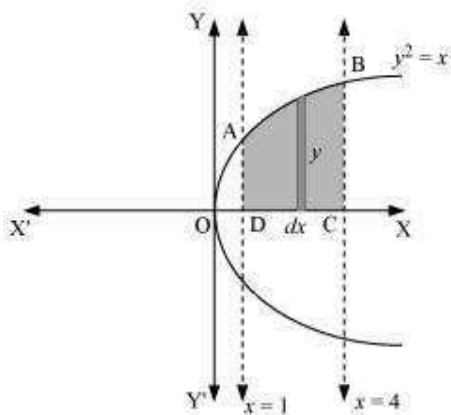## Question 1:

Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis.

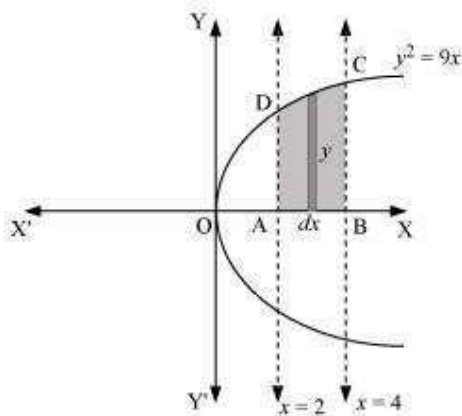Answer

The area of the region bounded by the curve, $y^2 = x$, the lines, $x = 1$ and $x = 4$, and the x-axis is the area ABCD.

$$\text{Area of ABCD} = \int_1^4 y\, dx$$

$$= \int_1^4 \sqrt{x}\, dx$$

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4$$

$$= \frac{2}{3} \left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right]$$

$$= \frac{2}{3} [8 - 1]$$

$$= \frac{14}{3} \text{ units}$$

Question 2:

Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

Answer

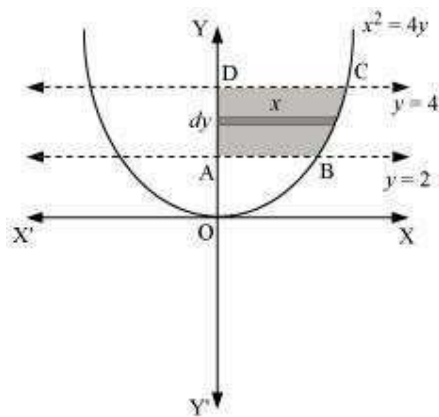The area of the region bounded by the curve, $y^2 = 9x$, $x = 2$, and $x = 4$, and the x-axis is the area ABCD.

Area of ABCD $= \int_2^4 y\,dx$

$\qquad = \int_2^4 3\sqrt{x}\,dx$

$\qquad = 3\left[\dfrac{x^{\frac{3}{2}}}{\dfrac{3}{2}}\right]_2^4$

$\qquad = 2\left[x^{\frac{3}{2}}\right]_2^4$

$\qquad = 2\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]$

$\qquad = 2\left[8 - 2\sqrt{2}\right]$

$\qquad = \left(16 - 4\sqrt{2}\right)$ units

Question 3:

Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

Answer

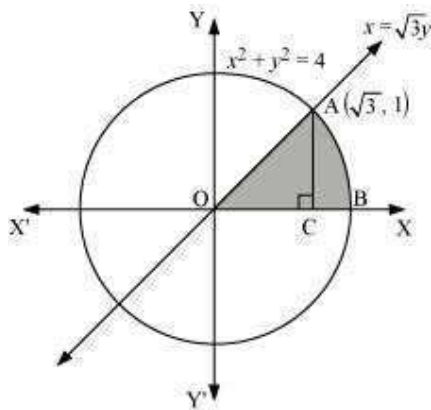The area of the region bounded by the curve, $x^2 = 4y$, $y = 2$, and $y = 4$, and the y-axis is the area ABCD.

$$\text{Area of ABCD} = \int_2^4 x\,dy$$

$$= \int_2^4 2\sqrt{y}\,dy$$

$$= 2\int_2^4 \sqrt{y}\,dy$$

$$= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$$

$$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]$$

$$= \frac{4}{3}\left[8 - 2\sqrt{2}\right]$$

$$= \left(\frac{32 - 8\sqrt{2}}{3}\right) \text{ units}$$

Question 4:

Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$

Answer

The area of the region bounded by the circle, $x^2 + y^2 = 4, x = \sqrt{3}y$, and the x-axis is the area OAB.

The point of intersection of the line and the circle in the first quadrant is $\left(\sqrt{3}, 1\right)$.

Area OAB = Area ΔOCA + Area ACB

Area of OAC $= \dfrac{1}{2} \times OC \times AC = \dfrac{1}{2} \times \sqrt{3} \times 1 = \dfrac{\sqrt{3}}{2}$ ...(1)

Area of ABC $= \displaystyle\int_{\sqrt{3}}^{2} y \, dx$

$$= \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\, dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} \right]_{\sqrt{3}}^{2}$$

$$= \left[2\times\frac{\pi}{2} - \frac{\sqrt{3}}{2}\sqrt{4-3} - 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \right]$$

$$= \left[\pi - \frac{\sqrt{3}\pi}{2} - 2\left(\frac{\pi}{3}\right) \right]$$

$$= \left[\pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \right]$$

$$= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right] \qquad ...(2)$$

Therefore, area enclosed by x-axis, the line $x = \sqrt{3}y$, and the circle $x^2 + y^2 = 4$ in the first

quadrant $= \dfrac{\sqrt{3}\pi}{2} + \dfrac{3\sqrt{\quad}}{3} - \dfrac{\pi}{2} = \dfrac{\pi}{3}$ units
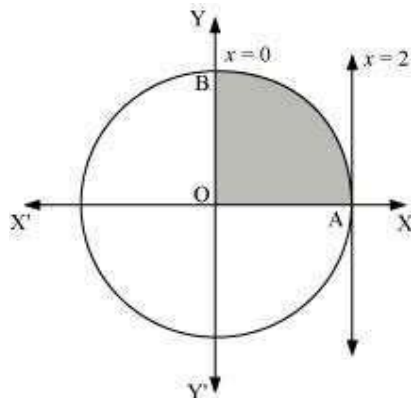
Question 5:

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

Answer

The area bounded by the circle and the lines, $x = 0$ and $x = 2$, in the first quadrant is represented as

$\therefore$ Area OAB $= \int_0^2 y \, dx$

$= \int_0^2 \sqrt{4 - x^2} \, dx$

$= \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2$

$= 2 \left(\frac{\pi}{2} \right)$

$= \pi$ units

Question 6:

Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

Answer

The smaller area enclosed by the circle, $x^2 + y^2 = 4$, and the line, $x + y = 2$, is represented by the shaded area ACBA as

It can be observed that,

Area ACBA = Area OACBO − Area (ΔOAB)

$= \int_0^2 \sqrt{4 - x^2} \, dx - \int_0^2 (2 - x) \, dx$

$= \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[2x - \frac{x^2}{2} \right]_0^2$

$= \left[2 \cdot \frac{\pi}{2} \right] - [4 - 2]$

$= (\pi - 2)$ units