

(*ii*) *voltage at points B and C*

Solution.

Resistance per 100 m of distributor

$$= 2 \times 0{\cdot}03 = 0{\cdot}06\,\Omega$$

Resistance of section AB, $R_{AB} = 0{\cdot}06 \times 150/100 = 0{\cdot}09\,\Omega$

Resistance of section BC, $R_{BC} = 0{\cdot}06 \times 50/100 = 0{\cdot}03\,\Omega$

Resistance of section CA, $R_{CA} = 0{\cdot}06 \times 100/100 = 0{\cdot}06\,\Omega$

(*i*) Let us suppose that a current I_A flows in section AB of the distributor. Then currents in sections BC and CA will be $(I_A - 120)$ and $(I_A - 200)$ respectively as shown in Fig. 13.36 (*i*).

According to Kirchhoff's voltage law, the voltage drop in the closed loop $ABCA$ is zero *i.e.*

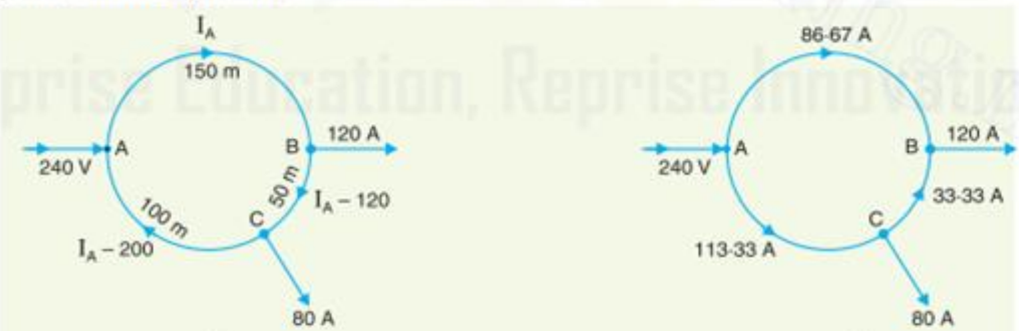
$$I_{AB} R_{AB} + I_{BC} R_{BC} + I_{CA} R_{CA} = 0$$

or $$0{\cdot}09 I_A + 0{\cdot}03\,(I_A - 120) + 0{\cdot}06\,(I_A - 200) = 0$$

or $$0{\cdot}18 I_A = 15{\cdot}6$$

∴ $$I_A = 15{\cdot}6/0{\cdot}18 = 86{\cdot}67 \text{ A}$$

The actual distribution of currents is as shown in Fig. 13.36 (*ii*) from where it is seen that B is the point of minimum potential.



(*i*) **Fig. 13.36** (*ii*)

Current in section AB, $I_{AB} = I_A =$ **86·67 A** *from A to B*

Current in section BC, $I_{BC} = I_A - 120 = 86{\cdot}67 - 120 = -\,33{\cdot}33$ A

$=$ **33.33 A** *from C to B*

Current in section CA, $I_{CA} = I_A - 200 = 86{\cdot}67 - 200 = -\,113{\cdot}33$ A

$=$ **113 ·33 A** *from A to C*

(*ii*) Voltage at point B, $V_B = V_A - I_{AB} R_{AB} = 240 - 86{\cdot}67 \times 0{\cdot}09 =$ **232·2 V**

Voltage at point C, $V_C = V_B + I_{BC}\,R_{BC}$

$= 232{\cdot}2 + 33{\cdot}33 \times 0{\cdot}03 =$ **233·2 V**

Example *A 2-wire d.c. distributor ABCDEA in the form of a ring main is fed at point A at 220 V and is loaded as under :*

10A at B ; 20A at C ; 30A at D and 10 A at E.

The resistances of various sections (go and return) are : AB = 0·1 Ω ; BC = 0·05 Ω ; CD = 0·01 Ω ; DE = 0·025 Ω and EA = 0·075 Ω. Determine :

(*i*) *the point of minimum potential*

(*ii*) *current in each section of distributor*

Solution. Fig. 13.37 (*i*) shows the ring main distributor. Let us suppose that current I flows in section AB of the distributor. Then currents in the various sections of the distributor are as shown in Fig. 13.37 (*i*).

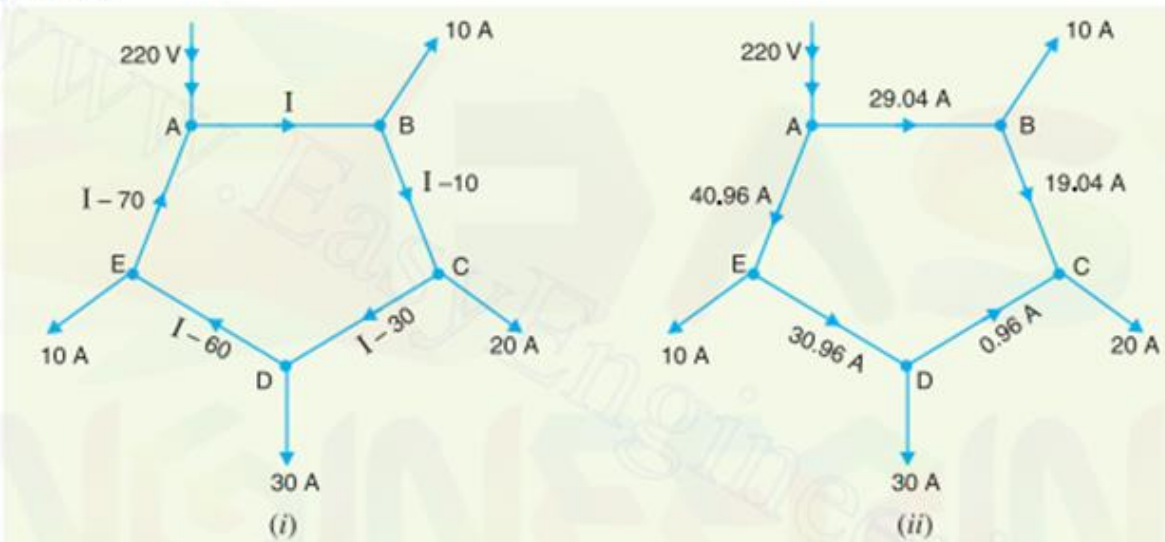


Fig. 13.37

(*i*) According to Kirchhoff's voltage law, the voltage drop in the closed loop $ABCDEA$ is zero *i.e.*

$$I_{AB}\,R_{AB} + I_{BC}\,R_{BC} + I_{CD}\,R_{CD} + I_{DE}\,R_{DE} + I_{EA}\,R_{EA} = 0$$

or $$0{\cdot}1I + 0{\cdot}05\,(I - 10) + 0{\cdot}01\,(I - 30) + 0{\cdot}025\,(I - 60) + 0{\cdot}075\,(I - 70) = 0$$

or $$0{\cdot}26\,I = 7{\cdot}55$$

∴ $$I = 7{\cdot}55/0{\cdot}26 = 29{\cdot}04 \text{ A}$$

The actual distribution of currents is as shown in Fig. 13.37 (*ii*) from where it is clear that C is the point of minimum potential.

∴ C is the point of minimum potential.

(*ii*) Current in section AB $= I =$ **29·04 A** *from A to B*

Current in section BC $= I - 10 = 29{\cdot}04 - 10 =$ **19·04 A** *from B to C*

Current in section CD $= I - 30 = 29{\cdot}04 - 30 = -\,0{\cdot}96$ A $=$ **0·96 A** *from D to C*

Current in section DE $= I - 60 = 29{\cdot}04 - 60 = -\,30{\cdot}96$ A $=$ **30·96 A** *from E to D*

Current in section EA $= I - 70 = 29{\cdot}04 - 70 = -\,40{\cdot}96$ A $=$ **40·96 A** *from A to E*

A.C. Distribution Calculations

A.C. distribution calculations differ from those of d.c. distribution in the following respects :

(*i*) In case of d.c. system, the voltage drop is due to resistance alone. However, in a.c. system, the voltage drops are due to the combined effects of resistance, inductance and capacitance.

(*ii*) In a d.c. system, additions and subtractions of currents or voltages are done arithmetically but in case of a.c. system, these operations are done vectorially.

(*iii*) In an a.c. system, power factor (p.f.) has to be taken into account. Loads tapped off form the distributor are generally at different power factors. There are two ways of referring power factor *viz*

(*a*) It may be referred to supply or receiving end voltage which is regarded as the reference vector.

(*b*) It may be referred to the voltage at the load point itself.

There are several ways of solving a.c. distribution problems. However, symbolic notation method has been found to be most convenient for this purpose. In this method, voltages, currents and impedances are expressed in complex notation and the calculations are made exactly as in d.c. distribution.

Methods of Solving A.C. Distribution Problems

In a.c. distribution calculations, power factors of various load currents have to be considered since currents in different sections of the distributor will be the vector sum of load currents and not the arithmetic sum. The power factors of load currents may be given (*i*) *w.r.t.* receiving or sending end voltage or (*ii*) *w.r.t.* to load voltage itself. Each case shall be discussed separately.

(*i*) **Power factors referred to receiving end voltage.** Consider an a.c. distributor AB with concentrated loads of I_1 and I_2 tapped off at points C and B as shown in Fig. 14.1. Taking the receiving end voltage V_B as the reference vector, let lagging power factors at C and B be $\cos\phi_1$ and $\cos\phi_2$ *w.r.t.* V_B. Let R_1, X_1 and R_2, X_2 be the resistance and reactance of sections AC and CB of the distributor.

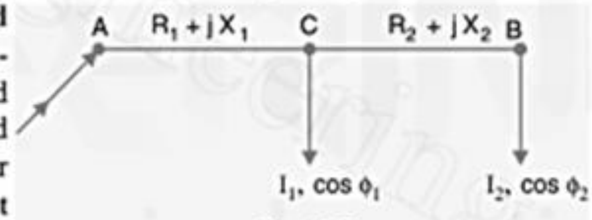


Fig. 14.1

Impedance of section AC, $\overrightarrow{Z_{AC}} = R_1 + jX_1$

Impedance of section CB, $\overrightarrow{Z_{CB}} = R_2 + jX_2$

Load current at point C, $\overrightarrow{I_1} = I_1(\cos\phi_1 - j\sin\phi_1)$

Load current at point B, $\overrightarrow{I_2} = I_2(\cos\phi_2 - j\sin\phi_2)$

Current in section CB, $\overrightarrow{I_{CB}} = \overrightarrow{I_2} = I_2(\cos\phi_2 - j\sin\phi_2)$

Current in section AC,
$$\overrightarrow{I_{AC}} = \overrightarrow{I_1} + \overrightarrow{I_2}$$
$$= I_1(\cos\phi_1 - j\sin\phi_1) + I_2(\cos\phi_2 - j\sin\phi_2)$$

Voltage drop in section CB, $\overrightarrow{V_{CB}} = \overrightarrow{I_{CB}}\,\overrightarrow{Z_{CB}} = I_2(\cos\phi_2 - j\sin\phi_2)(R_2 + jX_2)$

Voltage drop in section AC, $\overrightarrow{V_{AC}} = \overrightarrow{I_{AC}}\,\overrightarrow{Z_{AC}} = \left(\overrightarrow{I_1} + \overrightarrow{I_2}\right)Z_{AC}$

$$= [I_1(\cos\phi_1 - j\sin\phi_1) + I_2(\cos\phi_2 - j\sin\phi_2)]\,[R_1 + jX_1]$$

Sending end voltage, $\overrightarrow{V_A} = \overrightarrow{V_B} + \overrightarrow{V_{CB}} + \overrightarrow{V_{AC}}$

Sending end current, $\overrightarrow{I_A} = \overrightarrow{I_1} + \overrightarrow{I_2}$

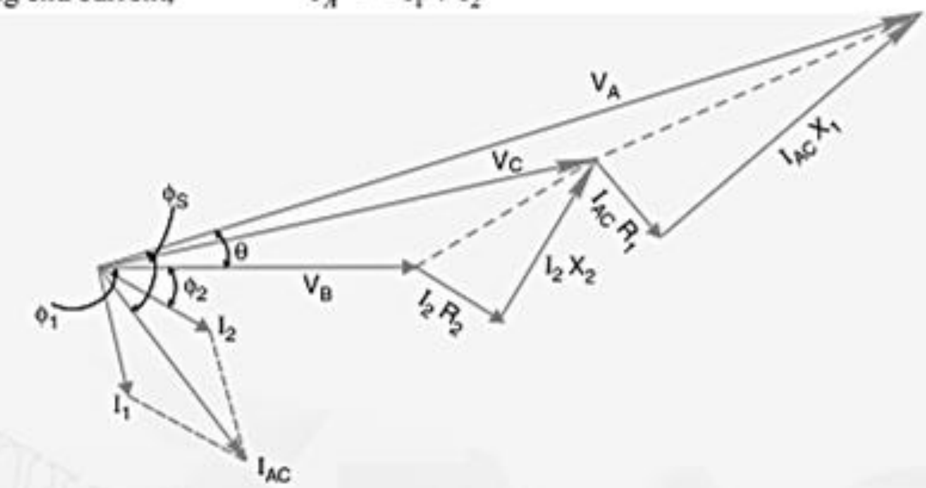


Fig. 14.2

The vector diagram of the a.c. distributor under these conditions is shown in Fig. 14.2. Here, the receiving end voltage V_B is taken as the reference vector. As power factors of loads are given *w.r.t.* V_B, therefore, I_1 and I_2 lag behind V_B by ϕ_1 and ϕ_2 respectively.

(*ii*) Power factors referred to respective load voltages. Suppose the power factors of loads in the previous Fig. 14.1 are referred to their respective load voltages. Then ϕ_1 is the phase angle between V_C and I_1 and ϕ_2 is the phase angle between V_B and I_2. The vector diagram under these conditions is shown in Fig. 14.3.

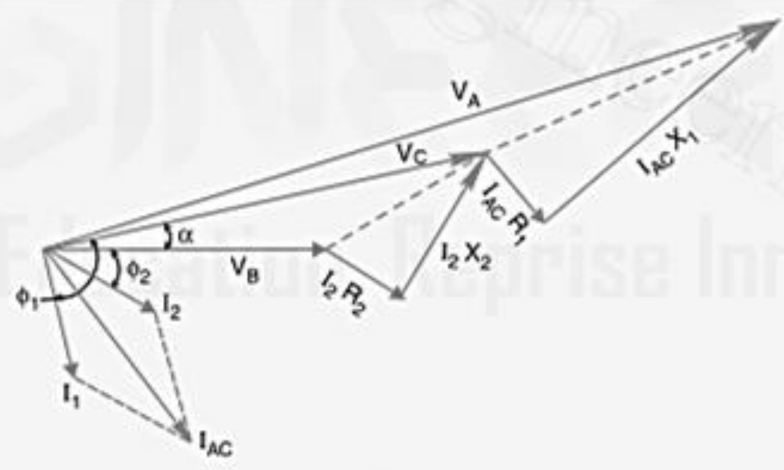


Fig. 14.3

Voltage drop in section $CB = \overrightarrow{I_2}\,\overrightarrow{Z_{CB}} = I_2(\cos\phi_2 - j\sin\phi_2)(R_2 + jX_2)$

Voltage at point $C = \overrightarrow{V_B} + \text{Drop in section } CB = V_C \angle \alpha$ (say)

Now $\overrightarrow{I_1} = I_1 \angle -\phi_1$ *w.r.t.* voltage V_C

$\therefore$ $\overrightarrow{I_1} = I_1 \angle -(\phi_1 - \alpha)$ *w.r.t.* voltage V_B

i.e. $\overrightarrow{I_1} = I_1[\cos(\phi_1 - \alpha) - j\sin(\phi_1 - \alpha)]$

Now $\overrightarrow{I_{AC}} = \overrightarrow{I_1} + \overrightarrow{I_2}$

$$= I_1 [\cos(\phi_1 - \alpha) - j \sin(\phi_1 - \alpha)] + I_2 (\cos\phi_2 - j \sin\phi_2)$$

Voltage drop in section $AC = \overrightarrow{I_{AC}}\ \overrightarrow{Z_{AC}}$

$\therefore$ Voltage at point $A = V_B$ + Drop in CB + Drop in AC

Example *A single phase a.c. distributor AB 300 metres long is fed from end A and is loaded as under :*

(*i*) *100 A at 0·707 p.f. lagging 200 m from point A*

(*ii*) *200 A at 0·8 p.f. lagging 300 m from point A*

The load resistance and reactance of the distributor is 0·2 Ω and 0·1 Ω per kilometre. Calculate the total voltage drop in the distributor. The load power factors refer to the voltage at the far end.

Solution. Fig. 14.4 shows the single line diagram of the distributor.

Impedance of distributor/km $= (0 \cdot 2 + j\, 0 \cdot 1)\,\Omega$

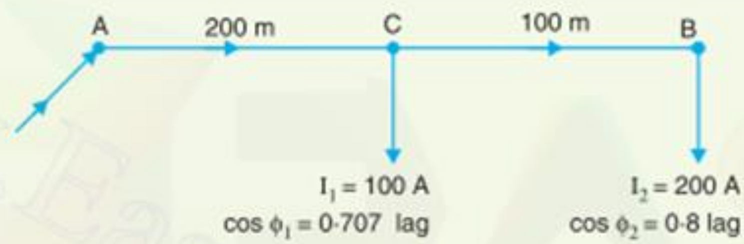


Fig. 14.4

Impedance of section AC, $\overrightarrow{Z_{AC}} = (0 \cdot 2 + j\, 0 \cdot 1) \times 200/1000 = (0 \cdot 04 + j\, 0 \cdot 02)\,\Omega$

Impedance of section CB, $\overrightarrow{Z_{CB}} = (0 \cdot 2 + j\, 0 \cdot 1) \times 100/1000 = (0 \cdot 02 + j\, 0 \cdot 01)\,\Omega$

Taking voltage at the far end B as the reference vector, we have,

Load current at point B, $\overrightarrow{I_2} = I_2 (\cos\phi_2 - j \sin\phi_2) = 200\,(0 \cdot 8 - j\, 0 \cdot 6)$
$= (160 - j\, 120)$ A

Load current at point C, $\overrightarrow{I_1} = I_1 (\cos\phi_1 - j \sin\phi_1) = 100\,(0 \cdot 707 - j\, 0 \cdot 707)$
$= (70 \cdot 7 - j\, 70 \cdot 7)$ A

Current in section CB, $\overrightarrow{I_{CB}} = \overrightarrow{I_2} = (160 - j\, 120)$ A

Current in section AC, $\overrightarrow{I_{AC}} = \overrightarrow{I_1} + \overrightarrow{I_2} = (70 \cdot 7 - j\, 70 \cdot 7) + (160 - j\, 120)$
$= (230 \cdot 7 - j\, 190 \cdot 7)$ A

Voltage drop in section CB, $\overrightarrow{V_{CB}} = \overrightarrow{I_{CB}}\ \overrightarrow{Z_{CB}} = (160 - j\, 120)\,(0 \cdot 02 + j\, 0 \cdot 01)$
$= (4 \cdot 4 - j\, 0 \cdot 8)$ volts

Voltage drop in section AC, $\overrightarrow{V_{AC}} = \overrightarrow{I_{AC}}\ \overrightarrow{Z_{AC}} = (230 \cdot 7 - j\, 190 \cdot 7)\,(0 \cdot 04 + j\, 0 \cdot 02)$
$= (13 \cdot 04 - j\, 3 \cdot 01)$ volts

Voltage drop in the distributor $= \overrightarrow{V_{AC}} + \overrightarrow{V_{CB}} = (13 \cdot 04 - j\, 3 \cdot 01) + (4 \cdot 4 - j\, 0 \cdot 8)$
$= (17 \cdot 44 - j\, 3 \cdot 81)$ volts

Magnitude of drop $= \sqrt{(17 \cdot 44)^2 + (3 \cdot 81)^2} =$ **17·85 V**

Example *A single phase distributor 2 kilometres long supplies a load of 120 A at 0·8 p.f. lagging at its far end and a load of 80 A at 0·9 p.f. lagging at its mid-point. Both power factors are* *referred to the voltage at the far end. The resistance and reactance per km (go and return) are 0·05 Ω and 0·1 Ω respectively. If the voltage at the far end is maintained at 230 V, calculate :*

(*i*) *voltage at the sending end*

(*ii*) *phase angle between voltages at the two ends.*

Solution. Fig. 14.5 shows the distributor *AB* with *C* as the mid-point

Impedance of distributor/km $= (0{\cdot}05 + j\,0{\cdot}1)\,\Omega$

Impedance of section *AC*, $\vec{Z}_{AC} = (0{\cdot}05 + j\,0{\cdot}1) \times 1000/1000 = (0{\cdot}05 + j\,0{\cdot}1)\,\Omega$

Impedance of section *CB*, $\vec{Z}_{CB} = (0{\cdot}05 + j\,0{\cdot}1) \times 1000/1000 = (0{\cdot}05 + j\,0{\cdot}1)\,\Omega$

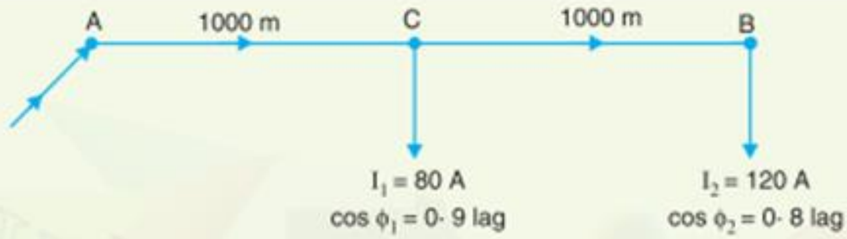


Fig. 14.5

Let the voltage V_B at point *B* be taken as the reference vector.

Then, $\overrightarrow{V_B} = 230 + j\,0$

(*i*) Load current at point *B*, $\overrightarrow{I_2} = 120\,(0{\cdot}8 - j\,0{\cdot}6) = 96 - j\,72$

Load current at point *C*, $\overrightarrow{I_1} = 80\,(0{\cdot}9 - j\,0{\cdot}436) = 72 - j\,34{\cdot}88$

Current in section *CB*, $\overrightarrow{I_{CB}} = \overrightarrow{I_2} = 96 - j\,72$

Current in section *AC*, $\overrightarrow{I_{AC}} = \overrightarrow{I_1} + \overrightarrow{I_2} = (72 - j\,34{\cdot}88) + (96 - j\,72)$

$= 168 - j\,106{\cdot}88$

Drop in section *CB*, $\overrightarrow{V_{CB}} = \overrightarrow{I_{CB}}\,\overrightarrow{Z_{CB}} = (96 - j\,72)\,(0{\cdot}05 + j\,0{\cdot}1)$

$= 12 + j\,6$

Drop in section *AC*, $\overrightarrow{V_{AC}} = \overrightarrow{I_{AC}}\,\overrightarrow{Z_{AC}} = (168 - j\,106{\cdot}88)\,(0{\cdot}05 + j\,0{\cdot}1)$

$= 19{\cdot}08 + j\,11{\cdot}45$

∴ Sending end voltage, $\overrightarrow{V_A} = \overrightarrow{V_B} + \overrightarrow{V_{CB}} + \overrightarrow{V_{AC}}$

$= (230 + j\,0) + (12 + j\,6) + (19.08 + j\,11.45)$

$= 261.08 + j\,17.45$

Its magnitude is $= \sqrt{(261{\cdot}08)^2 + (17{\cdot}45)^2} =$ **261·67 V**

(*ii*) The phase difference θ between V_A and V_B is given by :

$$\tan\theta = \frac{17{\cdot}45}{261{\cdot}08} = 0{\cdot}0668$$

∴ $$\theta = \tan^{-1} 0{\cdot}0668 = \mathbf{3{\cdot}82^\circ}$$

Example *A single phase distributor one km long has resistance and reactance per conductor of 0·1 Ω and 0·15 Ω respectively. At the far end, the voltage V_B = 200 V and the current is 100 A at a p.f. of 0·8 lagging. At the mid-point M of the distributor, a current of 100 A is tapped at a p.f.*

of 0·6 lagging with reference to the voltage V_M at the mid-point. Calculate :

(*i*) *voltage at mid-point*

(*ii*) *sending end voltage V_A*

(*iii*) *phase angle between V_A and V_B*

Solution. Fig. 14.6 shows the single line diagram of the distributor *AB* with *M* as the mid-point.

Total impedance of distributor $= 2(0·1 + j\,0·15) = (0·2 + j\,0·3)\,\Omega$

Impedance of section *AM*, $\overrightarrow{Z_{AM}} = (0·1 + j\,0·15)\,\Omega$

Impedance of section *MB*, $\overrightarrow{Z_{MB}} = (0·1 + j\,0·15)\,\Omega$

Let the voltage V_B at point *B* be taken as the reference vector.

Then, $\overrightarrow{V_B} = 200 + j\,0$

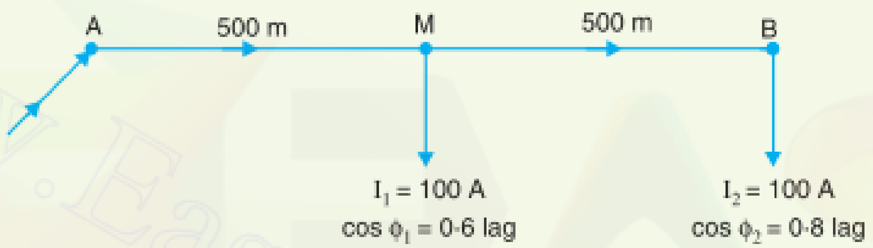


Fig. 14.6

(*i*) Load current at point *B*, $\overrightarrow{I_2} = 100\,(0·8 - j\,0·6) = 80 - j\,60$

Current in section *MB*, $\overrightarrow{I_{MB}} = \overrightarrow{I_2} = 80 - j\,60$

Drop in section *MB*, $\overrightarrow{V_{MB}} = \overrightarrow{I_{MB}}\,\overrightarrow{Z_{MB}}$

$= (80 - j\,60)\,(0·1 + j\,0·15) = 17 + j\,6$

∴ Voltage at point *M*, $\overrightarrow{V_M} = \overrightarrow{V_B} + \overrightarrow{V_{MB}} = (200 + j\,0) + (17 + j\,6)$

$= 217 + j\,6$

Its magnitude is $= \sqrt{(217)^2 + (6)^2}$ = **217·1 V**

Phase angle between V_M and V_B, $\alpha = \tan^{-1} 6/217 = \tan^{-1} 0·0276 = 1·58°$

(*ii*) The load current I_1 has a lagging p.f. of 0·6 *w.r.t.* V_M. It lags behind V_M by an angle $\phi_1 = \cos^{-1} 0·6 = 53·13°$

∴ Phase angle between I_1 and V_B, $\phi'_1 = \phi_1 - \alpha = 53·13° - 1·58 = 51·55°$

Load current at *M*, $\overrightarrow{I_1} = I_1\,(\cos\phi_1' - j\sin\phi_1') = 100\,(\cos 51·55° - j\sin 51·55°)$

$= 62·2 - j\,78·3$

Current in section *AM*, $\overrightarrow{I_{AM}} = \overrightarrow{I_1} + \overrightarrow{I_2} = (62·2 - j\,78·3) + (80 - j\,60)$

$= 142·2 - j\,138·3$

Drop in section *AM*, $\overrightarrow{V_{AM}} = \overrightarrow{I_{AM}}\,\overrightarrow{Z_{AM}} = (142·2 - j\,138·3)\,(0·1 + j\,0·15)$

$= 34·96 + j\,7·5$

Sending end voltage, $\overrightarrow{V_A} = \overrightarrow{V_M} + \overrightarrow{V_{AM}} = (217 + j\,6) + (34·96 + j\,7·5)$

$$= 251{\cdot}96 + j\,13{\cdot}5$$

Its magnitude is $= \sqrt{(251{\cdot}96)^2 + (13{\cdot}5)^2} = \mathbf{252{\cdot}32\ V}$

(*iii*) The phase difference θ between V_A and V_B is given by :

$$\tan\theta = 13{\cdot}5/251{\cdot}96 = 0{\cdot}05358$$

$$\therefore \qquad \theta = \tan^{-1} 0{\cdot}05358 = \mathbf{3{\cdot}07^{\circ}}$$

Hence supply voltage is 252·32 V and leads V_B by 3·07°.

Four-Wire Star-Connected Unbalanced Loads

We can obtain this type of load in two ways. First, we may connect a 3-phase, 4-wire unbalanced load to a 3-phase, 4-wire supply as shown in Fig. 14.10. Note that star point *N* of the supply is connected to the load star point *N′*. Secondly, we may connect single phase loads between any line and the neutral wire as shown in Fig.14.11. This will also result in a 3-phase, 4-wire **unbalanced load because it is rarely possible that single phase loads on all the three phases have the same magnitude and power factor. Since the load is unbalanced, the line currents will be different in magnitude and displaced from one another by unequal angles. The current in the neutral wire will be the phasor sum of the three line currents *i.e.*

Current in neutral wire, $I_N = I_R + I_Y + I_B$ ...*phasor sum*

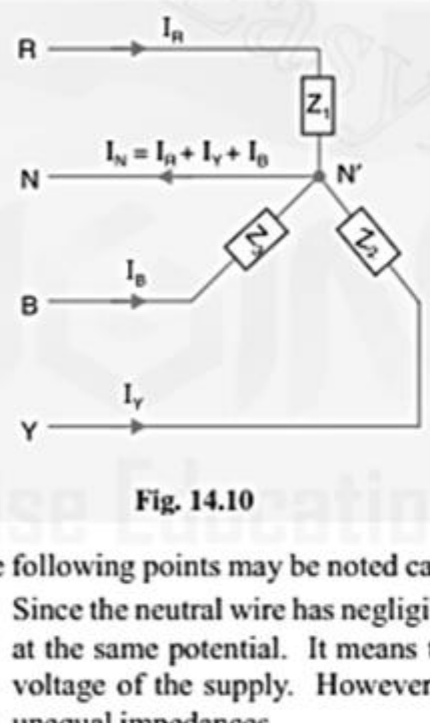


Fig. 14.10

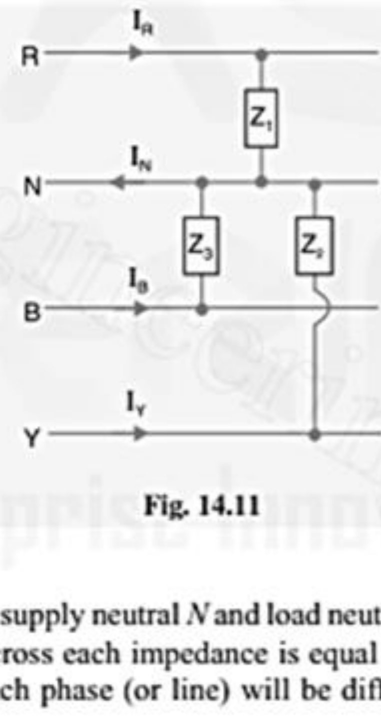


Fig. 14.11

The following points may be noted carefully :

(*i*) Since the neutral wire has negligible resistance, supply neutral *N* and load neutral *N′* will be at the same potential. It means that voltage across each impedance is equal to the phase voltage of the supply. However, current in each phase (or line) will be different due to unequal impedances.

(*ii*) The amount of current flowing in the neutral wire will depend upon the magnitudes of line currents and their phasor relations. In most circuits encountered in practice, the neutral current is equal to or smaller than one of the line currents. The exceptions are those circuits having severe unbalance.