LECTURE NOTES ON ENGINEERING MATHEMATIC-II

(COMMON FOR ALL BRANCH)

PREPARED BY,

SAMIRA KUMAR PATHI
LECTURER IN MATHEMATICS,
GOVT. POLYTECHNIC, GAJAPATI

ENGG. MATHEMATICS –II FOR DIPLOMA 2ND SEM STUDENTS

UNIT-I

SCALAR AND VECTOR

Scalar Quantity: Scalar Quantity is a physical quantity which has only magnitude.

Ex- mass, temperature, speed, distance

Vector Quantity: Vector quantity is a physical quantity which has both magnitude and direction.

Ex- Displacement, Velocity, acceleration, weight

Types of Vector Quantity:

Null vector:- Null vector is a vector whose magnitude is Zero

Parallel vector: Two Vectors are said to be parallel to each othe if they have same direction.

- If u and v are two non-zero vectors and u=cv, where c is a scalar then u and v are parallel to each other.

Unit Vector: A Vector which has unit magnitude is said to be unit vector.

Collinear Vector : vectors which lie along the same line or parallel lines are known as collinear vector.

Representation of vector:-

$\vec{a}$, unit vector $\hat{\imath}, \hat{\jmath}, \hat{k}$,$\hat{a}$

$\vec{b}$

- Any vector in three dimensional co-ordinate system can be represented as $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, where $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$ are the unit vectors along X-axis, Y-axis and Z-axis respectively.
- Any vector in two dimensional co-ordinate systems can be represented as $x\hat{\imath} + y\hat{\jmath}$where $\hat{\imath}$ & $\hat{\jmath}$ are the unit vectors along X-axis & Y-axis respectively.

Magnitude and direction of a vector

let $\vec{u} = x\hat{\imath} + y\hat{\jmath}$

magnitude- $\sqrt{x^2 + y^2} = |\vec{u}|$

$$direction\ \theta = \tan^{-1}\frac{y}{x}$$

Three dimension: - let $\vec{u} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$$|\vec{u}| = \sqrt{x^2 + y^2 + z^2},$$

direction$< \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}} >$

Ex: -find the magnitude and direction of $\vec{u} = 2\hat{\imath} + 3\hat{\jmath}$

magnitude- $\sqrt{x^2 + y^2} = |\vec{u}| = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$

$$direction\ \theta = \tan^{-1}\frac{3}{2}$$

$Algebra\ of\ vectors: -$

$\vec{u} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ & $\vec{v} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ then

$\vec{u} \pm \vec{v} = (x \pm a)\hat{\imath} + (y \pm b)\hat{\jmath} + (z \pm c)\hat{k}$

Ex:- $\vec{u} = 2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$ $\vec{v} = \hat{\imath} - 2\hat{\jmath} + 7\hat{k}$

$find\ \vec{u} + \vec{v}$ & $\vec{u} - \vec{v}$

$\vec{u} + \vec{v} = (2\hat{\imath} + 3\hat{\jmath} - 5\widehat{k}) + (\hat{\imath} - 2\hat{\jmath} + 7\hat{k})$

$$= (2+1)\hat{\imath} + (3-2)\hat{\jmath} + (-5+7)\hat{k} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$$

$$\vec{u} - \vec{v} = \hat{\imath} + 5\hat{\jmath} - 12\hat{k}$$

<u>Scalar Product or dot product of two vectors:-</u>

Let $\vec{u}$ & $\vec{v}$ are two vectors then their Scalar Product or dot product is $\vec{u}.\vec{v} = |\vec{u}|.|\vec{v}|cos\theta$

$Moreover\ if\ \vec{u} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ & $\vec{v} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}\ then$

$\vec{u}.\vec{v} = x.a + y.b + z.c$

$$\vec{u} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \ \& \ \vec{v} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

$$\vec{u}.\vec{v} = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\ \right)\left(a\hat{\imath} + b\hat{\jmath} + c\hat{k}\ \right)$$

$= x.a\ \hat{\imath}.\hat{\imath} + x.b\hat{\imath}.\hat{\jmath}$ +x.c$\hat{\imath}.\hat{k} + y.a\ \hat{\jmath}.\hat{\imath} + y.b\hat{\jmath}.\hat{\jmath}$ +y.c$\hat{\jmath}.\hat{k}$+ $z.a\ \hat{k}.\hat{\imath} + z.b\hat{k}.\hat{\jmath}$ +z.c$\hat{k}.\hat{k}$

$= x.a$ .1+0+0+0+y.b+0+0+0+z.c= $x.a + y.b + z.c$

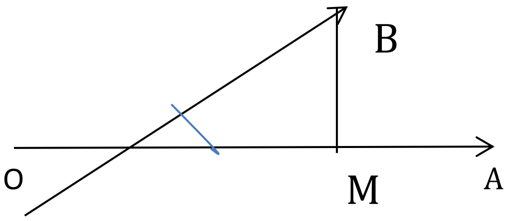
$$\hat{\imath}.\hat{\imath} = 1, \hat{\jmath}.\hat{\jmath} = 1, \hat{k}.\hat{k} = 1$$

$\hat{\imath}.\hat{\jmath}$ =$\hat{\imath}.\hat{k}$=$\hat{\jmath}.\hat{k}$=$\hat{k}.\hat{\imath}$=$\hat{k}.\hat{\jmath}$=$\hat{\jmath}.\hat{\imath} = 0$

Ex:- $\vec{u} = 2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ $\vec{v} = \hat{\imath} + 2\hat{\jmath} + 7\hat{k}$ find $\vec{u}.\vec{v}$

$$\vec{u}.\vec{v} = 2.1 + 3.2 + 5.7 = 2 + 6 + 35 = 43$$

Geometrical interpretation of Scalar product:-



let $\vec{u} = \overrightarrow{OA}$ and let $\vec{v} = \overrightarrow{OB}$ then $\vec{u}.\vec{v} = |\vec{u}|.|\vec{v}|cos\theta$

$= |\overrightarrow{OA}|.|\overrightarrow{OB}|.\frac{|\overrightarrow{OM}|}{|\overrightarrow{OB}|} = |\overrightarrow{OA}|.|\overrightarrow{OM}|$

$=> |\overrightarrow{OM}| = \frac{\vec{u}.\vec{v}}{|\overrightarrow{OA}|} = \frac{\vec{u}.\vec{v}}{|\vec{u}|}$

Therefore scalar projection of $\vec{v}$ *on* $\vec{u} = \frac{\vec{u}.\vec{v}}{|\vec{u}|}$

Also scalar projection of $\vec{u}$ *on* $\vec{v} = \frac{\vec{u}.\vec{v}}{|\vec{v}|}$

Example:- Find the scalar projection of $\vec{u} = 2\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ *on* $\vec{v} = \hat{\imath} - 2\hat{\jmath} + 7\hat{k}$

Solution: - Scalar projection of $\vec{u}$ *on* $\vec{v} = \frac{\vec{u}.\vec{v}}{|\vec{v}|}$

Now $\vec{u}.\vec{v} = 2 * 1 + (-3) * (-2) + 5 * 7 = 43$

$|\vec{v}| = \sqrt{1^2 + (-2)^2 + 7^2} = \sqrt{54} = 3\sqrt{6}$

Scalar projection of $\vec{u}$ *on* $\vec{v} = \frac{\vec{u}.\vec{v}}{|\vec{v}|} = \frac{43}{3\sqrt{6}}$

Example:- Find the scalar projection of $\vec{u} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ *on* $\vec{v} = \hat{\imath} + \hat{\jmath} + \hat{k}$

Solution: - Scalar projection of $\vec{u}$ *on* $\vec{v} = \frac{\vec{u}.\vec{v}}{|\vec{v}|}$

Now $\vec{u}.\vec{v} = 1 * 1 + (-2) * (1) + 3 * 1 = 2$

$|\vec{v}| = \sqrt{1^2 + (1)^2 + 1^2} = \sqrt{3} = \sqrt{3}$

Scalar projection of $\vec{u}$ *on* $\vec{v} = \frac{\vec{u}.\vec{v}}{|\vec{v}|} = \frac{2}{\sqrt{3}}$

Vector Product or Cross product of two vectors:-

Let $\vec{u}$ & $\vec{v}$ are two vectors then their Vector Product or cross product is $\vec{u}X\vec{v} = |\vec{u}|.|\vec{v}|.sin\theta$

$Moreover\ if\ \vec{u} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}\ \&\ \vec{v} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}\ then$

$$\vec{u}X\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ a & b & c \end{vmatrix}$$

$$\hat{\imath}X\hat{\imath} =, \hat{\jmath}X\hat{\jmath} = 1, \hat{k}X\hat{k} = 0$$

$\hat{\imath}X\hat{\jmath} = \hat{k}, \hat{\jmath}X\hat{k} = \hat{\imath}, \hat{k}X\hat{\imath} = \hat{\jmath}$

$\hat{j}X\hat{\imath}=-\hat{k}, \hat{k}X\hat{j}=-\hat{\imath}, \hat{\imath}X\hat{k}=-\hat{j}$

Ex:- $\vec{u} = 2\hat{\imath} + 3\hat{j} + 5\hat{k}$ & $\vec{v} = \hat{\imath} + 2\hat{j} + 7\hat{k}$ find $\vec{u}X\vec{v}$

Solution:- $\vec{u}X\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 1 & 2 & 7 \end{vmatrix}$

$= \hat{\imath}(3*7-2*5) - \hat{j}(2*7-1*5) + \hat{k}(2*2-1*3)$

$= 11\hat{\imath} - 9\hat{j} + \hat{k}$

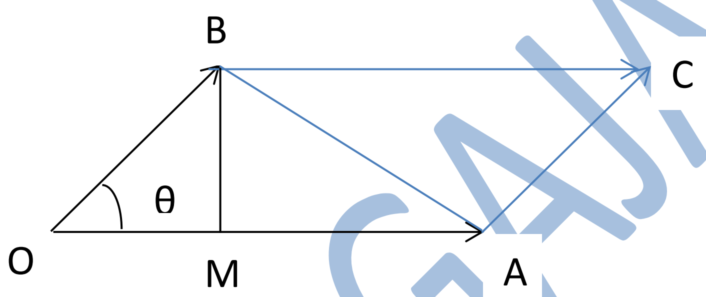
Ex:- $\vec{u} = \hat{\imath} - 3\hat{j} + 5\hat{k}$ & $\vec{v} = \hat{\imath} + \hat{j} + 2\hat{k}$ find $\vec{u}X\vec{v}$

$$\vec{u}X\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ 1 & -3 & 5 \\ 1 & 1 & 2 \end{vmatrix}$$

$= \hat{\imath}(-3*2-5*1) - \hat{j}(1*2-1*5) + \hat{k}(1*1-1*-3)$

$= -11\hat{\imath} + 3\hat{j} + \widehat{4k}$

Geometrical Interpretation of vector product:-



Let $\vec{u}$ & $\vec{v}$ *are two vectors which are represented by* $\overrightarrow{OA}$ *and* $\overrightarrow{OB}$ respectively in the pic.

We know that $\vec{u}X\vec{v} = |\vec{u}|.|\vec{v}|.sin\theta$

$=> \vec{u}X\vec{v} = OA.OB.\frac{BM}{OB} = OA.BM$

$=> \vec{u}X\vec{v} = area\ of\ the\ parallelogram\ OACB$

Vector Projection: -

Vector projection of $\vec{v}$ *on* $\vec{u} = \frac{\vec{u}.\vec{v}}{|\vec{u}|^2}\vec{u}$

Vector projection of $\vec{u}$ *on* $\vec{v} = \frac{\vec{u}.\vec{v}}{|\vec{v}|^2}\vec{v}$

Example:- Find the vector projection of $\vec{v} = \hat{\imath} + 2\hat{j} + 7\hat{k}$ *on* $\vec{u} = 2\hat{\imath} + 3\hat{j} + 5\hat{k}$

Solution: - we know that Vector projection of $\vec{v}$ $on\ \vec{u} = \frac{\vec{u}.\vec{v}}{|\vec{u}|^2}\vec{u}$

$\vec{u}.\vec{v} = 43$ & $|\vec{u}| = \sqrt{38}$

$$\frac{\vec{u}.\vec{v}}{|\vec{u}|^2}\vec{u} == \frac{43}{(\sqrt{38})^2}(2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) = \frac{43}{38}(2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) = \frac{43}{19}\hat{\imath} + \frac{129}{38}\hat{\jmath} + \frac{215}{38}\hat{k}$$

Ex- find area of a parallelogram whose sides are representer by the vectors $\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} + \hat{\jmath} + 2\hat{k}$

We know area of the parallelogram is magnitude of cross product of two vectors represented the sides of parallelogram

Here two side of vector given is let $\vec{u} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ $\quad \vec{v} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$

So area of parallelogram is

$$\vec{u}X\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -3 & 5 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \hat{\imath}(-3*2 - 5*1) - \hat{\jmath}(1*2 - 1*5) + \hat{k}(1*1 - 1*-3)$$

$$= -11\hat{\imath} + 3\hat{\jmath} + \widehat{4k}$$

Now area is magnitude of

$$|\vec{u}X\vec{v}| = \sqrt{(-11)^2 + 3^2 + 4^2} = \sqrt{121 + 9 + 16} = \sqrt{146}$$

Ex- find area of a triangle whose two sides are represented by the vectors $\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} + \hat{\jmath} + 2\hat{k}$

We know area of the triangle is magnitude of half of cross product of two vectors represented the sides of triangle

Here two side of vector given is let $\vec{u} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ $\quad \vec{v} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$

So area of triangle is

$$\frac{1}{2}\vec{u}X\vec{v} = \frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -3 & 5 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \frac{1}{2}(\hat{\imath}(-3*2 - 5*1) - \hat{\jmath}(1*2 - 1*5) + \hat{k}(1*1 - 1*-3))$$

$$= \frac{1}{2}(-11\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$$

Now area is magnitude of

$$\frac{1}{2}|\vec{u}X\vec{v}| = \frac{1}{2}\sqrt{(-11)^2 + 3^2 + 4^2} = \sqrt{121 + 9 + 16} = \frac{1}{2}\sqrt{146}$$

DIFFRENTIAL CALCULUS (DERIVATIVE)

DERIVATIVE OF A FUNCTION BY USING DEFINITION

Differentiation: - The procedure of finding derivative of a function is called as differentiation.

Find the derivative of x^2 by using definition.

$\lim\limits_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x}$

Here f(x) = x^2

$\frac{d}{dx}x^2 = \lim\limits_{\delta x \to 0} \frac{(x+\delta x)^2-x^2}{\delta x} = \lim\limits_{\delta x \to 0} \frac{(x+\delta x+x)(x+\delta x-x)}{\delta x}$

$= \lim\limits_{\delta x \to 0} \frac{(2x+\delta x)(\delta x)}{\delta x} = \lim\limits_{\delta x \to 0}(2x+\delta x) = 2x$

Find the derivative of x^3 by using definition.

Methods of differentiation:-

1) Parametric Function:-

y=f(x) (Explicit function)

f(x,y)=0 (implicit function)

y=f(t) and x=g(t) (parametric function)

Derivative of implicit function:-

Ex:- sinx.cosy+x.y=0 find $\frac{dy}{dx}$

$$sinx.cosy + x.y = 0 --- (1)$$

$$differentiating\ equation\ (1) with\ respect\ to\ x, we\ get$$

$$\frac{d}{dx}(sinx.\cos y + x.y) = \frac{d}{dx}0$$

$$=> \frac{d}{dx}(sinx.cosy) + \frac{d}{dx}(x.y) = 0$$

$$=> \left(\frac{d}{dx}sinx\right)cosy + sinx.\frac{d}{dx}(cosy) + \frac{d}{dx}(x).y + x.\frac{d}{dx}(y) = 0$$

$$=> cosx.cosy + sinx.(-siny).\frac{dy}{dx} + 1.y + x.\frac{dy}{dx} = 0$$

$$=> x.\frac{dy}{dx} - \sin x.\sin y\frac{dy}{dx} + cosx.cosy + y = 0$$

$$=> \frac{dy}{dx}(x - sinx\ siny) = -(cosx.cosy + y)$$

$$=> \frac{dy}{dx} = -\frac{cosx.cosy + y}{x - sinx.siny}$$

$ex: - a^x.e^y = 0\ then\ find\ \frac{dy}{dx}$ H.W

DERIVATIVE BY USING LOGARITHM

f= x^y

y = x^y

logy=logx^y

logy=ylog$x$

$\frac{d}{dx}$(logy)= $\frac{d}{dx}$ylogx

$\frac{1}{y}\frac{dy}{dx}$=$logx\frac{dy}{dx}$+y$\frac{d}{dx}(logx)$

$\frac{1}{y}\frac{dy}{dx}$=$logx\frac{dy}{dx}$+y$\frac{1}{x}$

$\frac{1}{y}\frac{dy}{dx}$-$logx\frac{dy}{dx}$= $\frac{y}{x}$

($\frac{1}{y}$ - $logx)\frac{dy}{dx}$= $\frac{y}{x}$

($\frac{1-ylogx}{y}$) $\frac{dy}{dx}$= $\frac{y}{x}$

$\frac{dy}{dx}$= $\frac{y}{x}$($\frac{y}{1-ylogx}$)

$\frac{dy}{dx}$= ($\frac{y^2}{x(1-ylogx)}$)

$y = x^{sinx}$

Taking log on both sides, we get

$$logy = \log(x^{sinx}) => \log y = sinx\ logx$$

$[\boldsymbol{as\ logx^y = y\ logx, \log(xy) = logx + logy, \log\frac{x}{y} = logx - logy}]$

Now differentiating both sides, we get

$$=> \frac{d}{dx}\log y = \frac{d}{dx}(sinx\ logx)$$

$$=> \frac{1}{y}\frac{dy}{dx} = \left(\frac{d}{dx} sinx\right) logx + sinx.\frac{d}{dx} logx$$

$$=> \frac{1}{y}\frac{dy}{dx} = cosx.logx + sinx.\frac{1}{x}$$

$$=> \frac{dy}{dx} = y\left(cosx.logx + sinx.\frac{1}{x}\right)$$

$$=> \frac{dy}{dx} = x^{sinx}(cosx.logx + sinx.\frac{1}{x})$$

Ex:-$y = x^x \ find \frac{dy}{dx}$ H.W

Ex:-$y = x^{tanx} \ find \frac{dy}{dx}$

$y = x^{tanx}$

Tanking log on both side, we get

$$logy = logx^{tanx} => logy = tanx\ logx$$

Differentiating both sides with respect to x, we get

$$=> \frac{d}{dx} logy = \frac{d}{dx} tanx\ logx$$

$$=> \frac{1}{y}\frac{dy}{dx} = \left[\frac{d}{dx}(tanx)\right].logx + tanx\left[\frac{d}{dx}\ logx\right]$$

$$=> \frac{1}{y}\frac{dy}{dx} = \sec^2 x\,.logx + tanx.\frac{1}{x}$$

$$=> \frac{dy}{dx} = y\left[\sec^2 x\,.logx + tanx.\frac{1}{x}\right]$$

$$=> \frac{dy}{dx} = x^{tanx}\,(\sec^2 x\,.logx + tanx.\frac{1}{x})$$

$$\frac{d}{dx}(cosy)\ \ think\ y = X$$

$$\frac{d}{dx}\ cosX = -sinX.\frac{dX}{dx} = -siny.\frac{dy}{dx}$$

$\frac{d}{dx}(\sin y) = \cos y\frac{dy}{dx}$

$\frac{d}{dx}(\cos x^3)$ Think $x^3 = X$

$\frac{d}{dx}\cos X = -\sin X . \frac{dX}{dx} = -\sin x^3 . \frac{d}{dx}x^3 = -\sin x^3 . 3x^2$

DIFFRENTIATION OF A FUNCTION WITH RESPECT TO ANOTHER FUNCTION OR PARAMETRIC FUNCTION

y=sint and x= cost then find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) * \left(\frac{dt}{dx}\right) = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dt} = \frac{d}{dt}\sin t = \cos t\,;$$

$$\frac{dx}{dt} = \frac{d}{dt}cos\ t = -\sin t$$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t} = -\cot t$

$y = t^2\ and\ x = a^t\ find\ \frac{dy}{dx}$

$$\frac{dy}{dt} = \frac{d}{dt}t^2 = 2t;$$

$$\frac{dx}{dt} = \frac{d}{dt}a^t = a^t lna$$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{a^t lna}$

$y = \sec t\ and\ x = cosec\ t\ find\ \frac{dy}{dx}$

Q-Diffrentiate sinx w.r.t cos x

y_1= sinx y_2= cos x

$\frac{dy_1}{dx}=\frac{d}{dx}(sinx) = \cos x$

$\frac{dy_2}{dx}=\frac{d}{dx}(\cos x) = -sinx$

$\frac{dy_1}{dy_2}=\frac{dy_1}{dx} \div \frac{dy_2}{dx} = \frac{\cos x}{-sinx}$= -cotx

ALTERNATELY

p= sinx   q= cos x

$\frac{dp}{dx}=\frac{d}{dx}(sinx) = \cos x$

$\frac{dq}{dx}=\frac{d}{dx}(\cos x) = -sinx$

$\frac{dp}{dq}=\frac{dp}{dx} \div \frac{dq}{dx} = \frac{\cos x}{-sinx}$= -cotx

Q-Diffrentiate 3^{sinx}w.r.t logx

$y_1=3^{sinx}$ y_2= logx

$\frac{dy_1}{dx}=\frac{d}{dx}(3^{sinx}) = 3^{sinx}$ .log3. $\frac{d}{dx}$(sinx)

= 3^{sinx} .log3.cosx

$\frac{dy_2}{dx}=\frac{d}{dx}(logx) = \frac{1}{x}$

$\frac{dy_1}{dy_2}=\frac{dy_1}{dx} \div \frac{dy_2}{dx} = \frac{3^{sinx}\text{ .log3.cosx}}{(\frac{1}{x})}$

= x .3^{sinx} .log3.cosx

Q- Q-Diffrentiate e^yw.r.t $\tan^{-1}(cosy)$

p= e^y q= $\tan^{-1}(cosy)$

$\frac{dp}{dy}=\frac{d}{dy}(e^y) = e^y$

$\frac{dq}{dy}=\frac{d}{dy}(\tan^{-1}(cosy)) = \frac{1}{1+cos^2y}\frac{d}{dy}$(cosy)

$=\frac{1}{1+cos^2y}(-siny)=\frac{(-siny)}{1+cos^2y}$

$\frac{dp}{dq}=\frac{dp}{dy} \div \frac{dq}{dy} = e^y \div \frac{(-siny)}{1+cos^2y} = \frac{(1+cos^2y)e^y}{(-siny)}$

Q- Diffrentiate $\sec^{-1}x$w.r.t $\cos^{-1}x$

$y_1=\sec^{-1}x$ $y_2=\cos^{-1}x$

$\frac{dy_1}{dx}=\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

$\frac{dy_2}{dx}=\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{dy_1}{dy_2}=\frac{dy_1}{dx} \div \frac{dy_2}{dx} = \frac{1}{x\sqrt{x^2-1}} \div \frac{-1}{\sqrt{1-x^2}}$

$=\frac{-\sqrt{1-x^2}}{x\sqrt{x^2-1}}$

Q- Diffrentiate $sinx$ w.r.t e^{x^2}

$y_1 = sinx$ $y_2 = e^{x^2}$

$\frac{dy_1}{dx} = \frac{d}{dx}(\sin x) = cosx$

$\frac{dy_2}{dx} = \frac{d}{dx}\left(e^{x^2}\right) = e^{x^2}.2x$

$\frac{dy_1}{dy_2} = \frac{dy_1}{dx} \div \frac{dy_2}{dx} = \frac{cosx}{e^{x^2}.2x}$

Find the derivative of sinx with respect to cosx.

Let y=sinx and z= cosx

To find $\frac{dy}{dz} = \frac{dy}{dx} * \frac{dx}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$

Here $\frac{dy}{dx} = cosx\ and\ \frac{dz}{dx} = -sinx$

$\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{cosx}{-sinx} = -cotx$

Ex:- Find the derivative of a^x *with respect to secx*.

Ex:- Find the derivative of x^3 *with respect to* e^x.

Ex:- Find the derivative of *logx with respect to tanx*.

<u>Successive derivative:-</u>

y=f(x) then $\frac{dy}{dx} - 1st\ derivative$

$\frac{d^2y}{dx^2} - 2nd\ sucessive\ derivative$

$\frac{d^3y}{dx^3} - 3rd\ successive\ derivative$ or 3rd order derivative

y=sinx find $\frac{d^2y}{dx^2}$ or second successive derivative or 2nd order derivative

y=sinx=> $\frac{dy}{dx} = cosx \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}(cosx) = -sinx$

$\frac{d^3y}{dx^3} = \frac{d}{dx}\frac{d^2y}{dx^2} = \frac{d}{dx}(-sinx) = -cosx$

y=cos x find $\frac{d^2y}{dx^2}$ & $\frac{d^3y}{dx^3}$

y=x

$\frac{dy}{dx} = 1; \frac{d^2y}{dx^2} = 0$

y=sinx

$\frac{dy}{dx} = cosx; \frac{d^2y}{dx^2} = -sinx; \frac{d^3y}{dx^3} = -cosx$

$\frac{d^4y}{dx^4} = -(-sinx) = sinx$

If there is any doubt on differentiation ask

Find the derivative of $\tan^2 x^2 - 2x^2 + 3$

Solution:- $\frac{d}{dx}(\tan^2 x^2 - 2x^2 + 3)$

$= \frac{d}{dx}(\tan^2 x^2) + \frac{d}{dx}(-2x^2) + \frac{d}{dx}(3)$

$= \frac{d}{dx}[\tan^2 x^2] - 2\frac{d}{dx}(x^2) + 0$

$= \frac{d}{dx}[\tan^2 x^2] - 2.2x + 0$

$= \frac{d}{dx}[\tan^2 x^2] - 2x^2 = 2tanx^2.\sec^2 x^2.2x - 2x^2$

Now $\frac{d}{dx}[\tan^2 x^2] = \frac{d}{dx}(tanx^2)^2$

$[tanx^2 = X]$ $= \frac{d}{dx}(X)^2 = 2X.\frac{dX}{dx} = 2tanx^2.\frac{d}{dx}(tanx^2)$

$[x^2 = X]$ $= 2tanX.\frac{d}{dx}(tanX) = 2tanX.\sec^2 X.\frac{dX}{dx}$

$= 2tanx^2.\sec^2 x^2.\frac{d}{dx}x^2 = 2tanx^2.\sec^2 x^2.2x$

PARTIAL DERIVATIVE

Partial Differentiation: - The procedure of finding partial derivative of a function is called as partial differentiation

Function with more than one dependent variable.

y=sinx

variables: - y (dependent) & x (independent)

z=sinx.cosy+cosx.siny

variable: - x,y(independent) & z(dependent)

$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(sinx.cosy + cosx.siny)$

$= \frac{\partial}{\partial x}(sinx.cosy) + \frac{\partial}{\partial x}(cosx.siny)$

$= \left(\frac{\partial}{\partial x}sinx\right).cosy + sinx\frac{\partial}{\partial x}(cosy) + \left[\frac{\partial}{\partial x}(cosx)\right]siny + cosx.\frac{\partial}{\partial x}(siny)$

$= cosx.cosy + sinx.0 + (-sinx)siny + cosx.0$

$= cosx.cosy - sinx.siny$

$\frac{d}{dx}(f(x).g(x)) =(\frac{d}{dx}(f(x)).g(x) + f(x).\frac{d}{dx}g(x)$

Q-z=sinx.cosy+cosx.siny

$\frac{\partial z}{\partial y} =\frac{\partial}{\partial y}(sinx.cosy + cosx.siny)$

$= \frac{\partial}{\partial y}(sinx.cosy) + \frac{\partial}{\partial y}(cosx.siny)$

$= \left(\frac{\partial}{\partial y}sinx\right).cosy + sinx\frac{\partial}{\partial y}(cosy) + \left[\frac{\partial}{\partial y}(cosx)\right]siny + cosx.\frac{\partial}{\partial y}(siny)$

$= 0.cosy + sinx.(-siny) + 0.siny + cosx.cosy$

$= cosx.cosy - sinx.siny$

$\frac{d}{dx}(f(x).g(x)) =(\frac{d}{dx}(f(x)).g(x) + f(x).\frac{d}{dx}g(x)$

Let z=$a^x.y + x.a^y$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(a^x.y + x.a^y) = \frac{\partial}{\partial x}(a^x.y) + \frac{\partial}{\partial x}(x.a^y)$

$= \left(\frac{\partial}{\partial x}.a^x\right).y + a^x.\frac{\partial}{\partial x}y + \left(\frac{\partial}{\partial x}x\right)a^y + x.\frac{\partial}{\partial x}.a^y$

$= a^x lna.y + a^x.0 + 1.a^y + x.0 = ya^x lna + a^y$

$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(a^x.y + x.a^y) = \frac{\partial}{\partial y}(a^x.y) + \frac{\partial}{\partial y}(x.a^y)$

$= \left(\frac{\partial}{\partial y}.a^x\right).y + a^x.\frac{\partial}{\partial y}y + \left(\frac{\partial}{\partial y}x\right)a^y + x.\frac{\partial}{\partial y}.a^y$

$= 0.y + a^x.1 + 0.a^y + x.a^y lna = xa^y lna + a^x$

h.w-Let z=$x^2y + x.y^2$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

Find $f_x(1,2)$, $f_y(1,2)$ if f=$\left(\frac{2x-3y}{x^2+y^2}\right)$

$f_x = \frac{\partial}{\partial x}\left(\frac{2x-3y}{x^2+y^2}\right)$

$f_x = \frac{(x^2+y^2)\frac{\partial}{\partial x}(2x-3y)-(2x-3y)\frac{\partial}{\partial x}(x^2+y^2)}{(x^2+y^2)^2}$

$f_x = \frac{(x^2+y^2)2-(2x-3y)2x}{(x^2+y^2)^2}$

$f_x(1,2) = \frac{(1^2+2^2)2-(2.1-3.2).2.1}{(1^2+2^2)^2}$

$f_x(1,2) = \frac{10-(-4)2}{(5)^2}$

$f_x(1,2) = \frac{18}{25}$

$f_y = \frac{\partial}{\partial y}\left(\frac{2x-3y}{x^2+y^2}\right)$

$f_y = \frac{(x^2+y^2)\frac{\partial}{\partial y}(2x-3y)-(2x-3y)\frac{\partial}{\partial y}(x^2+y^2)}{(x^2+y^2)^2}$

$f_y = \frac{(x^2+y^2)(-3)-(2x-3y)2y}{(x^2+y^2)^2}$

$f_y(1,2) = \frac{(1^2+2^2)(-3)-(2.1-3.2).2.2}{(1^2+2^2)^2}$

$f_y(1,2) = \frac{-15-(-4)4}{(5)^2}$

$f_y(1,2) = \frac{1}{25}$

find f_x and f_y of f= x^y+y^x

$f_x = \frac{\partial}{\partial x}(x^y+y^x)$

$f_x = yx^{y-1}+y^x logy$

$f_y = \frac{\partial}{\partial y}(x^y + y^x)$

$f_y = x^y logx + xy^{x-1}$

Q-find f_x and f_y of f= x^y

$f_x = \frac{\partial}{\partial x}(x^y)$

$f_x = yx^{y-1}$

$f_y = \frac{\partial}{\partial y} x^y$

$f_y = x^y logx$

Q- find f_x and f_y of f(x,y)= $5xy^2+4x$

$f_x = \frac{\partial}{\partial x}(5xy^2+4x)$

$f_x = 5.1.y^2+4.1$

$\mathbf{f_x = 5y^2+4 = \frac{\partial f}{\partial x}}$

$f_y = \frac{\partial}{\partial y}(5xy^2+4x)$

$f_y = 5.x.2y+0$

$\mathbf{f_y = 10xy = \frac{\partial f}{\partial y}}$

$\frac{\partial^2 f}{\partial x^2} = f_{xx} = \frac{\partial}{\partial x}(f_x) = \frac{\partial}{\partial x}(5y^2+4) = 0$

$\frac{\partial^2 f}{\partial y^2} = f_{yy} = \frac{\partial}{\partial y}(f_y) = \frac{\partial}{\partial y}(10xy) = 10x.1 = 10x$

$\frac{\partial^2 f}{\partial x\, \partial y} = f_{xy} = \frac{\partial}{\partial x}(f_y) = \frac{\partial}{\partial x}(10xy) = 10.1.y = 10y$

$\frac{\partial^2 f}{\partial y\, \partial x} = f_{yx} = \frac{\partial}{\partial y}(f_x) = \frac{\partial}{\partial y}(5y^2 + 4) = 10y$

INTEGRATION

The integration is the process of finding the antiderivative of a function. The integration is the inverse process of differentiation.

- The function F (x) is called anti-derivative or integral or primitive of the given function f (x) and c is known as the constant of integration or the arbitrary constant. The function f (x) is called the integrand and f (x)dx is known as the element of integration.

$\frac{d}{dx}(sinx)$=cosx $\int cosx\ dx = \sin x$+c

$\frac{d}{dx}(cosx)$= -sinx $\int sinx\ dx = -cosx$ +c

$\frac{d}{dx}(tanx)$=sec^{2}x $\int sec^2x\ dx = \tan x$+c

$\frac{d}{dx}(cotx)$= -cosec^{2}x $\int cosec^2x\ dx = -\cot x$+c

$\frac{d}{dx}(secx)$=secx. tanx

$\int secx.tanx\ dx = secx$+c

$\frac{d}{dx}(cosecx)$= -cosecx. Cot x

$\int cosecx.cotx\ \ dx = -cosecx$+c

$\frac{d}{dx}(e^x)$= e^x $\int e^x\ dx = e^x + c$

$\frac{d}{dx}(logx) = \frac{1}{x}$ $\int \frac{1}{x}\ dx = logx + c$

$\frac{d}{dx}\left(\frac{a^x}{loga}\right) = (a^x)$ $\int (a^x)\ dx = \frac{a^x}{loga} + c$

$$\frac{\frac{d}{dx}(x^{n+1})}{n+1} = \frac{(n+1)x^{(n+1)-1}}{n+1}$$

$$=$$

$\frac{d}{dx}\frac{(x^{n+1})}{n+1}$= x^n $\int x^n\ dx = \frac{(x^{n+1})}{n+1} + c$, n≠ -1

$\int x^1\ dx = \frac{(x^{1+1})}{1+1} + c$=$\frac{x^2}{2}$+c

$\int x^2\ dx = \frac{(x^{2+1})}{2+1} + c$=$\frac{x^3}{3}$+c

$\int x^{-2}\ dx = \frac{(x^{-2+1})}{-2+1} + c$=$\frac{x^{-1}}{-1}$=$\frac{-1}{x}$

$$\int x^{-1}\ dx = logx + c$$

$\int x^6\ dx = \frac{(x^{6+1})}{6+1} + c$=$\frac{x^7}{7}$+c

$\int x^7\ dx = \frac{(x^{7+1})}{7+1} + c$=$\frac{x^8}{8}$+c

$\int x^8\ dx = \frac{(x^{8+1})}{8+1} + c$=$\frac{x^9}{9}$+c

$\int x^{-8}\ dx = \frac{(x^{-8+1})}{-8+1} + c$=$\frac{x^{-7}}{-7}$+c

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$ $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$

$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$ $\int \frac{1}{\sqrt{1-x^2}}\,dx = -\cos^{-1}x + c$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$ $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$

$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$ $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + c$

Question $\int cos2x\,dx$

2x= t

$\frac{d}{dx}$2x= $\frac{d}{dx}$t=$\frac{dt}{dx}$

2=$\frac{dt}{dx}$, dx= dt/2

$$\int cos2x\,dx = \int cost\frac{dt}{2}$$
$$\frac{1}{2}\int costdt = \frac{1}{2}sint + c = \frac{1}{2}sin2x + c$$

question-

$$\int sin\ 3x\ dx$$

3x= t

$\frac{d}{dx}$3x= $\frac{d}{dx}$t=$\frac{dt}{dx}$

3=$\frac{dt}{dx}$, dx= dt/3

$$\int sin3x\ dx = \int sint\frac{dt}{3}$$
$$\frac{1}{3}\int sint\ dt = \frac{1}{3}(-cost) + c = \frac{-1}{3}cos3x + c$$

Question-$\int sec^2 3x\ dx$

3x= t

$\frac{d}{dx}$3x= $\frac{d}{dx}$t=$\frac{dt}{dx}$

3=$\frac{dt}{dx}$, dx= dt/3

$$= \int sec^2 3x\ dx = \int sec^2 t\frac{dt}{3}$$
$$= \frac{1}{3}\int sec^2 t\ dt = \frac{1}{3}(tant) + c = \frac{1}{3}tan3x + c$$

Question-$\int(x+1)^2$dx

=$\int(x^2 + 2x + 1)$dx

=$\frac{x^3}{3}$ +2$\frac{x^2}{2}$ +x+c

=$\frac{x^3}{3}$ + x^2 +x+c

Question-$\int(x+1)^2$dx

Let t= x+1

$\frac{d}{dx}$(x+1)= $\frac{d}{dx}$t=$\frac{dt}{dx}$

1=$\frac{dt}{dx}$

dt=dx

$=\int(t)^2dt = \frac{t^3}{3}$+c

$=\frac{(x+1)^3}{3}$+c

=(x^3+3x^2+3x+1)/3 +c

$=\frac{x^3}{3}+\frac{3x^2}{3}+\frac{3x}{3}+\frac{1}{3}+c$

$=\frac{x^3}{3}+x^2+x+c$

$\int(x+1)^2$dx

$=\int(x^2+2x+1)$dx

$=\frac{x^3}{3}$ +2$\frac{x^2}{2}$ +x+c

$=\frac{x^3}{3}+x^2$ +x+c

Question-$\int(2x+1)^9$dx

Let t= 2x+1

$\frac{d}{dx}$(2x+1)= $\frac{d}{dx}$t=$\frac{dt}{dx}$

2=$\frac{dt}{dx}$

dt/2 =dx

$\int(2x+1)^9$dx $=\int(\frac{t)^9dt}{2}=\frac{t^{10}}{2.10}$+c

$=\frac{(2x+1)^{10}}{20}$+c

QUESTION-$\int 2x^2+e^x$dx

$=\int 2x^2dx+\int e^x$dx

=2$\frac{x^3}{3}$+e^x+c

QUESTION-$\int(1-x)\sqrt{x}$ dx

$\int\sqrt{x}-x^{\frac{3}{2}}$ dx

$=\int\sqrt{x}$ dx -$\int x^{\frac{3}{2}}$ dx

$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$-$\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$+c

$=\frac{2x^{\frac{3}{2}}}{3}$ - $\frac{2x^{\frac{5}{2}}}{5}$+c

QUESTION-$\int\frac{x^3+3x+4}{\sqrt{x}}$ dx

$=\int\frac{x^3}{\sqrt{x}}+\frac{3x}{\sqrt{x}}+\frac{4}{\sqrt{x}}$dx

$=\int\frac{x^3}{\sqrt{x}}dx+\int\frac{3x}{\sqrt{x}}dx+\int\frac{4}{\sqrt{x}}dx$

$=\int x^{\frac{5}{2}}dx+\int 3x^{\frac{1}{2}}dx+\int 4x^{\frac{-1}{2}}dx$

$=\frac{x^{\frac{7}{2}}}{\frac{7}{2}}-3\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$+4$\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$+c

Question $\int secx(secx+tanx)\,dx$

$=\int secx(secx+tanx)\,dx$

$=\int sec^2x+secx.tanx)\,dx$

$=\int sec^2x\,dx+\int secx.tanx\,dx$

=tanx+secx+c

Question $\int\frac{sec^2x}{cosec^2x}dx$

$=\int \frac{sin^2x}{cos^2x}dx$
$=\int tan^2x\, dx$
$=\int sec^2x - 1\, dx$
$=\int sec^2x\, dx - \int 1\, dx$
=tanx-x+c

Formula $\int tanx\, dx$

$=\int tanx\, dx$
$=\int \frac{sinx}{cosx}\, dx$
t=cosx
$\frac{dt}{dx}$= -sinx
-dt=sinx.dx
$=\int \frac{sinx}{cosx}\, dx = \int \frac{-dt}{t}$
=- logt+c
= -log cosx+c
=log $(cosx)^{-1}$+c
=log $\frac{1}{cosx}$+c
= log |secx| +c
=log secx+c
$\int tanx\, dx$= log secx+c

$FORMULA \int cotx\, dx$= log sin x+c

$\int cotx\, dx = \int \frac{cosx}{sinx}\, dx$
t=sinx
$\frac{dt}{dx}$= cosx
dt=cosx.dx
$=\int \frac{cosx}{sinx}\, dx = \int \frac{dt}{t}$
= logt+c
= log sinx+c
$\int cotx\, dx$= log sin x+c

$FORMULA \int secx\, dx$

$\int secx\, dx = \int \frac{secx(secx+tanx)}{secx+tanx}\, dx$
$=\int \frac{sec^2x+secx.tanx}{secx+tanx}\, dx$
t= $secx + tanx$
$\frac{dt}{dx}$=$secx\,.tanx+sec^2x$
dt= $(secx\,.tanx+sec^2x)$dx
$=\int secx\, dx$

$=\int \frac{secx(secx+tanx)}{secx+tanx}\ dx$
$=\int \frac{dt}{t}$
=logt+c
=log$(secx + tanx) + c$
$\int secx\ dx$= log$(secx + tanx) + c$
$\int cosecx\ dx$= log$(cosecx - cotx) + c$ HW

Problems on integration by substitution

In this method of integration by substitution, any given integral is transformed into a simple form of integral by substituting the independent variable by others.

Evaluate $\int \frac{2x}{1+x^2}$dx
Take t= $1 + x^2$
$\frac{dt}{dx} = 2dx.$
dt=$2xdx$
$\int \frac{2x}{1+x^2}$dx
$=\int \frac{dt}{t}$
= logt+c
=log$(1 + x^2) + c$
q-*Evaluate* $\int \frac{1}{x+xlogx} dx$
$=\int \frac{1}{x(1+logx)} dx$
t=1+logx
$\frac{dt}{dx} = \frac{1}{x}.$
dt=$\frac{1}{x} dx$
$=\int \frac{1}{x(1+logx)} dx$
$=\int \frac{dt}{(t)}$
=logt+c
=log(1+logx)+c

question-$\int x\sqrt{x+2}\ \ dx$
$x + 2$=t
$\frac{dt}{dx}$=1
dt=dx
$=\int (t-2)\sqrt{t}\ \ dx$
$=\int t^{\frac{3}{2}} - 2\,t^{\frac{1}{2}}\, dx$
$=\frac{2t^{\frac{5}{2}}}{5} - 2.2\frac{t^{\frac{3}{2}}}{3}$+c
$=\frac{2t^{\frac{5}{2}}}{5} - 4\frac{t^{\frac{3}{2}}}{3}$+c
$=\frac{2(x+2)^{\frac{5}{2}}}{5} - 4\frac{(x+2)^{\frac{3}{2}}}{3}$+c

QUESTION-

$$\int (\sqrt{x} - \frac{1}{\sqrt{x}})^2 dx$$

$\int (\sqrt{x} - \frac{1}{\sqrt{x}})^2 dx$=$\int (x + \frac{1}{x} - 2\,) dx$

=$\frac{x^2}{2} + logx - 2x + c$

=$\int \frac{(logx)^2}{x} dx$

t=logx

$\frac{dt}{dx}$=$\frac{1}{x}$

dt=$\frac{1}{x} dx$

=$\int t^2 dt$

=$\frac{t^3}{3}$+c

=$\frac{(logx)^3}{3}$+c

Question-$\int x\sqrt{x+2}dx$

t=x+2,

$\frac{dt}{dx}$=1

dt=dx

=$\int (t-2)\sqrt{t}dt$

=$\int t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\, dt$

= $\frac{t^{\frac{5}{2}}}{(\frac{5}{2})} - 2\,\frac{t^{\frac{3}{2}}}{(\frac{3}{2})}$+c

=$\frac{(x+2)^{\frac{5}{2}}}{(\frac{5}{2})} - 2\,\frac{(x+2)^{\frac{3}{2}}}{(\frac{3}{2})}$+c

=$\frac{2(x+2)^{\frac{5}{2}}}{(5)} - 4\,\frac{(x+2)^{\frac{3}{2}}}{(3)}$+c

Question-$\int \frac{1}{x-\sqrt{x}}\quad dx$

=$\int \frac{1}{\sqrt{x}(\sqrt{x}-1)} dx$

t=$\sqrt{x} - 1$

$\frac{dt}{dx}$=$\frac{1}{2\sqrt{x}}$

dt=$\frac{1}{2\sqrt{x}} dx$

2 dt=$\frac{1}{\sqrt{x}} dx$

=$\int \frac{1}{\sqrt{x}(\sqrt{x}-1)} dx$

==$\int \frac{1}{(t)} 2dt$

=2$\int \frac{1}{(t)} dt$

2log t+c=2log($\sqrt{x} - 1) + c$

Question-$\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

t= $\tan^{-1} x$

dt=$\frac{1}{1+x^2}$dx

$=\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$

$=\int e^t dt$

$=e^t + c$

$=e^{\tan^{-1}x} + c$

Alternate q-$\int x\sqrt{x+2}\ \ dx$

$x+2$=t^2 (t=$\sqrt{(x+2)}$= $(x+2)^{\frac{1}{2}}$)

$\frac{d}{dx}(x+2) = \frac{d}{dx}$($t^2$)

1=$\frac{d}{dt}$(t^2) $\frac{dt}{dx}$

1=2t. $\frac{dt}{dx}$

dx=2t.dt

$=\int x\sqrt{x+2}\ \ dx$

$=\int (t^2-2)t\ \ 2tdt$

$==\int (t^3-2t)\ \ 2t.dt$

$=\int (2t^4-4t^2)\ \ dt$

$=\frac{2t^5}{5}-\frac{4t^3}{3}$+c

$=\frac{2(x+2)^{\frac{5}{2}}}{5}-\frac{4(x+2)^{\frac{3}{2}}}{3}$+c

QUESTION $\int \frac{e^{2x}-1}{e^{2x}+1}dx$

$$\int \frac{e^x(e^x-e^{-x})}{e^x(e^x+e^{-x})}dx$$

t= (e^x+e^{-x})

dt=(e^x-e^{-x})dx

$=\int \frac{e^x(e^x-e^{-x})}{e^x(e^x+e^{-x})}dx$

$=\int \frac{(e^x-e^{-x})}{(e^x+e^{-x})}dx$

$=\int \frac{1)}{t}dt$

$=logt + c$

=log(e^x+e^{-x})+c

question $\int \frac{1}{1+cotx}dx$

$=\int \frac{1}{1+\frac{cosx}{sinx}}dx$

$=\int \frac{sinx}{sinx+cosx}dx$

$=\frac{1}{2}\int \frac{sinx-cosx+cosx+sinx}{sinx+cosx}dx$

$=\frac{1}{2}\int \frac{sinx+cosx}{sinx+cosx}dx+\frac{1}{2}\int \frac{sinx-cosx}{sinx+cosx}dx$

$=\frac{1}{2}\int 1dx+\frac{1}{2}\int \frac{sinx-cosx}{sinx+cosx}dx$

$=\frac{1}{2}x+\frac{1}{2}\int \frac{sinx-cosx}{sinx+cosx}dx$

$=\frac{1}{2}x+\frac{1}{2}\int \frac{-1}{t}dt$ (t= $sinx+cosx, dt=(cosx-sinx)$dx

$-dt=(sinx-cosx)$dx)

$=\frac{1}{2}x+\frac{1}{2}(-logt)$+c

$=\frac{1}{2}x+\frac{1}{2}(-log(sinx+cosx)+c$

$=\frac{1}{2}(x-log(sinx+cosx)+c$

question $\int\frac{1}{1+tanx}dx$

$=\int\frac{1}{1+\frac{sinx}{cosx}}dx$

$=\int\frac{cosx}{sinx+cosx}dx$

$=\frac{1}{2}\int\frac{cosx-sinx+sinx+cosx}{sinx+cosx}dx$

$=\frac{1}{2}\int\frac{sinx+cosx}{sinx+cosx}dx+\frac{1}{2}\int\frac{cosx-sinx}{sinx+cosx}dx$

$=\frac{1}{2}\int 1dx+\frac{1}{2}\int\frac{cosx-sinx}{sinx+cosx}dx$

$=\frac{1}{2}x+\frac{1}{2}\int\frac{cosx-sinx}{sinx+cosx}dx$

$=\frac{1}{2}x+\frac{1}{2}\int\frac{1}{t}dt$ (t= $sinx+cosx, dt=(cosx-sinx)$dx

$=\frac{1}{2}x+\frac{1}{2}logt)$+c

$=\frac{1}{2}x+\frac{1}{2}log(sinx+cosx)$+c

$=\frac{1}{2}(x+log(sinx+cosx)$+c

question $\int\frac{sinx}{sinx+cosx}dx$

$=\frac{1}{2}\int\frac{sinx-cosx+cosx+sinx}{sinx+cosx}dx$

$=\frac{1}{2}\int\frac{sinx+cosx}{sinx+cosx}dx+\frac{1}{2}\int\frac{sinx-cosx}{sinx+cosx}dx$

$=\frac{1}{2}\int 1dx+\frac{1}{2}\int\frac{sinx-cosx}{sinx+cosx}dx$

$=\frac{1}{2}x+\frac{1}{2}\int\frac{sinx-cosx}{sinx+cosx}dx$

$=\frac{1}{2}x+\frac{1}{2}\int\frac{-1}{t}dt$ (t= $sinx+cosx, dt=(cosx-sinx)$dx

$-dt=(sinx-cosx)$dx)

$=\frac{1}{2}x+\frac{1}{2}(-logt)$+c

$=\frac{1}{2}x+\frac{1}{2}(-log(sinx+cosx)$+c

$=\frac{1}{2}(x-log(sinx+cosx)$+c

question $\int\frac{cosx}{sinx+cosx}dx$

$=\frac{1}{2}\int\frac{cosx-sinx+sinx+cosx}{sinx+cosx}dx$

$=\frac{1}{2}\int\frac{sinx+cosx}{sinx+cosx}dx+\frac{1}{2}\int\frac{cosx-sinx}{sinx+cosx}dx$

$=\frac{1}{2}\int 1dx+\frac{1}{2}\int\frac{cosx-sinx}{sinx+cosx}dx$

$=\frac{1}{2}x+\frac{1}{2}\int\frac{cosx-sinx}{sinx+cosx}dx$

$=\frac{1}{2}x+\frac{1}{2}\int\frac{1}{t}dt$ (t= $sinx+cosx, dt=(cosx-sinx)$dx

$=\frac{1}{2}x+\frac{1}{2}logt)$+c

$=\frac{1}{2}x+\frac{1}{2}log(sinx+cosx)$+c

$=\frac{1}{2}(x+log(sinx+cosx)$+c

question $\int \frac{1}{1-tanx} dx$

$=\int \frac{1}{1-\frac{sinx}{cosx}} dx$

$=\int \frac{cosx}{cosx-sinx} dx$

$=\frac{1}{2}\int \frac{cosx-sinx+sinx+cosx}{cosx-sinx} dx$

$=\frac{1}{2}\int \frac{cosx-sinx}{cosx-sinx} dx+\frac{1}{2}\int \frac{sinx+cosx}{cosx-sinx} dx$

$=\frac{1}{2}\int 1dx+\frac{1}{2}\int \frac{sinx+cosx}{cosx-sinx} dx$

$=\frac{1}{2}x+\frac{1}{2}\int \frac{sinx+cosx}{cosx-sinx} dx$

$=\frac{1}{2}x+\frac{1}{2}\int \frac{-1}{t} dt$ (t= $cosx - sinx, dt = (-sinx - cosx)$dx

$-dt = (sinx + cosx)$dx

$=\frac{1}{2}x+\frac{-1}{2}logt)$+c

$=\frac{1}{2}x+\frac{-1}{2}log(cosx - sinx)$+c

$=\frac{1}{2}(x + log(cosx - sinx)$+c

INTEGRATION BY USING TRIGNOMETRIC IDENTITIES

question$\int cos^2x\ dx$

$=\int \frac{1+cos2x}{2}\ dx$

$=\frac{1}{2}\int 1 + cos2x\ dx$

$=\frac{1}{2}(\int(1dx + \int cos2x\ \ dx)$

$=\frac{1}{2}(x + \frac{sin2x}{2})$ (t=2x,dt=2dx,dx=dt/2, $\int cost\frac{dt}{2}$)

question $\int sin^2x\ dx$

$=\int \frac{1-cos2x}{2}\ dx$

$=\frac{1}{2}\int 1 - cos2x\ dx$

$=\frac{1}{2}(\int(1dx - \int cos2x\ \ dx)$

$=\frac{1}{2}(x - \frac{sin2x}{2})$ (t=2x,dt=2dx,dx=dt/2, $\int cost\frac{dt}{2}$)

Question$\int sin^3x\ dx$

$\int sin^3x\ dx$

$=\int sin^2x.sinx\ dx$

$=\int(1 - cos^2x)sinx\ dx$

$=\int sinx\ dx - \int cos^2x\ .sinx\ dx$

$=-cosx - \int cos^2x\ .sinx\ dx$

$= -cosx - \int t^2.(-dt)$ (t=cosx,dt=-sinxdx)

$= -cosx + \int t^2dt$

$=-cosx + \frac{t^3}{3} + c=-cosx + \frac{cos^3x}{3} + c$

Question-

$\int cos^3x\ dx$

$=\int cos^2x.cosx\ dx$
$=\int(1-sin^2x)cosx\ dx$
$=\int cosx\ dx-\int sin^2x\,.cosx\ dx$
$=sinx-\ \int sin^2x\,.cosx\ dx$
$=sinx-\int t^2.dt)$ (t=sinx,dt=cosx dx)
$=sinx-\int t^2dt$
$=sinx-\frac{t^3}{3}+c=sinx+\frac{sin^3x}{3}+c$

questionI $=\int cos^4x\ dx$
$\int cos^4x\ dx=\int(cos^2x)^2$dx
$=\int(\frac{1+cos2x}{2})^2\ dx$
$=\frac{1}{4}\int(1+cos2x)^2\ dx$
$=\frac{1}{4}\int 1+cos^22x+2cos2x\ dx$
$=\frac{1}{4}(\int 1dx+\int 2cos2x\ \ dx+\int cos^22x\ dx)$
I$=\frac{1}{4}(x+\frac{2sin2x}{2}+\int cos^22x\ dx)$ (t=2x,dt=2dx,dx=dt/2, $\int cost\frac{dt}{2}$)
$I_1=\int cos^22x\ dx$
$=\int\frac{1+cos4x}{2}\ dx$
$=\frac{1}{2}\int 1+cos4x\ dx$
$=\frac{1}{2}(\int(1dx+\int cos4x\ \ dx)$
$I_1=\frac{1}{2}(x+\frac{sin4x}{4})$ (t=4x,dt=4dx,dx=dt/4, $\int cost\frac{dt}{4}$)
I $=\frac{1}{4}(x+\frac{2sin2x}{2}+\int cos^22x\ dx)$
$=\frac{1}{4}(x+\frac{2sin2x}{2}+I_1)$
$=\frac{1}{4}(x+\frac{2sin2x}{2}+\frac{1}{2}(x+\frac{sin4x}{4})$ +c
$=\frac{1}{4}x+\frac{sin2x}{4}+\frac{1}{8}(x+\frac{sin4x}{4})$ +c

Question-$\int sin^5x\ dx$
$=\int sin^5x\ dx$
$=\int sin^4x.\,sinx\ dx$
$=\int\ (1-cos^2x)^2.\,sinx\ dx$
T=cos x,dt= -sinx dx
$=\int\ (1-t^2)^2.(-1)\ dt$
$=\int(1+t^4-2t^2).(-1)\ dt$
$=\int(-1-t^4+2t^2)\ dt$
$=-t-\frac{t^5}{5}+2\frac{t^3}{3}+c$
$=-cosx-\frac{cos^5x}{5}+2\frac{cos^3x}{3}+c$

Home work
$\int cos^5x\ dx$
$\int sin^4x\ dx$

QUESTION-$\int sin2x.cos3x\ dx$

=$\int sin2x.cos3x\ dx$

SIN(A+B)+SIN(A-B)=2SINA.COSB

=$\frac{1}{2}\int 2\ sin2x.cos3x\ dx$

=$\frac{1}{2}\int sin(2x+3x)+sin(2x-3x)dx$

=$\frac{1}{2}\int (sin5x-sinx)dx$

=$\frac{1}{2}\left(-\frac{cos5x}{5}+cosx\right)+c$

QUESTION-$\int sin3x.cos4x\ dx$

=$\int sin3x.cos4x\ dx$

SIN(A+B)+SIN(A-B)=2SINA.COSB

=$\frac{1}{2}\int sin3x.cos4x\ dx$

=$\frac{1}{2}\int sin(3x+4x)+sin(3x-4x)dx$

=$\frac{1}{2}\int (sin7x-sinx)dx$

=$\frac{1}{2}\left(-\frac{cos7x}{7}+cosx\right)+c$

QUESTION-$\int \frac{cosx}{1+cosx}\ dx$

=$\int \frac{cosx}{1+cosx}\ dx$

=$\int \frac{2cos^2\frac{x}{2}-1}{2cos^2\frac{x}{2}}\ dx$

=$\int \frac{2cos^2\frac{x}{2}}{2cos^2\frac{x}{2}}-\frac{1}{2cos^2\frac{x}{2}}\ dx$

=$\int 1-\frac{1}{2}sec^2\frac{x}{2}\ dx$

=$\int 1dx-\int \frac{1}{2}sec^2\frac{x}{2}\ dx$

= x-$\frac{1}{2}$($\frac{tan\frac{x}{2}}{\frac{1}{2}}$)+c

$= x-\frac{1}{2}.2(tan\frac{x}{2}$)+c

$= x-(tan\frac{x}{2}$)+c

QUESTION-$\int \frac{cosx}{1+cosx}\ dx$

=$\int \frac{cosx}{1+cosx}\ dx$

=$\int \frac{cos^2\frac{x}{2}-sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}\ dx$

=$\int \frac{cos^2\frac{x}{2}}{2cos^2\frac{x}{2}}-\frac{1sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}\ dx$

=$\int \frac{1}{2}-\frac{1}{2}tan^2\frac{x}{2}\ dx$

=$\int \frac{1}{2}dx-\int \frac{1}{2}(sec^2\frac{x}{2}-1)\ dx$

=$\int \frac{1}{2}dx-((\int \frac{1}{2}(sec^2\frac{x}{2})dx-\int \frac{1}{2}\ dx)$

=$\int \frac{1}{2}dx-\left(\int \frac{1}{2}(sec^2\frac{x}{2}\right)dx+\int \frac{1}{2}\ dx)$

=$\int 1dx-\int \frac{1}{2}sec^2\frac{x}{2}\ dx$

= x-$\frac{1}{2}$($\frac{tan\frac{x}{2}}{\frac{1}{2}}$)+c

$= x - \frac{1}{2}.2(tan\frac{x}{2})$+c

$= x -(tan\frac{x}{2})$+c

Question-$\int sin^2(2x+5)dx$

2x+5=t

$\frac{dt}{dx}$=2

$\frac{dt}{2} = dx$

=$\int sin^2 t\frac{dt}{2}$

=$\int sin^2 t\frac{dt}{2}$

=$\frac{1}{2}\int sin^2 t\ dt$

=$\frac{1}{2}\int \frac{1-cos2t}{2}\ dt$

=$\frac{1}{4}\int 1 - cos2t\ dt$

=$\frac{1}{4}(\int(1dt - \int cos2t\ \ dt)$

=$\frac{1}{4}(t - \frac{sin2t}{2})$+c

=$\frac{1}{4}(2x+5 - \frac{sin2(2x+5)}{2})$+c

Some formulas by trigonometric substitution

$\int\frac{dx}{a^2-x^2} = \frac{1}{2a}log\left|\frac{a+x}{a-x}\right|$+c

$\int\frac{dx}{x^2-a^2} = \frac{1}{2a}log\left|\frac{x-a}{a+x}\right|$+c

$\int\frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$+c

$\int\frac{dx}{\sqrt{a^2-x^2}} = sin^{-1}\frac{x}{a}$+c

$\int\frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$+c

$\int\frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$+c

proof$\int\frac{dx}{a^2-x^2} = \frac{1}{2a}log\left|\frac{a+x}{a-x}\right|$**+c**

$$\int\frac{dx}{a^2-x^2}$$

$$\int\frac{dx}{(a-x)(a+x)}$$

$\frac{1}{2a}\int\frac{(a+x)+(a-x)}{(a-x)(a+x)}dx$

$$\frac{1}{2a}\int\frac{(a+x)}{(a-x)(a+x)}+\frac{(a-x)}{(a-x)(a+x)}dx$$

$$\frac{1}{2a}\int\frac{1}{(a-x)}+\frac{1}{(a+x)}dx$$

$\frac{1}{2a}(\int\frac{1}{(a-x)}dx + \int\frac{1}{(a+x)}dx)$

$$\frac{1}{2a}\left(-log(a-x)+log(a+x)\right)+c$$
$\frac{1}{2a}\left(log\frac{(a+x)}{(a-x)}\right)+c$
$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}log\ \left|\frac{a+x}{x+a}\right|$+c

(loga-logb= log$\frac{a}{b}$, $\text{loga}+\text{logb}=\ \log(a.b)$

$$question\int\frac{dx}{4-x^2}$$
$$=\int\frac{dx}{2^2-x^2}$$
$$=\int\frac{dx}{(2-x)(2+x)}$$

$=\frac{1}{2.2}\int\frac{(2+x)+(2-x)}{(2-x)(2+x)}dx$
$$=\frac{1}{4}\int\frac{(2+x)}{(2-x)(2+x)}+\frac{(2-x)}{(2-x)(2+x)}dx$$
$$=\frac{1}{4}\int\frac{1}{(2-x)}+\frac{1}{(2+x)}dx$$
$=\frac{1}{4}(\int\frac{1}{(2-x)}dx+\int\frac{1}{(2+x)}dx)$
$$=\frac{1}{4}\left(-log(2-x)+log(2+x)\right)+c$$
$=\frac{1}{4}\left(log\frac{(2+x)}{(2-x)}\right)+c$

$\int\frac{dx}{4-x^2}=\frac{1}{2a}log\ \left|\frac{a+x}{a-x}\right|$+c
$we\ know\int\frac{dx}{a^2-x^2}=\frac{1}{2a}log\ \left|\frac{a+x}{a-x}\right|$+c
Here $a^2\ =4$, a=2
$\int\frac{dx}{4-x^2}=\frac{1}{4}log\ \left|\frac{2+x}{2-x}\right|$+c

$question\int\frac{dx}{6-x^2}=\frac{1}{2a}log\ \left|\frac{a+x}{a-x}\right|$+c
$we\ know\int\frac{dx}{a^2-x^2}=\frac{1}{2a}log\ \left|\frac{a+x}{a-x}\right|$+c
Here $a^2\ =6$, a=$\sqrt{6}$
$\int\frac{dx}{6-x^2}=\frac{1}{2\sqrt{6}}log\ \left|\frac{\sqrt{6}+x}{\sqrt{6}-x}\right|$+c
$question\int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}$+c
$$\int\frac{dx}{x^2+a^2}$$
X=a tanθ , tan $\theta=\frac{x}{a}$, θ=$\tan^{-1}\frac{x}{a}$
$$\frac{dx}{d\theta}=asec^2\theta$$
$dx=asec^2\theta\ d\theta$

$$= \int \frac{asec^2\theta \; d\theta}{(\mathrm{a} \tan\theta\,)^2 + a^2}$$
$$= \int \frac{asec^2\theta \; d\theta}{a^2((\tan\theta)^2 + 1)}$$
$$= \int \frac{asec^2\theta \; d\theta}{a^2 sec^2\theta}$$
$$= \int \frac{1}{a} \; d\theta$$
$$= \frac{1}{a}\theta + c$$
$$= \frac{1}{\mathrm{a}} \tan^{-1}\frac{x}{a} + c$$
$\int \frac{dx}{\sqrt{x^2-a^2}} = log\ |x + \sqrt{x^2 - a^2}|$+c

$$\int \frac{dx}{\sqrt{x^2 - a^2}}$$
$x = asec\theta,$ $\quad sec\theta = \frac{x}{a}, .tan\theta = \sqrt{sec^2\theta - 1}$ $\quad tan\theta = \sqrt{\frac{x^2}{a^2} - 1}$=$\frac{\sqrt{x^2-a^2}}{a}$

$$\frac{dx}{d\theta} = asec\theta.tan\theta$$
$$dx = asec\theta\ .tan\theta\ d\theta$$
$$= \int \frac{asec\theta\ .tan\theta\ d\theta}{\sqrt{a^2 sec^2\theta - a^2}}$$
$$= \int \frac{asec\theta\ .tan\theta\ d\theta}{\sqrt{a^2(sec^2\theta - 1)}}$$
$$= \int \frac{asec\theta\ .tan\theta\ d\theta}{\sqrt{a^2(tan^2\theta)}}$$
$$= \int \frac{asec\theta\ .tan\theta\ d\theta}{atan\theta}$$
$$\int sec\theta\ .d\theta$$
=log$|sec\theta + tan\theta| + c$

=log$\left|\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{a}\right| + c$

=log$\left|\frac{x+\sqrt{x^2-a^2}}{a}\right| + c$

=log$\left|x + \sqrt{x^2 - a^2}\right| - loga + c$

=log$\left|x + \sqrt{x^2 - a^2}\right| + c$

question$\int \frac{dx}{\sqrt{x^2-3}}$

$$\int \frac{dx}{\sqrt{x^2 - 3}}$$
Here$a^2 = 3$, a=$\sqrt{3}$

$x = \sqrt{3}sec\theta,\quad sec\theta = \frac{x}{\sqrt{3}} tan\theta = \sqrt{sec^2\theta - 1}\quad tan\theta =$

$\sqrt{\frac{x^2}{3} - 1} = \frac{\sqrt{x^2-3}}{\sqrt{3}}$

$\frac{dx}{d\theta} = \sqrt{3}sec\theta.tan\theta$

$dx = \sqrt{3}sec\theta\,.tan\theta\,d\theta$

$$= \int \frac{\sqrt{3}sec\theta\,.tan\theta\,d\theta}{\sqrt{3sec^2\theta - 3}}$$

$$=\int \frac{\sqrt{3}sec\theta\,.tan\theta\,d\theta}{\sqrt{3(sec^2\theta-1)}}$$

$$= \int \frac{\sqrt{3}sec\theta\,.tan\theta\,d\theta}{\sqrt{3(tan^2\theta)}}$$

$$= \int \frac{\sqrt{3}sec\theta\,.tan\theta\,d\theta}{\sqrt{3}tan\theta}$$

$$= \int sec\theta\,.d\theta$$

$=\log|sec\theta + tan\theta| + c$

$=\log\left|\frac{x}{\sqrt{3}} + \frac{\sqrt{x^2-3}}{\sqrt{3}}\right| + c$

$=\log\left|\frac{x+\sqrt{x^2-3}}{\sqrt{3}}\right| + c$

$=\log\left|x + \sqrt{x^2 - 3}\right| - log\sqrt{3} + c$

$=\log\left|x + \sqrt{x^2 - 3}\right| + c$

$question \int \frac{dx}{\sqrt{x^2-3}}$

Here a^2=3 a=$\sqrt{3}$

We know $\int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$+c

$\int \frac{dx}{\sqrt{x^2-3}} = log\left|x + \sqrt{x^2 - 3}\right|$+c

$question \int \frac{dx}{\sqrt{x^2-4}}$

Here a^2=4 a=2

We know $\int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$+c

$\int \frac{dx}{\sqrt{x^2-4}} = log\left|x + \sqrt{x^2 - 4}\right|$+c

Question- $\int \sin^{-1}(cosx)dx$

$cosx = t$ $\quad$ sinx $= \sqrt{1 - cos^2x} = \sqrt{1 - t^2}$

$\frac{dt}{dx} = -sinx$

$\frac{dt}{-sinx} = dx\,,$

$\frac{dt}{-\sqrt{1-t^2}} = dx$

$$= \int \sin^{-1}(t)\frac{dt}{-\sqrt{1-t^2}}$$

$$= \int \frac{\sin^{-1}(t)\,dt}{-\sqrt{1-t^2}}$$

$u = \sin^{-1}(t)$, $\frac{du}{dt} = \frac{1}{\sqrt{1-t^2}}$

$$\frac{du}{1} = \frac{1}{\sqrt{1-t^2}}\,dt$$

$$= \int \frac{udu}{-1}$$

$$= \int -udu$$

$$= \frac{-u^2}{2} + c$$

$$= \frac{-(\sin^{-1} t)^2}{2} + c$$

$$= \frac{-(\sin^{-1}(cosx))^2}{2} + c$$

Question- $\int \tan^{-1}(secx + tanx)dx$

$$= \int \tan^{-1}\left(\frac{1}{cosx} + \frac{sinx}{cosx}\right) dx$$

$$= \int \tan^{-1}\left(\frac{1 + sinx}{cosx}\right) dx$$

$$= \int \tan^{-1}\left(\frac{sin^2\frac{x}{2} + cos^2\frac{x}{2} + 2sin\frac{x}{2}.cos\frac{x}{2}}{cosx}\right) dx$$

$$= \int \tan^{-1}\left(\frac{(sin\frac{x}{2} + cos\frac{x}{2})^2}{cos^2\frac{x}{2} - sin^2\frac{x}{2}}\right) dx$$

$$= \int \tan^{-1}\left(\frac{(sin\frac{x}{2} + cos\frac{x}{2})^2}{(sin\frac{x}{2} + cos\frac{x}{2})(cos\frac{x}{2} - sin\frac{x}{2})}\right) dx$$

$$= \int \tan^{-1}\left(\frac{(cos\frac{x}{2} + sin\frac{x}{2})}{(cos\frac{x}{2} - sin\frac{x}{2})}\right) dx$$

$$= \int \tan^{-1}\left(\frac{cos\frac{x}{2}(1 + \frac{sin\frac{x}{2}}{cos\frac{x}{2}})}{cos\frac{x}{2}(1 - \frac{sin\frac{x}{2}}{cos\frac{x}{2}})}\right) dx$$

$$= \int \tan^{-1}\left(\frac{(1 + tan\frac{x}{2})}{(1 - tan\frac{x}{2})}\right) dx$$

$$= \int \tan^{-1}\left(\frac{(tan\frac{\pi}{4} + tan\frac{x}{2})}{(1 - tan\frac{\pi}{4}.tan\frac{x}{2})}\right) dx$$

$$= \int \tan^{-1}\left(tan(\frac{\pi}{4} + \frac{x}{2}\right) dx$$

$$= \int \left(\frac{\pi}{4} + \frac{x}{2}\right) dx$$

$$= \frac{\pi}{4} x + \frac{1}{2}\frac{x^2}{2} + c$$
$$= \frac{\pi}{4} x + \frac{x^2}{4} + c$$

Question-

$$\int \frac{(x^3 + 4x^2 + 3x - 2)}{x + 2} dx$$

$$\int \frac{(x^3 + 4x^2 + 3x - 2)}{x + 2} dx$$

$$= \int \frac{(x^3 + 2x^2 + 2x^2 + 4x - x - 2)}{x + 2} dx$$
$$= \int \frac{x^2(x+2)+2x(x+2)-1(x+2)}{x+2} dx$$
$$= \int x^2 + 2x - 1\, dx$$
$$= \frac{x^3}{3} + x^2 - x + c$$

Question-$\int \sqrt{1 - sin2x}$

$\int \sqrt{1 - sin2x}$ dx

=$\int \sqrt{sin^2x + cos^2x - 2sinx.cosx}\, dx$

$$= \int \sqrt{(sinx + cosx)^2}\, dx$$
$$= \int sinx + cosx\, dx$$

=-cosx+sinx+c

Question $\int \frac{x^4+1}{x^2+1}\, dx$

$$\int \frac{x^4 + 1}{x^2 + 1}\, dx$$
$$= \int \frac{x^4 - 1 + 2}{x^2 + 1}\, dx$$
$$= \int \frac{x^4 - 1}{x^2 + 1} + \frac{2}{x^2 + 1}\, dx$$
$$= \int \frac{x^4 - 1}{x^2 + 1} dx + \int \frac{2}{x^2 + 1}\, dx$$
$$= \int \frac{(x^2 + 1)(x^2 - 1)}{x^2 + 1} dx + \int \frac{2}{x^2 + 1}\, dx$$
$$= \int (x^2 - 1)dx + 2\int \frac{1}{x^2 + 1}\, dx$$
$$= \frac{x^3}{3} - x + 2tan^{-1}\frac{x}{1} + c$$
$$= \frac{x^3}{3} - x + 2tan^{-1}x + c$$

Question- $\int \frac{1}{\sqrt{(x-a)(x-b)}} dx$

$$\int \frac{1}{\sqrt{(x-a)(x-b)}}dx = \int \frac{1}{\sqrt{x^2-ax-bx+ab}}dx$$

$$= \int \frac{1}{\sqrt{x^2-x(a+b)+ab}}dx$$

$$= \int \frac{1}{\sqrt{x^2-2.\frac{1}{2}x(a+b)+ab}}dx$$

$$= \int \frac{1}{\sqrt{x^2-2.x.\frac{(a+b)}{2}+ab}}dx$$

$$= \int \frac{1}{\sqrt{x^2-2.x.\frac{(a+b)}{2}+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab}}dx$$

$$= \int \frac{1}{\sqrt{\left(x-\frac{a+b}{2}\right)^2-\frac{(a+b)^2}{4}+ab}}dx$$

$$= \int \frac{1}{\sqrt{\left(x-\frac{a+b}{2}\right)^2-\frac{(a^2+b^2+2ab)}{4}+ab}}dx$$

$$= \int \frac{1}{\sqrt{\left(x-\frac{a+b}{2}\right)^2+\frac{(-a^2-b^2-2ab)+4ab}{4}}}dx$$

$$= \int \frac{1}{\sqrt{\left(x-\frac{a+b}{2}\right)^2+\frac{(-a^2-b^2+2ab)}{4}}}dx$$

$$= \int \frac{1}{\sqrt{\left(x-\frac{a+b}{2}\right)^2-\frac{(a^2+b^2-2ab)}{4}}}dx$$

$$= \int \frac{1}{\sqrt{\left(x-\frac{a+b}{2}\right)^2-\frac{(a-b)^2}{4}}}dx$$

$$= \int \frac{1}{\sqrt{\left(x-\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2}}dx$$

t= $x-\frac{a+b}{2}$

$\frac{dt}{dx} = 1$

dt=dx

$$= \int \frac{1}{\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}}dt$$

We know $\int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$+c

Here x=t,a $=\frac{a-b}{2}$

$= log\left|t+\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}\right|$+c

=$log\left|\left(x-\frac{a+b}{2}\right)+\sqrt{\left(x-\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2}\right|$+c

$question\int\frac{1}{\sqrt{(x-1)(x-3)}}dx$

$=\int\frac{1}{\sqrt{(x-1)(x-3)}}dx=\int\frac{1}{\sqrt{x^2-x-3x+3}}dx$

$=\int\frac{1}{\sqrt{x^2-4x+3}}dx$

$=\int\frac{1}{\sqrt{x^2-2.\frac{1}{2}4x+3}}dx$

$=\int\frac{1}{\sqrt{x^2-2.2\,x+3}}dx$

$=\int\frac{1}{\sqrt{x^2-2.x\,2+2^2-2^2+3}}dx$

$=\int\frac{1}{\sqrt{(x-2)^2-2^2+3}}dx$

$=\int\frac{1}{\sqrt{(x-2)^2-4+3}}dx$

$=\int\frac{1}{\sqrt{(x-2)^2-1}}dx$

t=x-2

dt/dx =1

dt=dx

$$\int\frac{1}{\sqrt{t^2-1}}dt$$

We know$\int\frac{dx}{\sqrt{x^2-a^2}}=log\left|x+\sqrt{x^2-a^2}\right|$+c

Here x=t, a=1

$=log\left|t+\sqrt{t^2-1^2}\right|$+c

$=log\left|x-2+\sqrt{(x-2)^2-1^2}\right|$+c

Question-$\int\frac{x+3}{x^2-2x-5}dx$

$=\int\frac{2(x+3)}{2(x^2-2x-5)}dx$

$=\int\frac{2x-2+2+6}{2(x^2-2x-5)}dx$

$=\int\frac{2x-2}{2(x^2-2x-5)}+\frac{8}{2(x^2-2x-5)}dx$

$=\int\frac{2x-2}{2(x^2-2x-5)}dx+\int\frac{8}{2(x^2-2x-5)}dx$

$=\int\frac{dt}{2t}+\int\frac{8}{2(x^2-2x-5)}dx$

$=\frac{1}{2}$log+$\int\frac{4}{(x^2-2x-5)}dx$

$=\frac{1}{2}$log(x^2-2x-5) +$\int\frac{4}{(x^2-2.\frac{1}{2}.2x-5)}dx$

$=\frac{1}{2}$log(x^2-2x-5) +$\int\frac{4}{(x^2-2.x.1-5)}dx$

$= \frac{1}{2}\log(x^2-2x-5) + \int \frac{4}{(x^2-2\,.x.1+1^2-1^2-5)}dx$

$= \frac{1}{2}\log(x^2-2x-5) + \int \frac{4}{(x-1)^2-6)}dx$

$= \frac{1}{2}\log(x^2-2x-5) + \int \frac{4}{{t_1}^2-6}dx \;\; (t_1 = x-1, dt_1 = dx)$

$= \frac{1}{2}\log(x^2-2x-5) + 4\int \frac{1}{{t_1}^2-6}dx$

$= \frac{1}{2}\log(x^2-2x-5) + 4\frac{1}{2\sqrt{6}}log\left|\frac{t_1-\sqrt{6}}{t_1+\sqrt{6}}\right|$+c

$= \int \frac{dx}{x^2-a^2} = \frac{1}{2a}log\left|\frac{x-a}{a+x}\right|$+c,here x=$t_1, a^2 = 6, a = \sqrt{6}$

$= \frac{1}{2}\log(x^2-2x-5) + 4\frac{1}{2\sqrt{6}}log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|$+c

Integration by partial fraction method

px^2+qx+r Partial fraction:

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{px^2+qx+r}{(ax^2+b)(cx+d)} = \frac{Ax+B}{ax^2+b} + \frac{C}{cx+d}$$

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$$

$$\frac{k}{(ax+b)(cx^2+d)^2} = \frac{A}{ax+b} + \frac{Bx+C}{(cx^2+d)} + \frac{Dx+E}{(cx^2+d)^2}$$

$$\frac{k}{(ax^2+b)(cx+d)^2} = \frac{Ax+B}{ax+b} + \frac{C}{cx+d} + \frac{D}{(cx+d)^2}$$

Question-$\int \frac{dx}{(x+1)(x+2)}$

$=\int \frac{dx}{(x+1)(x+2)}$

$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$

$$= \frac{Ax+2A+Bx+B}{(x+1)(x+2)} = \frac{(A+B)x+(2A+B)}{(x+1)(x+2)}$$

$$\frac{1}{(x+1)(x+2)} = \frac{(A+B)x+(2A+B)}{(x+1)(x+2)}$$

0.x+1=(A+B)x+(2A+B)

A+B=0 -----------(1)

(2A+B)=1------------(2)

By solving eq -1 & 2 we get

A=1 ,B= -1

$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{(-1)}{x+2}$

$$\int \frac{dx}{(x+1)(x+2)}$$

$$= \int \frac{1}{x+1} + \frac{(-1)}{x+2}\,dx$$

$$= \int \frac{1}{x+1}dx + \int \frac{(-1)}{x+2}\,dx$$

$$= log|x+1| + (-1)log|x+2| + c$$

$$= log|x+1| - log|x+2| + c$$

$$= log\frac{x+1}{x+2}+c$$

Question-

$$\int\frac{3x-2}{(x+1)^2(x+3)}dx$$

$$\frac{3x-2}{(x+1)^2(x+3)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x+3}=\frac{A(x+3)(x+1)+B(x+3)+C(x+1)^2}{(x+1)^2(x+3)}$$

$3x-2=A(x+3)(x+1)+B(x+3)+C(x+1)^2$---(1)

Putting x=-1 in eq-1

3.(-1)-2=0+B(-1+3)+0

$-5=2B$

$B=-\frac{5}{2}$

Putting x=-3 in eq-1

3.(-3)-2=0+0+C$(-3+1)^2$

$-11=4C$

$C=-\frac{11}{4}$

Putting x=0 in eq-1

3.(0)-2=3A+B(3)+C

$-2=3A+3B+C$

$-2=3A+3(-\frac{5}{2})+(-\frac{11}{4})$

$-2=3A-\frac{15}{2}-\frac{11}{4}$

$\frac{15}{2}+\frac{11}{4}-2=3A$

$\frac{30+11-8}{4}=3A$

$\frac{33}{4}=3A$

$\frac{11}{4}=A$

$$\frac{3x-2}{(x+1)^2(x+3)}=\frac{11}{4(x+1)}-\frac{5}{2(x+1)^2}-\frac{11}{4(x+3)}$$

$$\int\frac{3x-2}{(x+1)^2(x+3)}dx$$

$$=\int\frac{11}{4(x+1)}-\frac{5}{2(x+1)^2}-\frac{11}{4(x+3)}dx$$

$$=\int\frac{11}{4(x+1)}dx-\int\frac{5}{2(x+1)^2}dx-\int\frac{11}{4(x+3)}dx$$

$$=\frac{11}{4}log|x+1|-\frac{5}{2}\frac{(x+1)^{-1}}{-1}-\frac{11}{4}log|x+3|+c$$

$$=\frac{11}{4}log|x+1|-\frac{11}{4}log|x+3|-\frac{5}{2}\frac{(x+1)^{-1}}{-1}+c$$

$$= \frac{11}{4} log \frac{|x+1|}{|x+3|} - \frac{5}{2} \frac{(x+1)^{-1}}{-1} + c$$
$$= \frac{11}{4} log \frac{|x+1|}{|x+3|} + \frac{5}{2} \frac{1}{(x+1)} + c$$

$$question - \int \frac{x^2}{(x^2+1)(x^2+4)} dx$$

$$\frac{x^2}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+4)} = \frac{(Ax+B)(x^2+4)+(Cx+D)(x^2+1)}{(x^2+1)(x^2+4)}$$
$$(Ax+B)(x^2+4)+(Cx+D)(x^2+1) = x^2$$
$$Ax^3+4Ax+Bx^2+4B+Cx^3+Cx+Dx^2+D = x^2$$
$$x^3(A+C)+x(4A+C)+x^2(B+D)+(4B+D) = x^2$$

By comparing both sides we get

A+C=0---1

4A+C=0----2

$(B+D)=1$-----3

$(4B+D)=0$-----4

By solving equation 1&2 A=0,C=0

By solving equation 3&4 B=$\frac{-1}{3}$,D=$\frac{4}{3}$

$$\frac{x^2}{(x^2+1)(x^2+4)} = \frac{0.x-\frac{1}{3}}{(x^2+1)} + \frac{0.x+\frac{4}{3}}{(x^2+4)}$$
$$\frac{x^2}{(x^2+1)(x^2+4)} = \frac{-1}{3(x^2+1)} + \frac{4}{3(x^2+4)}$$
$$\int \frac{x^2}{(x^2+1)(x^2+4)} dx = \int \frac{-1}{3(x^2+1)} + \frac{4}{3(x^2+4)} dx$$
$$= \int \frac{-1}{3(x^2+1)} dx + \int \frac{4}{3(x^2+4)} dx$$
$$= -\frac{1}{3} tan^{-1}x + \frac{4}{3} . \frac{1}{2} tan^{-1}\frac{x}{2} + c$$
$$= -\frac{1}{3} tan^{-1}x + \frac{2}{3} tan^{-1}\frac{x}{2} + c$$

$$\int \frac{x}{(x^2+1)(x-1)} dx$$
$$\frac{x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} = \frac{(Ax+B)(x-1)+C(x^2+1)}{(x^2+1)(x-1)}$$

$(Ax+B)(x-1)+C(x^2+1) = x$-----1

By putting x=1 in eq-1

0+C(1+1)=1

2c=1

C=$\frac{1}{2}$

By putting x=0 in eq-1$(Ax+B)(x-1)+C(x^2+1) = x$-----1

$(0+B)(0-1)+C(0+1)$ =0

$-B+C$ =0

$-B+\frac{1}{2}$ =0

$B=\frac{1}{2}$

By putting x=-1 in eq-1$(Ax+B)(x-1)+C(x^2+1)=x$-----1

$(-A+B)(-2)+C(1+1)=-1$

$2A-2B+2C=-1$

$A-B+C=-\frac{1}{2}$

$A-\frac{1}{2}+\frac{1}{2}=-\frac{1}{2}$

$A=-\frac{1}{2}$

$$\int \frac{x}{(x^2+1)(x-1)}dx$$

$$\frac{x}{(x^2+1)(x-1)}=\frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}+\frac{\frac{1}{2}}{x-1}$$

$$\frac{x-1}{-2(x^2+1)}+\frac{1}{2(x-1)}$$

$$\int \frac{x}{(x^2+1)(x-1)}dx=\int \frac{x-1}{-2(x^2+1)}+\frac{1}{2(x-1)}dx$$

$$=\int \frac{x-1}{-2(x^2+1)}dx+\int \frac{1}{2(x-1)}dx$$

$$=\int \frac{x-1}{-2(x^2+1)}dx+\frac{1}{2}log|x-1|$$

$$=\int \frac{2(x-1)}{-4(x^2+1)}dx+\frac{1}{2}log|x-1|$$

$$=\int \frac{2x-2}{-4(x^2+1)}dx+\frac{1}{2}log|x-1|$$

$$=\int \frac{2x}{-4(x^2+1)}-\frac{1}{-2(x^2+1)}dx+\frac{1}{2}log|x-1|$$

$$=\int \frac{2x}{-4(x^2+1)}dx+\int \frac{1}{2(x^2+1)}dx+\frac{1}{2}log|x-1|$$

$$=-\frac{1}{4}log|x^2+1|+\frac{1}{2}tan^{-1}x+\frac{1}{2}log|x-1|+c$$

$$question \int \frac{1-x^2}{x(1-2x)}dx$$

$$\int \frac{1-x^2}{x(1-2x)}dx=\int \frac{\frac{1}{2}(x-2x^2)+1-\frac{1}{2}x}{x(1-2x)}dx$$

$$=\int \frac{\frac{1}{2}(x-2x^2)}{x(1-2x)}+\frac{1-\frac{1}{2}x}{x(1-2x)}dx$$

$$= \int \frac{\frac{1}{2}x(1-2x)}{x(1-2x)} + \frac{2-x}{2x(1-2x)}dx$$
$$= \int \frac{1}{2}dx + \frac{1}{2}\int \frac{2-x}{x(1-2x)}dx$$
$$= \frac{1}{2}x + \frac{1}{2}\int \frac{2-x}{x(1-2x)}dx$$
$$\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{1-2x} = \frac{A(1-2x)+Bx}{x(1-2x)}$$
$A(1-2x) + Bx = 2 - x$-------1

By putting x=0 in eq 1

$A(1) = 2$

A=2

By putting x=$\frac{1}{2}$ in eq 1

$$A\left(1-2.\frac{1}{2}\right) + \frac{B1}{2} = 2 - \frac{1}{2}$$
$$\frac{B}{2} = \frac{3}{2}$$
B=3
$$= \frac{1}{2}x + \frac{1}{2}\int \frac{2}{x} + \frac{3}{1-2x}dx$$
$$= \frac{1}{2}x + \int \frac{1}{x}dx + \int \frac{3}{2(1-2x)}dx$$
$$= \frac{1}{2}x + log|x| + \frac{3}{2}\left(\frac{1}{-2}\right) log|1-2x| + c$$
$$= \frac{1}{2}x + log|x| - \frac{3}{4}log|1-2x| + c$$

Integration by parts

ILATE Rule

Identify the function that comes first on the following list and select it as **'u'**

ILATE stands for:

I: Inverse trigonometric functions

L: Logarithmic functions

A: Algebraic functions.

T: Trigonometric functions, such as sin x, cos x, tan x etc.

E: Exponential functions.

If u and v are any two differentiable functions of a single variable x. Then, by the product rule of differentiation, we have;

$$\int \boldsymbol{u.vdx} = \boldsymbol{u}\int \boldsymbol{vdx} - \int (\int \boldsymbol{vdx})\frac{\boldsymbol{du}}{\boldsymbol{dx}}.\boldsymbol{dx}$$
$u = x \quad vdx = e^x dx$
$$\frac{du}{dx} = 1 \quad \int vdx = \int e^x dx = e^x + c$$

$$\int e^x.xdx = u\int vdx - \int (\int vdx)\frac{du}{dx}.dx$$

$$= x.e^x - \int e^x.1.dx$$

$$= x.e^x - e^x + c$$

question $\int x\ sinxdx$

$u = x \quad vdx = sinxdx$

$\frac{du}{dx} = 1 \quad \int vdx = \int sinx\ dx = -cosx + c$

$$\int x.sinx\ dx = u\int vdx - \int (\int vdx)\frac{du}{dx}.dx$$

$$= x.(-cosx) - \int (-cosx).1.dx$$

$$= -xcosx + \int cosxdx$$

$$= -xcosx + sinx + c$$

question $\int x\ cosx\ dx$

$u = x \quad vdx = cosx\ dx$

$\frac{du}{dx} = 1 \quad \int vdx = \int cosx\ dx = sinx + c$

$$\int x.sinx\ dx = u\int vdx - \int (\int vdx)\frac{du}{dx}.dx$$

$$= x.sinx - \int sinx\ .1.dx$$

$$= xsin\ x + cosx\ + c$$

Question $I = \int e^x.sinx\ dx$

$u = sinx \quad vdx = e^x dx$

$\frac{du}{dx} = cosx \quad \int vdx = \int e^x\ dx = e^x + c$

$$I = \int e^x.sinx\ dx = u\int vdx - \int (\int vdx)\frac{du}{dx}.dx$$

$$I = sinx.e^x - \int e^x\ cosx\ dx$$

$$I = sinx.e^x - I_1$$

$$I_1 = \int e^x\ cosx\ dx$$

$u_1 = cosx \quad v_1 dx = e^x dx$

$\frac{du_1}{dx} = -sinx \quad \int v_1 dx = \int e^x\ dx = e^x + c$

$$I_1 = \int e^x.cosx\ dx = u_1\int v_1 dx - \int (\int v_1 dx)\frac{du_1}{dx}.dx$$

$$I_1 = cosx.e^x - \int e^x.(-sinx)dx$$
$$I_1 = cosx.e^x + \int e^x.(sinx)dx$$
$$I_1 = cosx.e^x + \int e^x.(sinx)dx = cosx.e^x + I$$
$$I = sinx.e^x - (\,cosx.e^x + I)$$
$$I = sinx.e^x - \ cosx.e^x - I$$
$$I + I = sinx.e^x - \ cosx.e^x + c$$
$$2I = sinx.e^x - \ cosx.e^x + c$$
$$I = \frac{1}{2}(sinx.e^x - \ cosx.e^x) + c$$

Question $I = \int e^x.cosx\,dx$

$$\int e^x.cosx\,dx$$

$u = cosx \quad vdx = e^x dx$

$$\frac{du}{dx} = -sinx \qquad \int vdx = \int e^x\,dx = e^x + c$$
$$I = \int e^x.cosx\,dx = u\int vdx - \int (\int vdx)\frac{du}{dx}.dx$$
$$I = cosx.e^x + \int e^x sinx\,dx$$
$$I = cosx.e^x + I_1$$
$$I_1 = \int e^x\,sinx\,dx$$

$u_1 = sinx \quad v_1 dx = e^x dx$

$$\frac{du_1}{dx} = cosx \int v_1 dx = \int e^x\,dx = e^x + c$$
$$I_1 = \int e^x.sinx\,dx = u_1\int v_1 dx - \int (\int v_1 dx)\frac{du_1}{dx}.dx$$
$$I_1 = sinx.e^x - \int e^x.cosxdx$$
$$I_1 = sinx.e^x - \int e^x.(cosx)dx$$
$$I_1 = sinx.e^x - \int e^x.(cosx)dx = sinx.e^x - I$$
$$I = cosx.e^x + I_1$$
$$I = cosx.e^x + \ sinx.e^x - I$$
$$I + I = cosx.e^x + \ sinx.e^x + c$$
$$2I = cosx.e^x + sinx.e^x + c$$
$$I = \frac{1}{2}(cosx.e^x + sinx.e^x) + c$$

Question $I = \int e^x.(cosx + sinx)dx$

$$I = \int e^x.cosx + \int e^x sinx\,dx$$
$$I = \frac{1}{2}(sinx.e^x - \ cosx.e^x) + \frac{1}{2}(sinx.e^x + \ cosx.e^x) + c$$
$$I = \frac{1}{2}sinx.e^x - \frac{1}{2}cosx.e^x + \frac{1}{2}sinx.e^x + \frac{1}{2}cosx.e^x + c$$

$I = \frac{1}{2} sinx.e^x + \frac{1}{2} sinx.e^x + c$
$I = sinx.e^x$ +c

Question $I = \int e^x.(cosx + sinx)dx$

$$\int e^x.(f(x) + f'(x))dx = e^x f(x) + c$$

$$\int e^x.(cosx - sinx)dx = \int e^x.(cosx + (-sinx))dx$$

Here we can see if f(x)= cosx
F'(x)=-sinx so it is in the form $\int e^x.(f(x) + f'(x))dx$
So answer is $e^x f(x) + c = e^x cosx + c$
Question-$\int e^x.(\frac{1}{x} + logx)dx$
Here we can see if f(x)= logx
F'(x)= $\frac{1}{x}$ so it is in the form $\int e^x.(f(x) + f'(x))dx$
So answer is $e^x f(x) + c = e^x logx + c$

SOME FORMULAS

$\int \sqrt{x^2 + a^2}\, dx = \frac{1}{2} x\sqrt{x^2 + a^2} + \frac{a^2}{2} log\left|x + \sqrt{x^2 + a^2}\right| + c$

$$\int \sqrt{x^2 - a^2}\, dx = \frac{1}{2} x\sqrt{x^2 - a^2} - \frac{a^2}{2} log\left|x + \sqrt{x^2 - a^2}\right| + c$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} x\sqrt{a^2 - x^2} - \frac{a^2}{2} sin^{-1}\frac{x}{a} + c$$

Question-$\sqrt{x^2 + 2x + 5}\, dx$

$\sqrt{x^2 + 2x + 5}\, dx$

$= \sqrt{x^2 + 2.\frac{1}{2}2x + 5\, dx}$

$= \sqrt{x^2 + 2.1\,.x + 1 - 1 + 5}\, dx$

$= \sqrt{x^2 + 2.1\,.x + 1^2 + 4}\, dx$

$= \sqrt{(x + 1)^2 + 4}\, dx$

$= \sqrt{t^2 + 4}\, dt$

$\int \sqrt{t^2 + 2^2}\, dx = \frac{1}{2} t\sqrt{t^2 + 4} + \frac{4}{2} log\left|t + \sqrt{t^2 + 4}\right| + c$

$= \frac{1}{2}(x + 1)\sqrt{(x + 1)^2 + 4} + \frac{4}{2} log\left|(x + 1) + \sqrt{(x + 1)^2 + 4}\right| + c$

Definite Integral

The definite integral of a real-valued function f(x) with respect to a real variable x on an interval [a, b] is expressed as

$$\int_a^b f(x)dx = F(b) - F(a)$$

Here,

a = Lower limit

b = Upper limit

f(x) = Integrand

dx = Integrating agent

Thus, $\int_a^b$ f(x) dx is read as the definite integral of f(x) with respect to dx from a to b.

$$\int f(x)dx = F(x) + c$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

e.g-

$$\int_{-1}^{2} xdx = \left[\frac{x^2}{2}\right]_{-1}^{2} = \frac{2^2}{2} - \frac{(-1)^2}{2} = 2 - \frac{1}{2} = \frac{3}{2}$$

question-$\int_{-0}^{\frac{\pi}{4}} sin^3 2t.cos2t\ dt$

$$\int sin^3 2t.cos2t.dt$$

$$u = sin2t$$

$$\frac{du}{dt} = cos2t.2$$

$$\frac{du}{2} = cos2t.dt$$

$$\int u^3.\frac{du}{2} = \frac{u^4}{8} + c = \frac{sin^4 2t}{8} + c$$

$$\int_{-0}^{\frac{\pi}{4}} sin^3 2t.cos2t\ dt = \left[\frac{sin^4 2t}{8}\right]_0^{\frac{\pi}{4}} = \frac{sin^4 .\frac{\pi}{2}}{8} - \frac{sin^4 0}{8} = \frac{1}{8} - 0 = \frac{1}{8}$$

Alternate way $\int_{-0}^{\frac{\pi}{4}} sin^3 2t.cos2t\ dt$

$$\int_0^{\frac{\pi}{4}} sin^3 2t.cos2t.dt$$

$u = sin2t$ if t=0then u=sin0=0,t=$\frac{\pi}{4}$ $then\ u = sin\ 2.\frac{\pi}{4} = sin\frac{\pi}{2} = 1$

$\frac{du}{dt} = cos2t.2$

$\frac{du}{2} = cos2t.dt$

$$= \int_0^1 u^3.\frac{du}{2}$$

$$= \left[\frac{u^4}{4.2}\right]_0^1 = \frac{1}{8} - 0 = \frac{1}{8}$$

Eg-

$$\int_0^1 \frac{x}{x^2+1}dx$$

$t = x^2 + 1$ if x=0then t=1,x=1 $then\ t = 1 + 1 = 2$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x.dx$

$$\int_0^1 \frac{x}{x^2+1}dx = \int_1^2 \frac{1}{t}.\frac{dt}{2}$$

$$= \frac{1}{2}[log|t|]_1^2 = \frac{1}{2}(log2 - log1) = \frac{1}{2}log\frac{2}{1} = \frac{1}{2}log2$$

SOME PROPERTIES OF DEFINITE INTEGRAL

$\int_a^b f(x)dx = \int_a^b f(t)dt$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \quad a < c < b$

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx \,.if\ f(2a-x) = f(x)$$

=0 $\quad if\ f(2a-x) = -f(x)$

$$\int_{-a}^{a} f(x)dx = 2\int_0^{a} f(x)dx\ ,\quad if\ f(-x) = f(x)$$

=0 $\quad if\ f(-x) = -f(x)$

Ex- $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} sin^2xdx$

=2 $\int_0^{\frac{\pi}{4}} sin^2xdx$ (since it is an even function $\int_{-a}^{a} f(x)dx = 2\int_0^{a} f(x)dx\ ,\quad if\ f(-x) = f(x)$)

$$= 2\int_0^{\frac{\pi}{4}} \frac{1-cos2x}{2}dx$$

$$= \int_0^{\frac{\pi}{4}} 1 - cos2xdx$$

=$\left[x - \frac{sin2x}{2}\right]_0^{\frac{\pi}{4}}$

$$= \left[\frac{\pi}{4} - \frac{sin2\frac{\pi}{4}}{2} - 0 + 0\right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} - \frac{sin\frac{\pi}{2}}{2} = \frac{\pi}{4} - \frac{1}{2}$$

Ex-$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} sin^3 3x\ dx$

=0, (since it is an odd function $\int_{-a}^{a} f(x)dx = 2\int_0^{a} f(x)dx\ ,\quad if\ f(-x) = -f(x)$)

Ex-$I = \int_0^{\pi} \frac{xsinx}{1+cos^2x}dx$

$= \int_0^{\pi} \frac{(\pi-x)sin(\pi-x)}{1+cos^2(\pi-x)}dx \qquad applying \int_0^{a} f(x)dx = \int_0^{a} f(a-x)dx$

$$I = \int_0^{\pi} \frac{(\pi-x)sinx}{1+cos^2x}dx$$

$$I = \int_0^{\pi} \frac{\pi sinx}{1+cos^2x}dx - \int_0^{\pi} \frac{xsinx}{1+cos^2x}dx$$

$$I = \int_0^{\pi} \frac{\pi sinx}{1+cos^2x}dx - I$$

$$2I = \int_0^{\pi} \frac{\pi sinx}{1+cos^2x}dx = \pi\int_0^{\pi} \frac{sinx}{1+cos^2x}dx$$

$$2I = \pi\int_0^{\pi} \frac{sinx}{1+cos^2x}dx$$

Putting t= cosx, for x=0 t=cos 0 =1

$\frac{dt}{dx} = -sinx$ For x= $\pi, t = cos\pi = -1$

$-dt = sinxdx$

$$2I = \pi\int_1^{-1} \frac{-dt}{1+t^2}$$

$$I = -\frac{\pi}{2}\int_{1}^{-1}\frac{dt}{1+t^2}$$

$$I = -\frac{\pi}{2}[tan^{-1}t]_1^{-1}$$

$$I = -\frac{\pi}{2}[tan^{-1}(-1) - tan^{-1}(1)]$$

$$I = -\frac{\pi}{2}\left[tan^{-1}\left(tan(-\frac{\pi}{4}\right) - tan^{-1}(tan\frac{\pi}{4})\right]$$

$$I = -\frac{\pi}{2}\left[-\frac{\pi}{4} - \frac{\pi}{4}\right]$$

$$I = -\frac{\pi}{2}\left[-\frac{\pi}{2}\right] = \frac{\pi^2}{2}$$

Ex- $\int_0^{\frac{\pi}{2}}\frac{sin^4x}{sin^4x+cos^4x}dx$

$I = \int_0^{\frac{\pi}{2}}\frac{sin^4x}{sin^4x+cos^4x}dx$-------eq-1

$$I = \int_0^{\frac{\pi}{2}}\frac{sin^4(\frac{\pi}{2}-x)}{sin^4(\frac{\pi}{2}-x)+cos^4(\frac{\pi}{2}-x)}dx \qquad applying \int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$I = \int_0^{\frac{\pi}{2}}\frac{cos^4(x)}{cos^4(x)+sin^4(x)}dx$ -----eq-2

By adding eq-1 & 2 we get

$$I = \int_0^{\frac{\pi}{2}}\frac{sin^4x}{sin^4x+cos^4x}dx + \int_0^{\frac{\pi}{2}}\frac{cos^4(x)}{cos^4(x)+sin^4(x)}dx$$

$$I = \int_0^{\frac{\pi}{2}}\frac{sin^4x+cos^4(x)}{cos^4(x)+sin^4(x)}dx$$

$$I = \int_0^{\frac{\pi}{2}}1\,dx$$

$$I = [x]_0^{\frac{\pi}{2}} = \left[\frac{\pi}{2}-0\right] = \frac{\pi}{2}$$

Ex- $\int_0^4 |x|\,dx$

$$= \int_0^4 |x|\,dx$$

$$= \int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4 = \frac{16}{2} - 0 = 8$$

Ex-$\int_{-2}^{4}|x|\,dx$

$$=\int_{-2}^{4}|x|\,dx$$
$$=\int_{-2}^{0}|x|dx+\int_{0}^{4}|x|dx$$
$$=\int_{-2}^{0}|x|dx+\int_{0}^{4}|x|dx$$
$$=\int_{-2}^{0}-x\,dx+\int_{0}^{4}x\,dx$$
$$=-\left[\frac{x^2}{2}\right]_{-2}^{0}+\left[\frac{x^2}{2}\right]_{0}^{4}$$
$$=-\left(0-\frac{4}{2}\right)+\left(\frac{16}{2}-0\right)$$
$$=2+8=10$$

Ex-$\int_{0}^{4}|x-1|\,dx$

$$\int_{0}^{4}|x-1|\,dx$$
$$=\int_{0}^{1}|x-1|dx+\int_{1}^{4}|x-1|dx$$
$$=\int_{0}^{1}-(x-1)dx+\int_{1}^{4}x-1\,dx$$
$$=-\left[\frac{x^2}{2}-x\right]_{0}^{1}+\left[\frac{x^2}{2}-x\right]_{1}^{4}$$
$$=-\left(\frac{1}{2}-1-0\right)+\left(\frac{16}{2}-4-\left(\frac{1}{2}-1\right)\right)$$
$$=-\frac{1}{2}+1+8-4-\frac{1}{2}+1$$
$$=-\frac{1}{2}-\frac{1}{2}+6=-1+6=5$$

Ex-$\int_{0}^{4}[x]\,dx$

$$=\int_{0}^{4}[x]\,dx$$
$$=\int_{0}^{1}[x]\,dx+\int_{1}^{2}[x]\,dx+\int_{2}^{3}[x]\,dx+\int_{3}^{4}[x]\,dx$$
$$=\int_{0}^{1}0\,dx+\int_{1}^{2}1\,dx+\int_{2}^{3}2\,dx+\int_{3}^{4}3\,dx$$
$$=0+[x]_1^2+2[x]_2^3+3[x]_3^4$$
$$=2-1+2(3-2)+3(4-3)$$
$$=1+2+3=6$$

Ex-$\int_{-1}^{1}[x-1]\,dx$

$= \int_{-1}^{0} [x-1]\, dx + \int_{0}^{1} [x-1]\, dx$

$= \int_{-1}^{0} -2\, dx + \int_{0}^{1} -1\, dx$

$= -2[x]_{-1}^{0} - 1[x]_{0}^{1}$

$= -2(0+1) - 1(1-0)$

$= -2 - 1 = -3$

Ex- $\int_{0}^{2} [x^2]\, dx$

$= \int_{0}^{1} [x^2]\, dx + \int_{1}^{\sqrt{2}} [x^2]\, dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]\, dx + \int_{\sqrt{3}}^{2} [x^2]\, dx$

$= \int_{0}^{1} 0\, dx + \int_{1}^{\sqrt{2}} 1\, dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\, dx + \int_{\sqrt{3}}^{2} 3\, dx$

$= 0 + [x]_{1}^{\sqrt{2}} + 2[x]_{\sqrt{2}}^{\sqrt{3}} + 3[x]_{\sqrt{3}}^{2}$

$= \sqrt{2} - 1 + 2(\sqrt{3} - \sqrt{2})| + 3(2 - \sqrt{3)}$

$= \sqrt{2} + 2\sqrt{3} - 2\sqrt{2} - 3\sqrt{3)} + 6 - 1$

$= -\sqrt{3} - \sqrt{2} + 5$

$= 5 - \sqrt{3} - \sqrt{2}$

AREA BOUNDED BY A CURVE AND A LINE

Ex-Find area of the region bounded by the curve
$y^2 = x$ *and the lines* $x = 1$ *and* $x = 4$. *and the* $x-$*axis in the first quadrant.*

Ans- $\int_1^4 y\,dx$

Here $y^2 = x$

$y = \sqrt{x}$

$\int_1^4 \sqrt{x}\,dx = [\frac{2}{3}x^{\frac{3}{2}}]_1^4 = \frac{2}{3}\left(4^{\frac{3}{2}} - 1\right) = \frac{2}{3}(8-1) = \frac{14}{3}$

Ex- Find area of the region bounded by the curve $x^2 = 4y$ *and the lines* $y = 2$ *and* $y = 4$ *and the* $y-$*axis in the first quadrant.*

Ans- $\int_2^4 \boldsymbol{x\,dy}$

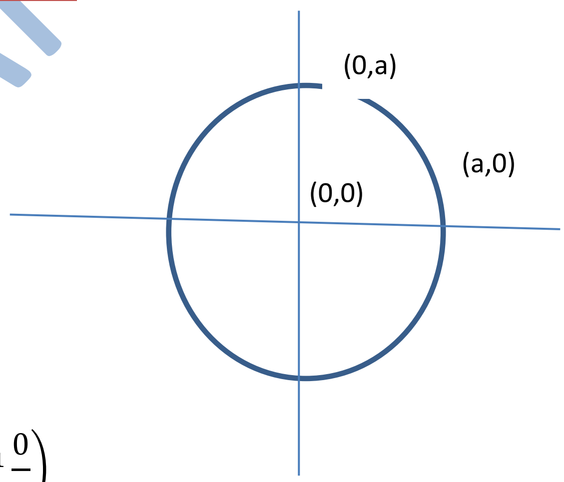
Here $\boldsymbol{x^2 = 4y}$

$\boldsymbol{x = 2\sqrt{y}}$

$$\int_2^4 2\sqrt{y}\,dy = 2[\frac{2}{3}y^{\frac{3}{2}}]_2^4 = \frac{4}{3}\left(4^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) = \frac{4}{3}(8-\sqrt{8})$$

Area bounded by a circle

Ex- Find area bounded by a circle $\boldsymbol{x^2 + y^2 = a^2}$.



Ans-

$\int_0^a x\,dy$

$4\int_0^a \sqrt{a^2 - x^2}\,dx$

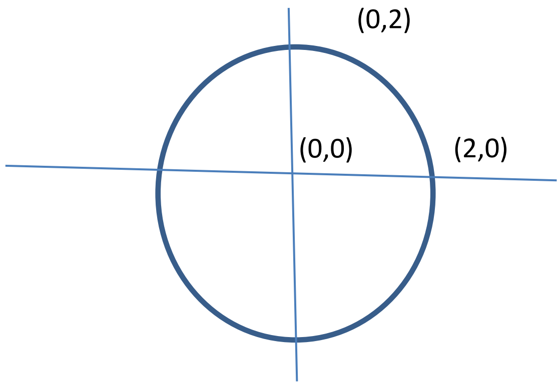
$4\left[\frac{1}{2}x\sqrt{a^2 - x^2} - \frac{a^2}{2}sin^{-1}\frac{x}{a}\right]_0^a$

$4\left(\frac{1}{2}a\sqrt{a^2 - a^2} - \frac{a^2}{2}sin^{-1}\frac{a}{a}\right) - 4\left(\frac{1}{2}0\sqrt{a^2 - 0^2} - \frac{a^2}{2}sin^{-1}\frac{0}{a}\right)$

$4\left(\frac{1}{2}a\sqrt{a^2 - a^2} - \frac{a^2}{2}sin^{-1}\frac{a}{a}\right)$

$4\left(-\frac{a^2}{2}sin^{-1}(1)\right) = 4\left(\frac{a^2}{2}\frac{\pi}{2}\right) = \pi a^2$

Ex- Find area bounded by a circle $x^2 + y^2 = 4$.



Ans-

$\int_0^2 y\,dx$

$4\int_0^2 \sqrt{a^2 - x^2}\,dx$

$$4\left[\frac{1}{2}x\sqrt{2^2-x^2}-\frac{x^2}{2}sin^{-1}\frac{x}{2}\right]_0^a$$

$$4\left(\frac{1}{2}2\sqrt{2^2-2^2}-\frac{2^2}{2}sin^{-1}\frac{2}{2}\right)-4\left(\frac{1}{2}0\sqrt{2^2-0^2}-\frac{2^2}{2}sin^{-1}\frac{0}{2}\right)$$

$$4\left(\frac{1}{2}2\sqrt{2^2-2^2}-\frac{2^2}{2}sin^{-1}\frac{2}{a}\right)$$

$$4\left(-\frac{2^2}{2}sin^{-1}(1)\right)=4\left(\frac{2^2}{2}\frac{\pi}{2}\right)=4\pi$$

Diffrential Equation

The order of a differential equation is the order of the highest order derivative involved in the differential equation. The degree of a differential equation is the exponent of the highest order derivative involved in the differential equation

Order and Degree of a Diffrential Equation

$\frac{dy}{dx} = 1$	order= 1	degree=1
$\frac{d^2y}{dx^2} - 3x = 0$	order= 2	degree=1
$(\frac{dy}{dx})^2 = 1$	order=1	degree=2
$\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 - 3y + 2 = 0$	order=2	degree=1
$\frac{d^3y}{dx^3} + (\frac{dy}{dx})^2 - 3x = 0$	order=3	degree=1
$(\frac{d^2y}{dx^2})^2 - 3 = 0$	order= 2	degree=2
$(\frac{dy}{dx})^2 + (\frac{d^2y}{dx^2})^3 = 1$	order=2	degree=3
$(\frac{d^2y}{dx^2})^2 + (\frac{dy}{dx})^3 - 3y + 2 = 0$	order= 2	degree=2
$\left(\frac{d^3y}{dx^3}\right)^3 + (\frac{dy}{dx})^2 - 3x = 0$	order= 3	degree=3
$\left(\frac{d^3y}{dx^3}\right)^2 + (\frac{dy}{dx})^4 - 4 = 0$	order= 3	degree=2
$(\frac{d^2y}{dx^2})^2 + (\frac{dy}{dx})^3 + (\frac{dy}{dx})^4 - 3y + 2 = 0$	order=2	degree=2

Differential Equation solving by variable separable method

$solve\ \frac{dy}{dx} = 1$

$\frac{dy}{dx} = 1$

$dy = dx$

Integrate both side

$\int 1.dy = \int dx$

$y = x + c$

$Ex - solve\ \frac{dy}{dx} + sinx = 3$

$\frac{dy}{dx} + sinx = 3$

$\frac{dy}{dx} = 3 - sinx$

$dy = (3 - sinx)dx$

Integrating both side

$\int 1.dy = \int (3 - sinx)dx$

$y = 3x + cosx + c$

$Ex - solve\ \frac{dy}{dx} + sinx = 0$

$\frac{dy}{dx} + sinx = 0$

$\frac{dy}{dx} = -sinx$

$dy = (-sinx)dx$

Integrating both side

$\int 1.dy = \int (-sinx)dx$

$y = cosx + c$

$\mathrm{Ex - solve}\ \frac{\mathrm{dy}}{\mathrm{dx}}\mathrm{x} + \mathrm{y} = 0$

$\frac{\mathrm{dy}}{\mathrm{dx}}\mathrm{x} + \mathrm{y} = 0$

$\frac{\mathrm{dy}}{\mathrm{dx}}\mathrm{x} = -\mathrm{y}$

$\frac{\mathrm{dy}}{\mathrm{y}} = -\frac{\mathrm{dx}}{\mathrm{x}}$

Integrating both side

$\int \frac{\mathrm{dy}}{\mathrm{y}} = -\int \left(\frac{\mathrm{dx}}{\mathrm{x}}\right)\mathrm{dx}$

$\mathrm{logy} = -\mathrm{logx} + \mathrm{c}$

$\mathrm{logy} + \mathrm{logx} = \mathrm{c}$

$\log \mathrm{xy} = \mathrm{c}$

$xy = e^c$

Ex- $if\ y = e^x + 1 \quad verify \quad y'' - y' = 0$

$y' = \frac{dy}{dx} = e^x$

$y'' = \frac{d^2y}{dx^2} = e^x$

$y'' - y' = e^x - e^x = 0(proved)$

Ex-form differential Equation of the given Equation $y = e^x(acosx + bsinx)$

$y' = \frac{dy}{dx} = e^x(acosx + bsinx) + e^x(-asinx + bcosx)$

$y' = \frac{dy}{dx} = y + e^x(-asinx + bcosx)$

$y'' = \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-asinx + bcosx) + e^x(-acosx - bsinx)$

$y'' = \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-asinx + bcosx) - e^x(acosx + bsinx)$

$y'' = \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-asinx + bcosx) - y$

$y'' = \frac{d^2y}{dx^2} = \frac{dy}{dx} + y' - y - y \qquad (y' - y = e^x(-asinx + bcosx))$

$$y'' = \frac{d^2y}{dx^2} = y' + y' - 2y$$
$$y'' = 2y' - 2y$$
$$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2y$$

Ex- Find the general solution of the differential Equation $\frac{dy}{dx} = \frac{x+1}{2-y}$?

$$\frac{dy}{dx} = \frac{x+1}{2-y}$$
$$(2-y)dy = (x+1)dx$$

Integrating both side

$$\int (2-y)dy = \int (x+1)dx$$
$$2y - \frac{y^2}{2} = \frac{x^2}{2} + x + c$$

Ex- Find the general solution of the differential Equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$?

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$
$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

Integrating both side

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$
$$tan^{-1}y = tan^{-1}x + c$$

Ex- Find the general solution of the differential Equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ at (0,0)

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$
$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

Integrating both side

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$
$$tan^{-1}y = tan^{-1}x + c$$

At (0,0)

$$tan^{-1}0 = tan^{-1}0 + c$$
$$0 = 0 + c$$

C=0

$$tan^{-1}y = tan^{-1}x$$

Ex- Find the general solution of the differential Equation $\frac{dy}{dx} = \sqrt{4-y^2}$?

$$\frac{dy}{dx} = \sqrt{4-y^2}$$
$$\frac{dy}{\sqrt{4-y^2}} = dx$$

Integrating both side

$\int \frac{dy}{\sqrt{4-y^2}} = \int dx$

$sin^{-1}\frac{x}{2} = x + c$

$Ex - \frac{dy}{dx} = sin^{-1}x$

$\frac{dy}{dx} = sin^{-1}x$

$\int dy = \int 1.sin^{-1}xdx$

$u = sin^{-1}x \qquad \int v\,dx = \int 1\,dx$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \int v\,dx = x + c,$

$$\int 1.sin^{-1}xdx = u.\int v\,dx - \int \frac{du}{dx}\int v\,dx$$

$$= xsin^{-1}x - \int \frac{1}{\sqrt{1-x^2}}\,x\,dx$$

$$= xsin^{-1}x + \int \frac{1}{\sqrt{t}}\frac{dt}{2}$$

$$= xsin^{-1}x + \frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= xsin^{-1}x + \sqrt{1-x^2} + c$$

$\int dy = \int 1.sin^{-1}xdx$

$\int dy = xsin^{-1}x + \sqrt{1-x^2}$

$y = xsin^{-1}x + \sqrt{1-x^2} + c$(solved)

$Ex = (x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x; y = 1 when\ x = 0$

$(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$

$dy = \frac{2x^2 + x}{(x^3 + x^2 + x + 1)}dx$

$\int dy = \int \frac{2x^2 + x}{(x^3 + x^2 + x + 1)}dx$

$y = \int \frac{2x^2 + x}{(x^3 + x^2 + x + 1)}dx$

$y = \int \frac{x(2x + 1)}{(x^2(x + 1) + 1(x + 1))}dx$

$y = \int \frac{x(2x + 1)}{(x + 1)(x^2 + 1)}dx$

$y = \int \frac{1}{x + 1} + \frac{3x - 2}{2(x^2 + 1)}dx$

$$y = log|x+1| + \frac{1}{2}\int \frac{3x}{(x^2+1)} - \frac{2}{(x^2+1)} dx$$

$$y = log|x+1| + \frac{1}{2}(\frac{3}{2}log|x^2+1| - 2tan^{-1}x| + c$$

$$y = log|x+1| + \frac{3}{4}log|x^2+1| - tan^{-1}x + c$$

$$1 = log|0+1| + \frac{3}{4}log|0^2+1| - tan^{-1}0 + c$$

$$1 = 0 + 0 - 0 + c$$

C=1

$$y = log|x+1| + \frac{3}{4}log|x^2+1| - tan^{-1}x + 1$$

LINEAR DIFFRENTIAL EQUATION

It is of the following form

$$\frac{dy}{dx} + Py = Q, wher\ P, Q\ are\ function\ of\ x$$

Then solution is $ye^{\int pdx} = \int e^{\int pdx} Qdx + c$

$$\frac{dx}{dy} + Px = Q, wher\ P, Q\ are\ function\ of\ y$$

Then solution is $xe^{\int pdy} = \int e^{\int pdy} Qdy$ +c

Ex= $\frac{dy}{dx} + y = sinx$

It is in linear form

$$\frac{dy}{dx} + Py = Q,$$

$$\frac{dy}{dx} + y = sinx$$

Here P= 1, $Q = sinx$

$$\int pdx = \int 1dx = x,$$

$$e^{\int pdx} = e^x$$

$$ye^{\int pdx} = \int e^{\int pdx} Qdx$$

$$ye^x = \int e^x sinxdx = I$$

$$ye^x = I$$

$$I = \int e^x sinxdx$$

$u = sinx$ $\quad \int vdx = \int e^x dx = e^x + k$

$$\frac{du}{dx} = cosx$$

$$du = cosx\ dx$$

$$I = u\int vdx - \int\int vdx.\frac{du}{dx}dx$$

$$I = sinx\,.e^x - \int e^x cosx\,dx = sinx.e^x - I_1$$

$$I_1 = \int e^x cosx\,dx$$

$$u_1 = cosx \qquad \int v_1 dx = \int e^x dx = e^x + k$$

$$\frac{du_1}{dx} = -sinx$$

$$I_1 = u_1\int v_1 dx - \int\int v_1 dx.\frac{du_1}{dx}dx$$

$$I_1 = cosx\,.e^x + \int e^x sinx\,dx = e^x.cosx + I$$

$$I = sinx.e^x - I_1 = sinx.e^x - (e^x.cosx + I)$$

$$I = sinx.e^x - e^x.cosx - I$$

$$2I = sinx.e^x - e^x.cosx = e^x(sinx - cosx)$$

$$I = \frac{e^x}{2}(sinx - cosx)$$

$ye^x = I$+c

$$ye^x = \frac{e^x}{2}(sinx - cosx) + c$$

Ex- Find general solution of the differential Equation

$$x\frac{dy}{dx} + 2y = x^2$$

Sol- $x\frac{dy}{dx} + 2y = x^2$

$$\frac{dy}{dx} + \frac{2}{x}\,y = x$$

Here P=$\frac{2}{x}, Q = x$

$$\int pdx = \int \frac{2}{x}dx = 2\,logx$$

$$e^{\int pdx} = e^{2logx} = e^{logx^2} = x^2$$

$$ye^{\int pdx} = \int e^{\int pdx}Qdx + c$$

$$yx^2 = \int x^2 xdx = \int x^3\,dx = \frac{x^4}{4} + c$$

$$yx^2 = \frac{x^4}{4} + c$$

$$ex - \frac{dy}{dx} + 2ytanx = sinx\,, y = 0\ when\ x = \frac{\pi}{3}$$

$$\frac{dy}{dx} + 2ytanx = sinx$$

$$here\ P = 2tanx, Q = sinx$$

$$e^{\int Pdx} = e^{\int 2tanx\,dx} = e^{2log|secx|} = e^{log\,(secx)^2} = sec^2x$$

$$ye^{\int pdx} = \int e^{\int pdx} Q dx + c$$

$$ysec^2x = \int sec^2x \, sinx \, dx + c$$

$$ysec^2x = \int \frac{sinx}{cos^2x} dx + c = \int tanx.secx \; dx + c$$

$ysec^2x = -\int \frac{dt}{t^2} + c$ (t=cosx)

$$ysec^2x = \frac{1}{t} + c$$

$$ysec^2x = \frac{1}{cosx} + c$$

$$ysec^2x = secx \; + c$$

$$y = 0 \; when \; x = \frac{\pi}{3}$$

$$0sec^2\frac{\pi}{3} = sec\frac{\pi}{3} \; + c$$

$$0 = 2 + c$$

C= - 2

$$ysec^2x = secx - 2$$

Ex- what will be the integrating factor of the differential Equation

$$x\frac{dy}{dx} - y = 2x^2$$

I.F= $e^{\int Pdx}$

$$\frac{dy}{dx} - \frac{1}{x}y = 2x$$

$$P = -\frac{1}{x}$$

$$e^{\int Pdx} = e^{\int -\frac{1}{x}dx} = e^{-logx} = e^{logx^{-1}} = x^{-1} = \frac{1}{x}$$

Some question on Partial Fraction

$$\int \frac{x^2}{(x^2+1)(x^2+4)}dx$$

$$\frac{x^2}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1} + \frac{cx+D}{x^2+4} = \frac{(Ax+B)(x^2+4)+(Cx+D)(x^2+1)}{(x^2+1)(x^2+4)}$$

$$(Ax+B)(x^2+4)+(Cx+D)(x^2+1) = x^2$$

$$Ax^3+4Ax+Bx^2+4B+Cx^3+Cx+Dx^2+D = x^2$$

$$x^3(A+C)+x^2(B+D)+x(C+4A)+(D+4B) = x^2$$

$A+C = 0$------1

$B+D = 1$--------2

$C+4A = 0$-------3

$D+4B = 0$-------4

By solving these equation we get

A=0 ,B= -1/3 ,C= 0 ,D=4/3 ,

$$\frac{x^2}{(x^2+1)(x^2+4)} = \frac{-1}{3(x^2+1)} + \frac{4}{3(x^2+4)}$$

$$\int \frac{x^2}{(x^2+1)(x^2+4)}dx$$

$$\int \frac{-1}{3(x^2+1)} + \frac{4}{3(x^2+4)}dx = -\frac{1}{3}tan^{-1}x + \frac{4}{6}tan^{-1}2 + c$$

(Prepared by Sri Samira Kumar Pathi ,Lecturer in mathematics, G.P. Gajapati)

LIMIT AND CONTINUITY

Consider the function $f(x) = \frac{x^2 - 1}{x - 1}$

You can see that the function f(x) is not defined at x = 1 as $x - 1$ is in the denominator. Take the value of x very nearly equal to but not equal to 1 as given in the tables below. In this case $x - 1 \neq 0$ as $x \neq 1$.

$\therefore$ We can write $f(x) = \frac{x^2 - 1}{x - 1} = \frac{(x+1)(x-1)}{(x-1)} = x + 1$, because $x - 1 \neq 0$ and so division by $(x - 1)$ is possible.

Table -1

x	f(x)
0.5	1.5
0.6	1.6
0.7	1.7
0.8	1.8
0.9	1.9
0.91	1.91
:	:
:	:
0.99	1.99
:	:
:	:
0.9999	1.9999

Table - 2

x	f(x)
1.9	2.9
1.8	2.8
1.7	2.7
1.6	2.6
1.5	2.5
:	:
:	:
1.1	2.1
1.01	2.01
1.001	2.001
:	:
:	:
1.00001	2.00001

In the above tables, you can see that as x gets closer to 1, the corresponding value of f (x) also gets closer to 2.

However, in this case f(x) is not defined at x = 1. The idea can be expressed by saying that the limiting value of f(x) is 2 when x approaches to 1.

Let us consider another function f (x) =2x. Here, we are interested to see its behavior near the point 1 and at x = 1. We find that as x gets nearer to 1, the corresponding value of f (x) gets closer to 2 at x = 1 and the value of f (x) is also 2.

So from the above findings, what more can we say about the behaviour of the function near $x = 2$ and at $x = 2$?

In this lesson we propose to study the behaviour of a function near and at a particular point where the function may or may not be defined.

In the introduction, we considered the function $f(x) = \frac{x^2 - 1}{x - 1}$. We have seen that as x approaches 1, f(x) approaches 2. In general, if a function f(x) approaches L when x approaches 'a', we say that L is the limiting value of f(x)

Symbolically it is written as

$$\lim_{x \to a} f(x) = L$$

Now let us find the limiting value of the function $(5x - 3)$ when x approaches 0.

i.e. $$\lim_{x \to 0} (5x - 3)$$

For finding this limit, we assign values to x from left and also from right of 0.

x	−0.1	−0.01	−0.001	−0.0001..........
5x − 3	−3.5	−3.05	−3.005	−3.0005 ....

x	0.1	0.01	0.001	0.0001..........
5x − 3	−2.5	−2.95	−2.995	−2.9995 ....

It is clear from the above that the limit of $(5x-3)$ as $x \to 0$ is -3

i.e., $$\lim_{x\to 0}(5x-3) = -3$$

This is illustrated graphically in the Fig.

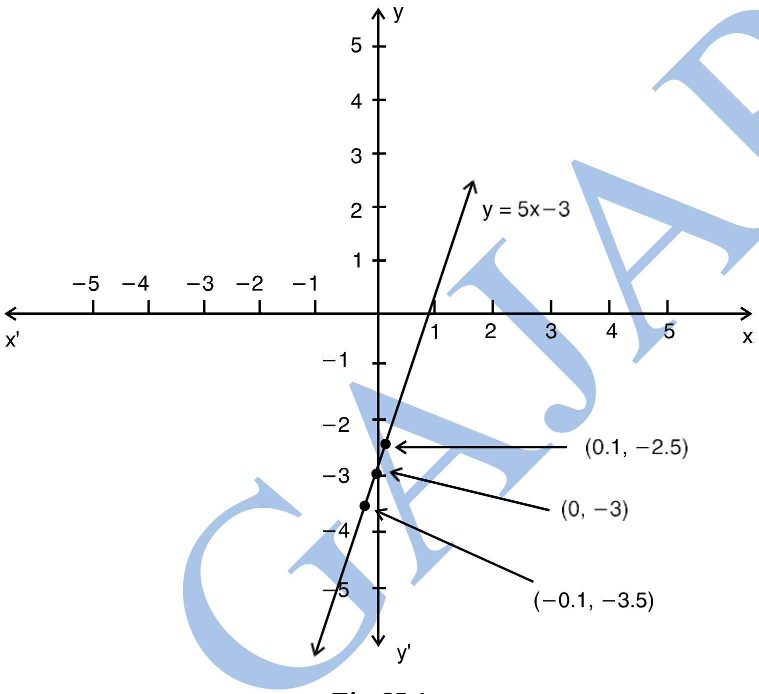


Fig. 25.1

The method of finding limiting values of a function at a given point by putting the values of the variable very close to that point may not always be convenient.

We, therefore, need other methods for calculating the limits of a function as x (independent variable) ends to a finite quantity, say a

Consider an example : Find $\lim_{x\to 3} f(x)$, where $f(x) = \dfrac{x^2-9}{x-3}$

We can solve it by the method of substitution. Steps of which are as follows :

Remarks : It may be noted that f (3) is not defined, however, in this case the limit of the

You have already seen that $x \to a$ means x takes values which are very close to 'a', i.e. either the value is greater than 'a' or less than 'a'.

In case x takes only those values which are less than 'a' and very close to 'a' then we say x is approaches 'a' from the left and we write it as $x \to a^-$. Similarly, if x takes values which are greater than 'a' and very close to 'a' then we say x is approaching 'a' from the right and we write it as $x \to a^+$.

Thus, if a function f (x) approaches a limit l_1, as x approaches 'a' from left, we say that the left hand limit of f (x) as $x \to a$ is l_1.

We denote it by writing

$$\lim_{x \to a^-} f(x) = l_1 \quad \text{or} \quad \lim_{h \to 0} f(a-h) = l_1,\ h > 0$$

Similarly, if f (x) approaches the limit l_2, as x approaches 'a' from right we say, that the right hand limit of f (x) as $x \to a$ is l_2.

We denote it by writing

$$\lim_{x \to a^+} f(x) = l_2 \quad \text{or} \quad \lim_{h \to 0} f(a+h) = l_2,\ h > 0$$

Working Rules

Finding the right hand limit i.e.,

$$\lim_{x \to a^+} f(x)$$

Put $x = a + h$

Find $\lim_{h \to 0} f(a+h)$

Finding the left hand limit, i.e,

$$\lim_{x \to a^-} f(x)$$

Put $x = a - h$

Find $\lim_{h \to 0} f(a-h)$

Note : In both cases remember that h takes only positive values.

Limit of a function y=f(x) at x=a Consider an example :

Find $\lim_{x\to 1} f(x)$, where $f(x) = x^2 + 5x + 3$

Here $\lim_{x\to 1^+} f(x) = \lim_{h\to 0}\left[(1+h)^2 + 5(1+h) + 3\right]$

$= \lim_{h\to 0}\left[1 + 2h + h^2 + 5 + 5h + 3\right]$

$= 1 + 5 + 3 = 9$..........(i)

and $\lim_{x\to 1^-} f(x) = \lim_{h\to 0}\left[(1-h)^2 + 5(1-h) + 3\right]$

$= \lim_{x\to 0}\left[1 - 2h + h^2 + 5 - 5h + 3\right]$

$= 1 + 5 + 3 = 9$ .....(ii)

From (i) and (ii), $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$

Now consider another example :

Evaluate: $\lim_{x\to 3} \frac{|x-3|}{x-3}$

Here $\lim_{x\to 3^+} \frac{|x-3|}{x-3} = \lim_{h\to 0} \frac{|(3+h)-3|}{[(3+h)-3]}$

$= \lim_{h\to 0} \frac{|h|}{h} = \lim_{h\to 0} \frac{h}{h}$ (as h>0, so |h| = h)

$=1$..................(iii)

and $\lim_{x\to 3^-} \frac{|x-3|}{x-3} = \lim_{h\to 0} \frac{|(3-h)-3|}{[(3-h)-3]}$

$= \lim_{h\to 0} \frac{|-h|}{-h} = \lim_{h\to 0} \frac{h}{-h}$ (as h > 0, so |-h| = h)

$= -1$ .....(iv)

$\therefore$ From (iii) and (iv), $\lim_{x\to 3^+} \frac{|x-3|}{x-3} \neq \lim_{x\to 3^-} \frac{|x-3|}{x-3}$

Thus, in the first example right hand limit = left hand limit whereas in the second example right hand limit $\neq$ left hand limit.

Hence the left hand and the right hand limits may not always be equal.

Basic Theorem on Limit

1. $\lim\limits_{x\to a} cx = c \lim\limits_{x\to a} x$, c being a constant.

To verify this, consider the function $f(x) = 5x$.

We observe that in $\lim\limits_{x\to 2} 5x$, 5 being a constant is not affected by the limit.

$$\therefore \quad \lim_{x\to 2} 5x = 5 \lim_{x\to 2} x$$
$$= 5 \times 2 = 10$$

2. $$\lim_{x\to a}\left[g(x)+h(x)+p(x)+\ldots\ldots\right] = \lim_{x\to a} g(x) + \lim_{x\to a} h(x) + \lim_{x\to a} p(x) + \ldots\ldots$$

where $g(x), h(x), p(x), \ldots\ldots$ are any function.

3. $$\lim_{x\to a}\left[f(x)\cdot g(x)\right] = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$$

If $\lim\limits_{x\to a} f(x) = \ell$ and $\lim\limits_{x\to a} g(x) = m$, then

(i) $\lim\limits_{x\to a} kf(x) = k \lim\limits_{x\to a} f(x) = k\ell$ where k is a constant.

(ii) $\lim\limits_{x\to a}\left[f(x) \pm g(x)\right] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x) = \ell \pm m$

(iii) $\lim\limits_{x\to a}\left[f(x)\cdot g(x)\right] = \lim\limits_{x\to a} f(x) \cdot \lim\limits_{x\to a} g(x) = \ell \cdot m$

(iv) $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} = \dfrac{\ell}{m}$, provided $\lim\limits_{x\to a} g(x) \neq 0$

The above results can be easily extended in case of more than two functions.

E Find $\lim\limits_{x\to 1} f(x)$, where

$$f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x \neq 1 \\ 1, & x = 1 \end{cases}$$

Solution : $f(x) = \dfrac{x^2-1}{x-1} = \dfrac{(x-1)(x+1)}{x-1} = (x+1) \quad [\because x \neq 1]$

$$\therefore \quad \lim_{x\to 1} f(x) = \lim_{x\to 1}(x+1) = 1 + 1 = 2$$

Evaluate : $\lim\limits_{x \to 3} \dfrac{\sqrt{12-x}-x}{\sqrt{6+x}-3}$.

Solution : Rationalizing the numerator as well as the denominator, we get

$$\lim_{x \to 3} \frac{\sqrt{12-x}-x}{\sqrt{6+x}-3} = \lim_{x \to 3} \frac{\left(\sqrt{12-x}-x\right)\left(\sqrt{12-x}+x\right)\cdot\left(\sqrt{6+x}+3\right)}{\sqrt{6+x}-3\left(\sqrt{6+x}+3\right)\left(\sqrt{12-x}+x\right)}$$

$$= \lim_{x \to 3} \frac{\left(12-x-x^2\right)}{6+x-9} \cdot \lim_{x \to 3} \frac{\sqrt{6+x}+3}{\sqrt{12-x}+x}$$

$$= \lim_{x \to 3} \frac{-(x+4)(x-3)}{(x-3)} \cdot \lim_{x \to 3} \frac{\sqrt{6+x}+3}{\sqrt{12-x}+x} \quad [\because\ x \neq 3]$$

$$= -(3+4)\cdot\frac{6}{6} = -7$$

Note : Whenever in a function, the limits of both numerator and denominator are zero, you should simplify it in such a manner that the denominator of the resulting function is not zero.

However, if the limit of the denominator is 0 and the limit of the numerator is non zero, then the limit of the function does not exist.

Let us consider the example given below :

Find $\lim_{x \to 0} \frac{1}{x}$, if it exists.

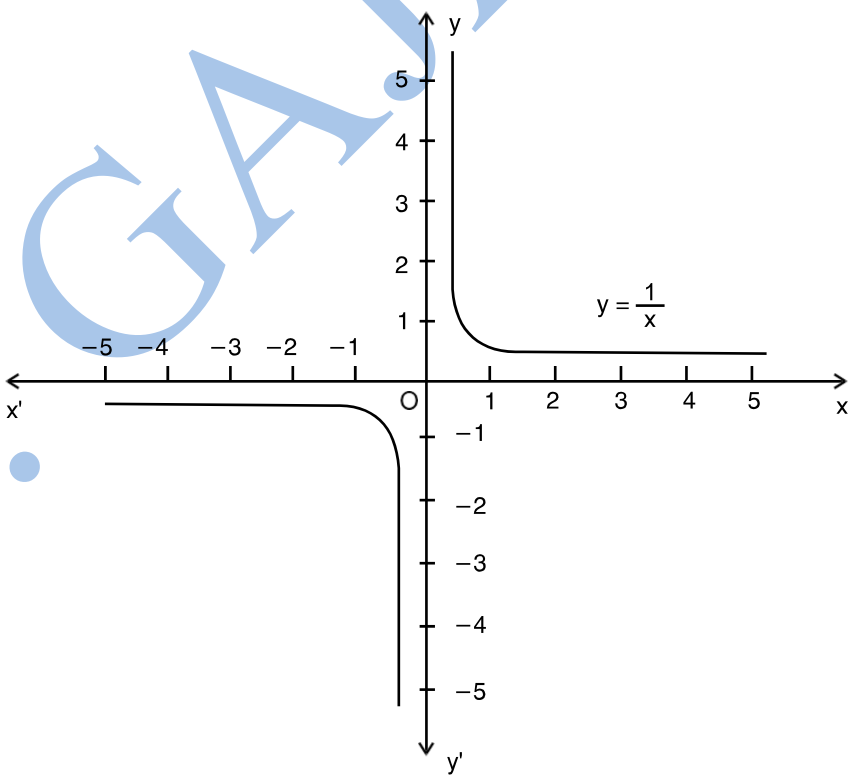
Solution : We choose values of x that approach 0 from both the sides and tabulate the correspondling values of $\frac{1}{x}$.

x	−0.1	−.01	−.001	−.0001
$\frac{1}{x}$	−10	−100	−1000	−10000

x	0.1	.01	.001	.0001
$\frac{1}{x}$	10	100	1000	10000

We see that as $x \to 0$, the corresponding values of $\frac{1}{x}$ are not getting close to any number.

Hence, $\lim_{x \to 0} \frac{1}{x}$ does not exist. This is illustrated by the graph in Fig. 20.2



Evaluate : $\lim_{x \to 0} (|x| + |-x|)$

Solution : Since |x| has different values for $x \geq 0$ and x<0, therefore we have to find out both left hand and right hand limits.

$$\lim_{x\to 0^-} (|x| + |-x|) = \lim_{h\to 0} (|0-h| + |-(0-h)|)$$

$$= \lim_{h\to 0} (|-h| + |-(-h)|)$$

$$= \lim_{h\to 0} h + h = \lim_{h\to 0} 2h = 0 \qquad ...(i)$$

and

$$\lim_{x\to 0^+} (|x| + |-x|) = \lim_{h\to 0} (|0+h| + |-(0+h)|)$$

$$= \lim_{x\to 0} h + h = \lim_{h\to 0} 2h = 0 \qquad ...(ii)$$

From (i) and (ii),

$$\lim_{x\to 0^-} (|x| + |-x|) = \lim_{h\to 0^+} [|x| + |-x|]$$

Thus,

$$\lim_{h\to 0} [|x| + |-x|] = 0$$

Note : We should remember that left hand and right hand limits are specially used when (a) the functions under consideration involve modulus function, and (b) function is defined by more than one rule.

Example-Find the vlaue of 'a' so that

$\lim_{x\to 1} f(x)$ exist, where $f(x) = \begin{cases} 3x+5 & , x \leq 1 \\ 2x+a, & x > 1 \end{cases}$

Solution :

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} (3x+5) \qquad [\because f(x) = 3x+5 \text{ for } x \leq 1]$$

$$= \lim_{h\to 0} [3(1-h)+5]$$

$$= 3 + 5 = 8 \ldots\ldots\ldots \text{(i)}$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} (2x+a) \qquad [\because f(x) = 2x+a \text{ for } x > 1]$$

$$= \lim_{h\to 0} (2(1+h)+a)$$

$$= 2 + a \ldots\ldots\ldots \text{(ii)}$$

We are given that $\lim_{x\to 1} f(x)$ will exists provided

(i) Prove that (a) $\lim_{x\to 0} \sin x = 0$ **and** **(b)** $\lim_{x\to 0} \cos x = 1$

Proof : Consider a unit circle with centre B, in which $\angle C$ is a right angle and $\angle ABC = x$ radians.

Now $\sin x = AC$ and $\cos x = BC$

As x decreases, A goes on coming nearer and nearer to C.

i.e., when $x \to 0, A \to C$

or when $x \to 0, AC \to 0$

and $BC \to AB$, i.e., $BC \to 1$

$\therefore$ When $x \to 0$ $\sin x \to 0$ and $\cos x \to 1$

Thus we have

$$\lim_{x\to 0} \sin x = 0 \quad \text{and} \quad \lim_{x\to 0} \cos x = 1$$

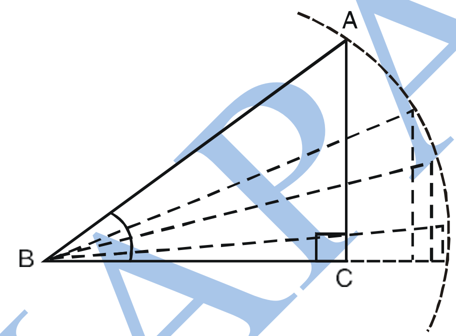


Fig. 25.3

(ii) Prove that $\lim_{x\to 0} \frac{\sin x}{x} = 1$

Proof : Draw a circle of radius 1 unit and with centre at the origin *O*. Let B (1,0) be a point on the circle. Let A be any other point on the circle. Draw $AC \perp OX$.

Let $\angle AOX = x$ radians, where $0<x<\frac{\pi}{2}$

Draw a tangent to the circle at B meeting OA produced at D. Then $BD \perp OX$.

Area of $\Delta AOC <$ area of sector $OBA <$ area of ΔOBD.

or $\frac{1}{2} OC \times AC < \frac{1}{2} x(1)^2 < \frac{1}{2} OB \times BD$

$\left[\because \text{ area of triangle} = \frac{1}{2} \text{base} \times \text{height and area of sector} = \frac{1}{2} \theta r^2\right]$

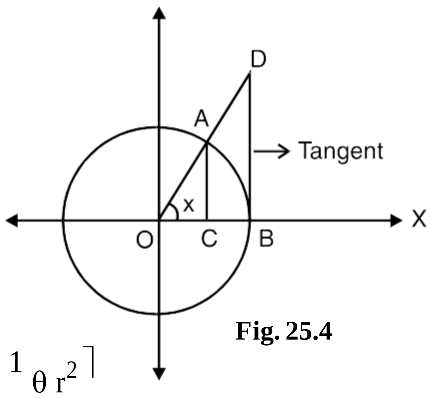


Fig. 25.4

$\therefore \frac{1}{2} \cos x \sin x < \frac{1}{2} x < \frac{1}{2} \cdot 1 \cdot \tan x$

$\left[\because \cos x = \frac{OC}{OA}, \sin x = \frac{AC}{OA} \text{ and } \tan x = \frac{BD}{OB}, OA = 1 = OB\right]$

i.e., $\cos x < \frac{x}{\sin x} < \frac{\tan x}{\sin x}$ [Dividing throughout by $\frac{1}{2}\sin x$]

or $$\cos x < \frac{x}{\sin x} < \frac{1}{\cos x}$$

or $$\frac{1}{\cos x} > \frac{\sin x}{x} < \cos x$$

i.e., $$\cos x < \frac{\sin x}{x} < \frac{1}{\cos x}$$

Taking limit as $x \to 0$, we get

$$\lim_{x\to 0} \cos x < \lim_{x\to 0} \frac{\sin x}{x} < \lim_{x\to 0} \frac{1}{\cos x}$$

or $$1 < \lim_{x\to 0} \frac{\sin x}{x} < 1 \quad \left[\because \lim_{x\to 0} \cos x = 1 \text{ and } \lim_{x\to 0} \frac{1}{\cos x} = \frac{1}{1} = 1\right]$$

Thus, $$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Note : In the above results, it should be kept in mind that the angle x must be expressed in radians.

$$\mathbf{\lim_{x\to 0} (1 + x)^{\frac{1}{x}} = e}$$

$$\mathbf{\lim_{x\to 0} \frac{\log(1+x)}{x} = \lim_{x\to 0} \frac{1}{x} \log(1+x) = \lim_{x\to 0} \log(1+x)^{1/x}}$$

Evaluate : $\lim_{x\to 0} \frac{\sin 3x}{x}$.

Solution : $$\lim_{x\to 0} \frac{\sin 3x}{x} = \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot 3 \qquad \text{[Multiplying and dividing by 3]}$$

$$= 3 \lim_{3x\to 0} \frac{\sin 3x}{3x} \qquad [\because \text{ when } x \to 0, 3x \to 0]$$

$$= 3.1 \quad \left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= 3$$

Thus, $$\lim_{x\to 0} \frac{\sin 3x}{x} = 3$$

CONTINUITY OF A FUNCTION AT A POINT

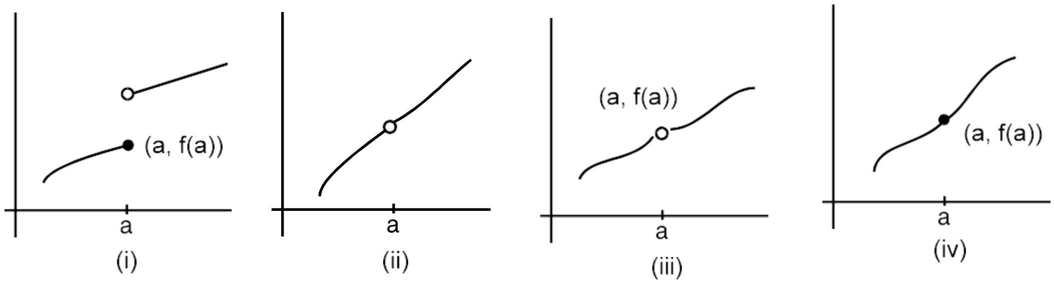


Fig. 25.5

Let us observe the above graphs of a function.

We can draw the graph (iv) without lifting the pencil but in case of graphs (i), (ii) and (iii), the pencil has to be lifted to draw the whole graph.

In case of (iv), we say that the function is continuous at $x = a$. In other three cases, the function is not continuous at $x = a$. i.e., they are discontinuous at $x = a$.

In case (i), the limit of the function does not exist at $x = a$.

In case (ii), the limit exists but the function is not defined at $x = a$.

In case (iii), the limit exists, but is not equal to value of the function at $x = a$.

In case (iv), the limit exists and is equal to value of the function at $x = a$.

Ex-Examine the continuity of the function $f(x) = x - a$ at $x = a$.

Solution : $\lim_{x\to a} f(x) = \lim_{h\to 0} f(a+h)$

$$= \lim_{h\to 0}[(a+h)-a]$$

$= 0$ ....(i)

Also $f(a) = a - a = 0$ .....(ii)

From (i) and (ii),

$$\lim_{x\to a} f(x) = f(a)$$

Thus $f(x)$ is continuous at $x = a$.

Ex-Show that $f(x) = c$ is continuous.

Solution : The domain of constant function c is R.Let 'a' be any arbitrary real number.

$\therefore$ $\lim_{x\to a} f(x) = c$ and $f(a) = c$

$\therefore$ $\lim_{x\to a} f(x) = f(a)$

$\therefore$ $f(x)$ is continuous at $x = a$. But 'a' is arbitrary. Hence $f(x) = c$ is a constant function.

.

Consider the function $f(x) = \frac{7}{2}x$. We know that x is a constant function. Let 'a' be an arbitrary real number.

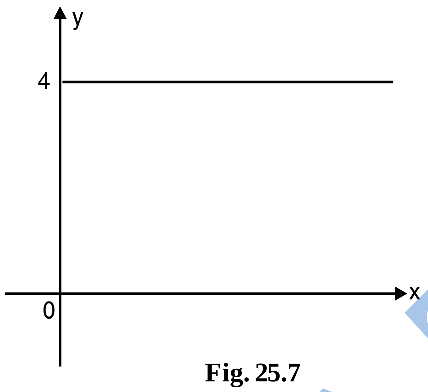


Fig. 25.7

$$\lim_{x \to a} f(x) = \lim_{h \to 0} f(a+h)$$

$$= \lim_{h \to 0} \frac{7}{2}(a+h)$$

$$= \frac{7}{2}a \qquad .....(i)$$

Also $\qquad f(a) = \frac{7}{2}a \qquad .....(ii)$

$\therefore$ From (i) and (ii),

$$\lim_{x \to a} f(x) = f(a)$$

$\therefore$ $f(x) = \frac{7}{2}x$ is continuous at $x = a$.

As $\frac{7}{2}$ is constant, and x is continuous function at $x = a$, $\frac{7}{2}x$ is also a continuous function at $x = a$.

(i) Consider the function $f(x) = x^2 + 2x$. We know that the function x^2 and $2x$ are continuous.

Now $\qquad \lim_{x \to a} f(x) = \lim_{h \to 0} f(a+h)$

$$= \lim_{h \to 0} \left[(a+h)^2 + 2(a+h) \right]$$

$$= \lim_{h \to 0} \left[a^2 + 2ah + h^2 + 2a + 2ah \right]$$

$$= a^2 + 2a \qquad .....(i)$$

Also $\qquad f(a) = a^2 + 2a \qquad .....(ii)$

$\therefore$ From (i) and (ii), $\lim_{x \to a} f(x) = f(a)$

$\therefore$ f(x) is continuous at $x = a$.

Thus we can say that if x^2 and $2x$ are two continuous functions at $x = a$ then $(x^2 + 2x)$ is also continuous at $x = a$.

(ii) Consider the function $f(x) = (x^2 + 1)(x + 2)$. We know that $(x^2 + 1)$ and $(x + 2)$ are two continuous functions.

Also $\qquad f(x) = (x^2 + 1)(x + 2)$

$$= x^3 + 2x^2 + x + 2$$

As $x^3, 2x^2, x$ and 2 are continuous functions, therefore.

$x^3 + 2x^2 + x + 2$ is also a continuous function.

$\therefore$ We can say that if $\left(x^2+1\right)$ and (x+2) are two continuous functions then $\left(x^2+1\right)$ (x + 2) is also a continuous function.

(iii) Consider the function $f(x) = \dfrac{x^2-4}{x+2}$ at x = 2. We know that $\left(x^2-4\right)$ is continuous at x = 2. Also (x + 2) is continuous at x = 2.

Again
$$\lim_{x\to 2}\frac{x^2-4}{x+2} = \lim_{x\to 2}\frac{(x+2)(x-2)}{x+2}$$
$$= \lim_{x\to 2}(x-2)$$
$$= 2-2=0$$

Also
$$f(2) = \frac{(2)^2-4}{2+2}$$
$$= \frac{0}{4} = 0$$

$\therefore$ $\lim_{x\to 2} f(x) = f(2)$. Thus f (x) is continuous at x = 2.

$\therefore$ If $x^2 - 4$ and $x+2$ are two continuous functions at $x=2$, then $\dfrac{x^2-4}{x+2}$ is also continuous.

(iv) Consider the function $f(x) = |x-2|$. The function can be written as

$$f(x) = \begin{cases} -(x-2), & x < 2 \\ (x-2), & x \geq 2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h),\ h>0$$
$$= \lim_{h\to 0}\left[(2-h)-2\right]$$
$$= 2-2=0$$
$$\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h),\ h>0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$
$$= \lim_{x\to 2}\left[(2+h)-2\right]$$
$$= 2-2=0 \qquad \ldots\ldots\text{(ii)}$$

Also $f(2) = (2-2) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$

$\therefore$ From (i), (ii) and (iii), $\lim_{x\to 2} f(x) = f(2)$

Thus, $|x-2|$ is continuous at x = 2.

Note

(i) $\lim\limits_{x\to a} kf(x) = k \lim\limits_{x\to a} f(x) = k\ell$

(ii) $\lim\limits_{x\to a} [f(x) \pm g(x)] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x) = \ell \pm m$

(i) $\lim\limits_{x\to a} [f(x)g(x)] = \lim\limits_{x\to a} f(x) \lim\limits_{x\to a} g(x) = \ell m$

(ii) $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} = \frac{\ell}{m}$, provided $\lim\limits_{x\to a} g(x) \neq 0$

• LIMIT OF IMPORTANT FUNCTIONS

(i) $\lim\limits_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

(ii) $\lim\limits_{x\to 0} \sin x = 0$

(iii) $\lim\limits_{x\to 0} \cos x = 1$

(iv) $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

(v) $\lim\limits_{x\to 0} (1+x)^{\frac{1}{x}} = e$

(vi) $\lim\limits_{x\to 0} \frac{\log(1+x)}{x} = 1$

(vii) $\lim\limits_{x\to 0} \frac{e^x - 1}{x} = 1$