LECTURE NOTES

ON

FLUID MECHANICS(Th-3)

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CONTENTS:

COURSE OUTCOME:

At the end of the course students will be able to:

CO	Statement
C223.1	Identify various fluid properties & relationship between them.
C223.2	Select best method for estimation of fluid pressure measurement & flow measurement.
C223.3	Derive Continuity equation, Bernoulli's theorem & apply them in fluid flow problem.
C223.4	Calculate major & minor head losses for viscous flow through pipes.
C223.5	Derive work done, efficiencies on different types of vanes when fluid is impacted to vanes.

UNIT I

PROPERTIES OF FLUIDS AND FLUID STATICS

Introduction to Fluid Mechanics

Definition of a fluid

A fluid is defined as a substance that deforms continuously under the action of a shear stress, however small magnitude present. It means that a fluid deforms under very small shear stress, but a solid may not deform under that magnitude of the shear stress.

Fig.L-1.1a: Deformation of solid under a constant shear force

By contrast a solid deforms when a constant shear stress is applied, but its deformation does not continue with increasing time. In Fig.L1.1, deformation pattern of a solid and a fluid under the action of constant shear force is illustrated. We explain in detail here deformation behaviour of a solid and a fluid under the action of a shear force.

In Fig.L1.1, a shear force ***F*** is applied to the upper plate to which the solid has been bonded, a shear stress resulted by the force equals to $\tau = \frac{F}{A}$, where *A* is the contact area of the upper plate. We know that in the case of the solid block the deformation is proportional to the shear stress *t* provided the elastic limit of the solid material is not exceeded.

When a fluid is placed between the plates, the deformation of the fluid element is illustrated in Fig.L1.3. We can observe the fact that the deformation of the fluid element continues to increase as long as the force is applied. The fluid particles in direct contact with the plates move with the same speed of the plates. This can be interpreted that there is no slip at the boundary. This fluid behavior has been verified in numerous experiments with various kinds of fluid and boundary material.

In short, ***a fluid continues in motion under the application of a shear stress and can not sustain any shear stress when at rest.***

Fluid as a continuum

In the definition of the fluid the molecular structure of the fluid was not mentioned. As we know the fluids are composed of molecules in constant motions. For a liquid, molecules are closely spaced compared with that of a gas. In most engineering applications the average or macroscopic effects of a large number of molecules is considered. We thus do not concern about the behavior of individual molecules. The fluid is treated as an infinitely divisible substance, a continuum at which the properties of the fluid are considered as a continuous (smooth) function of the space variables and time.

To illustrate the concept of fluid as a continuum consider fluid density as a fluid property at a small region. Density is defined as mass of the fluid molecules per unit volume. Thus the mean density within the small region **C** could be equal to mass of fluid molecules per unit volume. When the small region **C** occupies space which is larger than the cube of molecular spacing, the number of the molecules will remain constant. This is the limiting volume $\delta v'$ above which the effect of molecular variations on fluid properties is negligible.

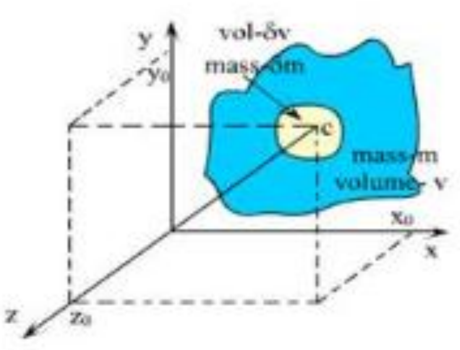


Fig. L-1.2(a):Small region in fluid domain

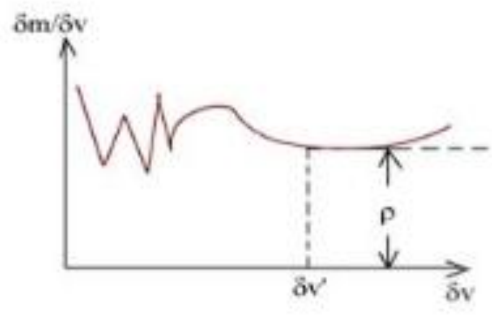


Fig. L-1.2(b): Variation of density with respect to volume of the region

The density of the fluid is defined as

$$\rho = \lim_{\delta v \to \delta v'} \frac{\delta m}{\delta v}$$

Note that the limiting volume $\delta v'$ is about $10^{-9} mm^3$ for all liquids and for gases at atmospheric temperature. Within the given limiting value, air at the standard condition has approximately 3×10^7 molecules. It justifies in defining a nearly constant density in a region which is larger than the limiting volume.

In conclusion, since most of the engineering problems deal with fluids at a dimension which is larger than the limiting volume, the assumption of fluid as a continuum is valid. For example the fluid density is defined as a function of space (for Cartesian coordinate system, x, y, and z) and time (t) by $\rho = \rho(x, y, z, t)$. This simplification helps to use the differential calculus for solving fluid problems.

Properties of fluid

Some of the basic properties of fluids are discussed below-

Density : As we stated earlier the density of a substance is its mass per unit volume. In fluid mechanic it is expressed in three different ways-

Mass density r is the mass of the fluid per unit volume (given by Eq.L1.1)

Unit- kg/m^3
Dimension- ML^{-3}
Typical values: water- *1000* *kg/* m^3
Air- $1.23\ kg/m^3$ at standard pressure and temperature (STP)

Specific weight, w: - As we express a mass *M* has a weight ***W=Mg*** . The specific weight of the fluid can be defined similarly as its weight per unit volume.

$$w = \rho g \qquad \text{L-2.1}$$

Unit: N/m^3
Dimension: $ML^{-2}T^{-2}$

Typical values; water- $9.810 N/m^3$
Air- $12.07 N/m^3$ (STP)

Relative density (Specific gravity), ***S*** :-

Specific gravity is the ratio of fluid density (specific weight) to the fluid density (specific weight) of a standard reference fluid. For liquids water at $4^0 C$ is considered as standard fluid.

$$S_{liquid} = \frac{\rho_{liquid}}{\rho_{water\ at\ 4^0C}} \qquad \text{L-2.2}$$

Similarly for gases air at specific temperature and pressure is considered as a standard reference fluid.

$$S_{gas} = \frac{\rho_{gas}}{\rho_{gas\ at\ STP}} \qquad \text{L-2.3}$$

Units: pure number having no units.

Dimension:- $M^oL^oT^o$

Typical vales : - Mercury- 13.6

Water-1

Specific volume v_s : - Specific volume of a fluid is mean volume per unit mass *i.e.* the reciprocal of mass density.

$$v_s = \frac{1}{\rho} \qquad \text{L-2.4}$$

Units:- m^3/kg

Dimension: $M^{-1}L^3$

Typical values: - Water - $10^{-3} m^3/kg$

Air- $1.23 \times 10^{-3} m^3/kg$

Viscosity

In section L1 definition of a fluid says that under the action of a shear stress a fluid continuously deforms, and the shear strain results with time due to the deformation. Viscosity is a fluid property, which determines the relationship between the fluid strain rate and the applied shear stress. It can be noted that in fluid flows, shear strain rate is considered, not shear strain as commonly used in solid mechanics. Viscosity can be inferred as a quantative measure of a fluid's resistance to the flow. For example moving an object through air requires very less force compared to water. This means that air has low viscosity than water.

Let us consider a fluid element placed between two infinite plates as shown in fig (Fig-2.1). The upper plate moves at a constant velocity δu under the action of constant shear force δF. The shear stress, *t* is expressed as

$$\tau = \lim_{\delta A \to 0} \frac{\delta F}{\delta A} = \frac{dF}{dA}$$

where, δA is the area of contact of the fluid element with the top plate. Under the action of shear force the fluid element is deformed from position *ABCD* at time *t* to position AB'C'D' at time $t + \delta t$ (fig-L2.1). The shear strain rate is given by

Shear strain rate $= \lim_{\delta t \to 0} \frac{\delta \alpha}{\delta t} = \frac{d\alpha}{dt}$ L2.6

Where α is the angular deformation

From the geometry of the figure, we can define

For small $\delta\alpha$, $\tan \delta\alpha = \frac{\delta u \ \delta t}{\delta y}$

Therefore,

$$\frac{\delta\alpha}{\delta t} = \frac{\delta u}{\delta y}$$

The limit of both side of the equality gives $\frac{d\alpha}{dt} = \frac{du}{dy}$ L-2.5

The above expression relates shear strain rate to velocity gradient along the *y* -axis.

Newton's Viscosity Law

Sir Isaac Newton conducted many experimental studies on various fluids to determine relationship between shear stress and the shear strain rate. The experimental finding showed that a linear relation between them is applicable for common fluids such as water, oil, and air. The relation is

$$\tau \ \alpha \ \frac{d\alpha}{dt}$$

Substituting the relation gives in equation(L-2.5)

$$\tau \, \alpha \, \frac{du}{dy} \qquad \text{L-2.6}$$

Introducing the constant of proportionality

$$\tau = \mu \frac{du}{dy}$$

where μ is called absolute or dynamic viscosity. Dimensions and units for μ are $ML^{-1}T^{-1}$ and $N-s/m^2$, respectively. [In the absolute metric system basic unit of co-efficient of viscosity is called poise. 1 poise = $N-s/m^2$]

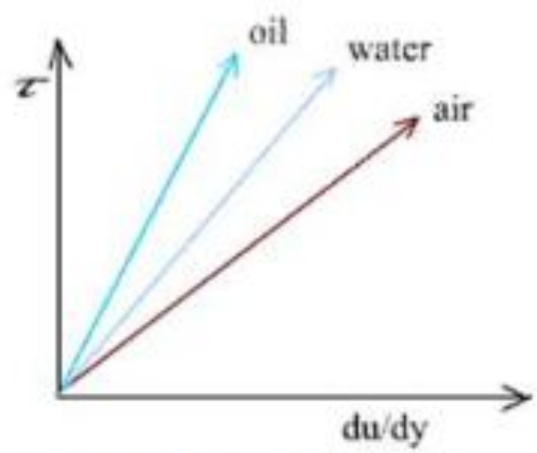


Fig.L-2.2: Relationship between shear stress and velocity gradient of Newtonian fluids

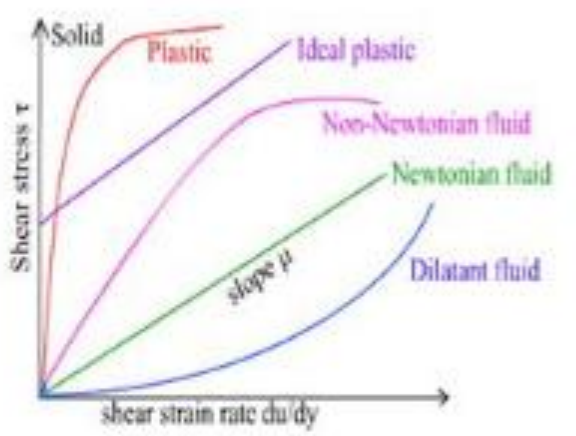


Fig.L-2.3: Relationship between shear stress and shear strain rate of diferent fluids

Typical relationships for common fluids are illustrated in Fig-L2.3.

The fluids that follow the linear relationship given in equation (L-2.7) are called Newtonian fluids.

Kinematic viscosity v

Kinematic viscosity is defined as the ratio of dynamic viscosity to mass density

$$\nu = \frac{\mu}{\rho} \qquad \text{L-2.8}$$

Units: m^2/s

Dimension: L^2T^{-1}

Typical values: water $1.14\times10^{-6}m^2s^{-1}$ air $1.46\times10^{-5}m^2/s$

Non - Newtonian fluids

Fluids in which shear stress is not linearly related to the rate of shear strain are non– Newtonian fluids. Examples are paints, blot, polymeric solution, etc. Instead of the dynamic viscosity apparent viscosity, μ_{ap} which is the slope of shear stress versus shear strain rate curve, is used for these types of fluid.

Based on the behavior of μ_{ap}, non-Newtonian fluids are broadly classified into the following groups –

a. *Pseudo plastics* (shear thinning fluids): μ_{ap} decreases with increasing shear strain rate. For example polymer solutions, colloidal suspensions, latex paints, pseudo plastic.
b. *Dilatants* (shear thickening fluids) μ_{ap} increases with increasing shear strain rate.

   Examples: Suspension of starch and quick sand (mixture of water and sand).

c. *Plastics* : Fluids that can sustain finite shear stress without any deformation, but once shear stress exceeds the finite stress τ_y, they flow like a fluid. The relation between the shear stress and the resulting shear strain is given by

$$\tau = \tau_y + \mu_{ap}\left(\frac{du}{dy}\right)^n \qquad \text{L-2.9}$$

Fluids with n = 1 are called Bingham plastic. some examples are clay suspensions, tooth paste and fly ash.

d. *Thixotropic* fluid(Fig. L-2.4): μ_{ap} decreases with time under a constant applied shear stress.

Example: Ink, crude oils.

e. *Rheopectic fluid* : μ_{ap} increases with increasing time.

Example: some typical liquid-solid suspensions.

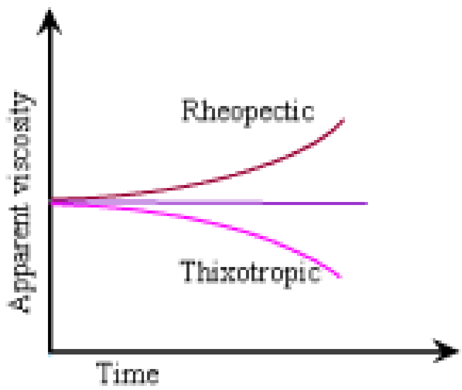
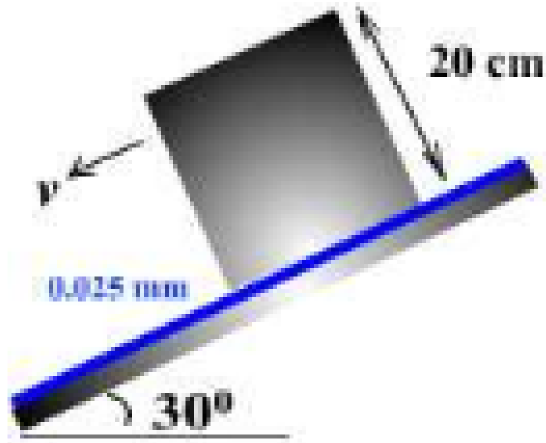


Fig. L-2.4: Thixotropic and Rheopectic fluids

Example

As shown in the figure a cubical block of 20 cm side and of 20 kg weight is allowed to slide down along a plane inclined at 30^0 to the horizontal on which there is a film of oil having viscosity $2.16x10^{-3}$ N-s/m^2 .What will be the terminal velocity of the block if the film thickness is 0.025mm?



Given data : Weight = 20 kg

Block dimension = 20x20x20 cm^3

Driving force along the plane $F = W \sin 30^0 = 98.1N$

Shear force $\tau = F / A = 2452.5 N / m^2$

Contact area, $A = 0.2 \times 0.2 m^2$

Also, $\tau = \mu \frac{dv}{dy}$

Answer: 28.38m/s.

Example

If the equation of a velocity profile over a plate is *v = 5y 2 + y* (where *v* is the velocity in m/s) determine the shear stress at *y =0* and at *y =7.5cm* . Given the viscosity of the liquid is 8.35 poise.

Solution

Given Data: Velocity profile $v = 5y^2 + y$

$\mu = 8.35 poise$

$$\text{Velocity gradient, } \frac{dv}{dy} = 10y + 1$$

$$\tau = \mu . \frac{dv}{dy} = \mu .(10y + 1)$$

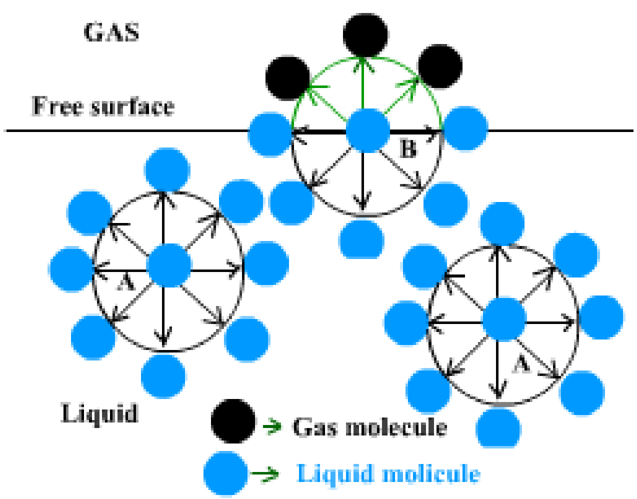
Substituting $y = 0$ *and* $y = 0.075$ on the above equation, we get shear stress at respective depths.

Answer: 0.835 ; $1.46\ N/m^2$

Surface tension and Capillarity

Surface tension

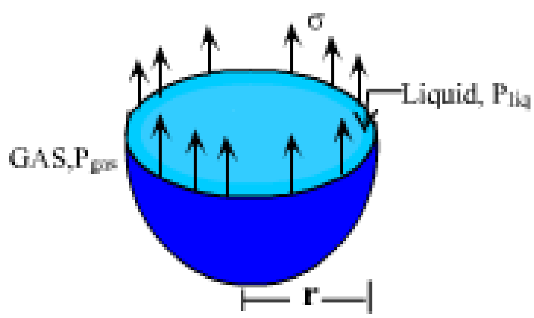
In this section we will discuss about a fluid property which occurs at the interfaces of a liquid and gas or at the interface of two immiscible liquids. As shown in Fig (L - 3.1) the liquid molecules- 'A' is under the action of molecular attraction between like molecules (cohesion). However the molecule ‘B' close to the interface is subject to molecular attractions between both like and unlike molecules (adhesion). As a result the cohesive forces cancel for liquid molecule 'A'. But at the interface of molecule 'B' the cohesive forces exceed the adhesive force of the gas. The corresponding net force acts on the interface; the interface is at a state of tension similar to a stretched elastic membrane. As explained, the corresponding net force is referred to as surface tension, δ. In short it is apparent tensile stresses which acts at the interface of two immiscible fluids.



Dimension: MT^{-2}
Unit: N/m
Typical values: Water $0.074\,N/m$ at 20° C with air.

Note that surface tension decreases with the liquid temperature because intermolecular cohesive forces decreases. At the critical temperature of a fluid surface tension becomes zero; i.e. the boundary between the fluids vanishes.

Pressure difference at the interface



Surface tension on a droplet

In order to study the effect of surface tension on the pressure difference across a curved interface, consider a small spherical droplet of a fluid at rest.

Since the droplet is small the hydrostatic pressure variations become negligible. The droplet is divided into two halves as shown in Fig.L-3.2. Since the droplet is at rest, the sum of the forces acting at the interface in any direction will be zero. Note that the only forces acting at the interface are pressure and surface tension. Equilibrium of forces gives

$$\left(P_{\text{liq}} - P_{\text{gas}}\right)\pi r^2 = \sigma\left(2\pi r\right) \qquad \text{L - 3.1}$$

Solving for the pressure difference and then denoting $\Delta P = P_{liq} - P_{gas}$ we can rewrite equation (L- 3.1) as

$$\Delta P = \frac{2\sigma}{r}$$

Contact angle and welting

As shown in fig. a liquid contacts a solid surface. The line at which liquid gas and solid meet is called the contact line. At the contact line the net surface tension depending upon all three materials - liquid, gas, and solid is evident in the contact angle, θ_c. A force balance on the contact line yields:

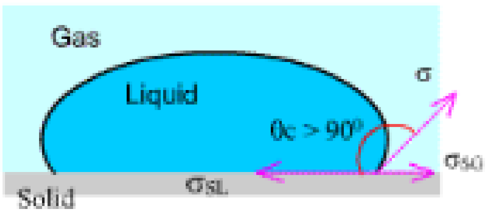


Fig : L-3.3: Contact line for wetting condition

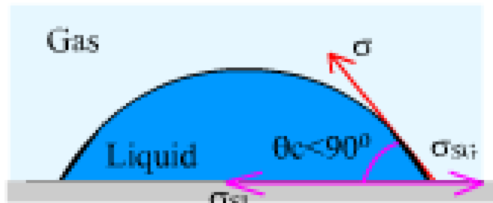


Fig : L-3.4: Contact line for non-wetting condition

$$\sigma_{gas} - \sigma_{solid} = \sigma \cos \theta_c$$

here σ_{gas} is the surface tension of the gas-solid interface, σ_{solid} is the surface tension of solid-liquid interface, and σ is the surface tension of liquid-gas interface.

Typical values:

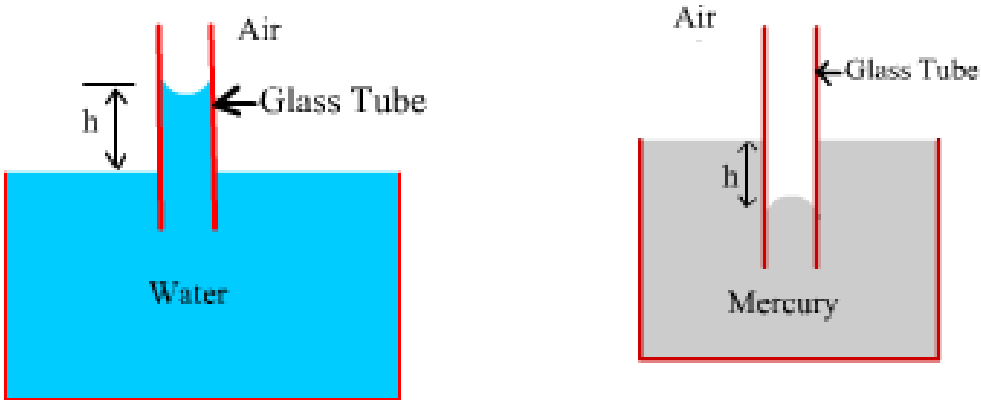
$\theta_c \approx 0^\circ$ for air-water- glass interface

$\theta_c \approx 140^\circ$ for air-mercury–glass interface

If the contact angle $\theta_c < 90^\circ$ the liquid is said to wet the solid. Otherwise, the solid surface is not wetted by the liquid, when $\theta_c > 90^\circ$.

Capillarity

If a thin tube, open at the both ends, is inserted vertically in to a liquid, which wets the tube, the liquid will rise in the tube (fig : L -3.4). If the liquid does not wet the tube it will be depressed below the level of free surface outside. Such a phenomenon of rise or fall of the liquid surface relative to the adjacent level of the fluid is called capillarity. If θ_c is the angle of contact between liquid and solid, *d* is the tube diameter, we can determine the capillary rise or depression, *h* by equating force balance in the z-direction (shown in Fig : L-3.5), taking into account surface tension, gravity and pressure. Since the column of fluid is at rest, the sum of all of forces acting on the fluid column is zero.



The pressure acting on the top curved interface in the tube is atmospheric, the pressure acting on the bottom of the liquid column is at atmospheric pressure because the lines of constant pressure in a liquid at rest are horizontal and the tube is open.

Upward force due to surface tension $= \sigma \cos\theta_c \pi d$

Weight of the liquid column $= \rho g . \pi \dfrac{d^2}{4} h$

Thus equating these two forces we find

$$\sigma \cos\theta_c \pi d = \rho g \pi \frac{d^2}{4} h$$

The expression for *h* becomes

$$h = \frac{4\sigma \cos\theta_c}{\rho g d} \qquad \text{L -3.2}$$

Typical values of capillary rise are

a. Capillary rise is approximately 4.5 mm for water in a glass tube of 5 mm diameter.
b. Capillary depression is approximately - 1.5 mm (depression) for mercury in the same tube.
c. Capillary action causes a serious source of error in reading the levels of the liquid in small pressure measuring tubes. Therefore the diameter of the measuring tubes should be large enough so that errors due to the capillary rise should be very less. Besides this,

capillary action causes the movement of liquids to penetrate cracks even when there is no significant pressure difference acting to move the fluids in to the cracks.

d. In figure (Fig : L - 3.6), a two-dimensional model for the capillary rise of a liquid in a crack width, *b*, is illustrated. The height of the capillary rise can also be computed by equating force balance as explained in the previous section.

Capillary rise, $$h = \frac{2\sigma \cos\theta_c}{b\rho g}$$ L-3.3

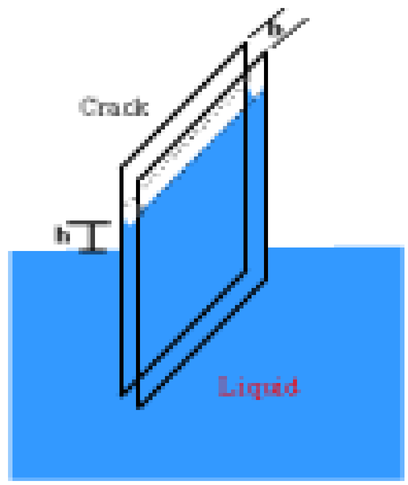


Fig. L-3.6: Capillary rise in a Crack

Vapour Pressure

Since the molecules of a liquid are in constant motion, some of the molecules in the surface layer having sufficient energy will escape from the liquid surface, and then changes from liquid state to gas state. If the space above the liquid is confined and the number of the molecules of the liquid striking the liquid surface and condensing is equal to the number of liquid molecules at any time interval becomes equal, an equilibrium exists. These molecules exerts of partial pressure on the liquid surface known as vapour pressure of the liquid, because degree of molecular activity increases with increasing temperature. The vapour pressure increases with temperature. Boiling occurs when the pressure above a liquid becomes equal to or less then the vapour pressure of the liquid. It means that boiling of water may occur at room temperature if the pressure is reduced sufficiently.

For example water will boil at 60 ° C temperature if the pressure is reduced to 0.2 atm.

Cavitation

In many fluid problems, areas of low pressure can occur locally. If the pressure in such areas is equal to or less then the vapour pressure, the liquid evaporates and forms a cloud of vapour bubbles. This phenomenon is called cavitation. This cloud of vapour bubbles is swept in to an area of high pressure zone by the flowing liquid. Under the high pressure the bubbles collapses. If this phenomenon occurs in contact with a solid surface, the high pressure developed by collapsing bubbles can erode the material from the solid surface and small cavities may be formed on the surface.

The cavitation affects the performance of hydraulic machines such as pumps, turbines and propellers.

Compressibility and the bulk modulus of elasticity

When a fluid is subjected to a pressure increase the volume of the fluid decreases. The relationship between the change of pressure and volume is linear for many fluids. This relationship may be defined by a proportionality constant called bulk modulus.

Consider a fluid occupying a volume *V* in the piston and cylinder arrangement shown in figure. If the pressure on the fluid increase from *p* to $p+\delta p$ due to the piston movement as a result the volume is decreased by ΔV. We can express the bulk modulus of elasticity

$$k = -\frac{\delta p}{\delta v / v} \qquad \text{L - 4.1}$$

The negative sign indicates the volume decreases as pressure increases. As in the limit as $\delta p \rightarrow 0$ then

$$k = -\frac{dp}{dv / v} \qquad \text{L - 4.2}$$

Since $-\frac{dv}{v} = \frac{dp}{p}$ the equation can be rearranged as

$$k = \frac{dp}{d\rho / \rho} \qquad \text{L - 4.3}$$

Dimension :- $ML^{-1}T^{-2}$

Unit :- N/m^2

Typical values:-

Air - 1.03 x 10 5 N/m^2

water $2.05 \times 10^9\ N/m^2$ at standard temperature and pressure as compared to that of Mild steel $2.06 \times 10^{11}\ N/m^2$.

The above typical values show that the air is about 20,000 times more compressible than water while water is about 100 times more compressible than mild steel.

Basic Equations

To analysis of any fluid problem, the knowledge of the basic laws governing the fluid flows is required. The basic laws, applicable to any fluid flow, are:

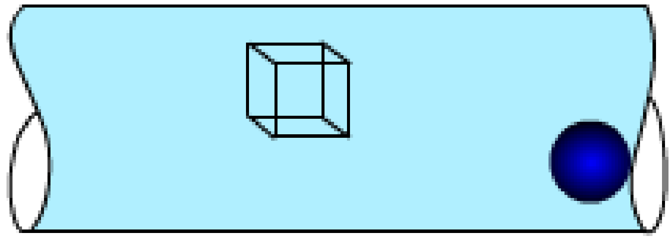
a. Conservation of mass. (Continuity)
b. Linear momentum. (Newton 's second law of motion)
c. Conservation of energy (First law of Thermodynamics)

Besides these governing equations, we need the state relations like $\rho = \rho(P,T)$ and appropriate boundary conditions at solid surface, interfaces, inlets and exits. Note that all basic laws are not always required to any one problem. These basic laws, as similar in solid mechanics and thermodynamics, are to be reformulated in suitable forms so that they can be easily applied to solve wide variety of fluid problems.

System and control volume

A system refers to a fixed, identifiable quantity of mass which is separated from its surrounding by its boundaries. The boundary surface may vary with time however no mass crosses the system boundary. In fluid mechanics an infinitesimal lump of fluid is considered as a system and is referred as a fluid element or a particle. Since a fluid particle has larger dimension than the limiting volume (refer to section fluid as a continuum). The continuum concept for the flow analysis is valid.

control volume is a fixed, identifiable region in space through which fluid flows. The boundary of the control volume is called control surface. The fluid mass in a control volume may vary with time. The shape and size of the control volume may be arbitrary.



System and control volume

When a fluid is at rest, the fluid exerts a force normal to a solid boundary or any imaginary plane drawn through the fluid. Since the force may vary within the region of interest, we conveniently define the force in terms of the pressure, P, of the fluid. The pressure is defined as the *force per unit area.*

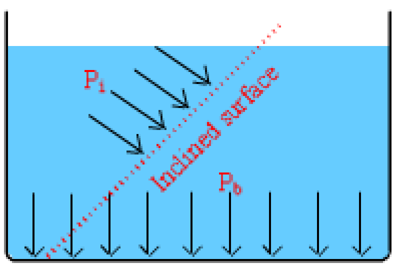


Fig : L - 6.1: Pressure variation at the bottom surface P_b and at the inclined surface P_i

In Fig : L - 6.1 pressure variation of a fluid at different locations is illustrated.

Commonly the pressure changes from point to point. We can define the pressure at a point as

$$P = \lim_{\delta A \to 0} \frac{\delta F}{\delta A} = \frac{dF}{dA} \qquad \text{L - 6.1}$$

where dA is the area on which the force dF acts. It is a scalar field and varies spatially and temporally as given *P = P (x, y, z, t)*

Pascal's Law : Pressure at a point

The Pascal's law states that *the pressure at a point in a fluid at rest is the same in all directions* . Let us prove this law by considering the equilibrium of a small fluid element shown in Fig : L - 6.2

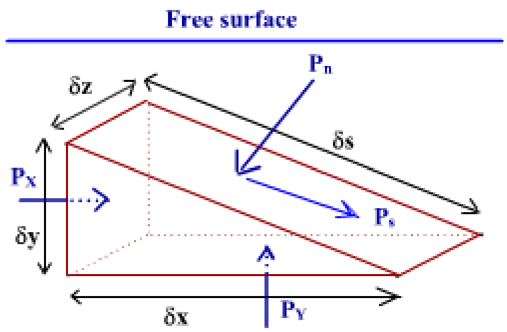


Fig : L -6.2: A fluid element with force components

Since the fluid is at rest, there will be no shearing stress on the faces of the element.

The equilibrium of the fluid element implies that sum of the forces in any direction must be zero. For the x-direction:

Force due to P_x is $P_x \cdot \delta y \cdot \delta z$

Component of force due to P_n

$$= -P_n \cdot \delta n \cdot \delta z \cdot \frac{\delta y}{\delta n}$$
$$= -P_n \cdot \delta y \cdot \delta z$$

Summing the forces we get,

$$P_x \cdot \delta y \cdot \delta z - P_n \cdot \delta y \cdot \delta z = 0$$
$$\text{then } P_x = P_n$$

Similarly in the y-direction, we can equate the forces as given below

Force due to P_y = $P_y \cdot \delta x \cdot \delta z$

Component of force due to P_n

$$= -P_n \cdot \delta n \cdot \delta z \cdot \frac{\delta x}{\delta n}$$

$$= -P_n \cdot \delta x \cdot \delta z$$

$$\text{Weight of the fluid element} = -\text{Specific weight} \times \text{volume of the element}$$

$$= -\rho \cdot g \cdot \frac{1}{2} \cdot \delta x \cdot \delta y \cdot \delta z$$

The negative sign indicates that weight of the fluid element acts in opposite direction of the z-direction.

Summing the forces yields

$$P_y \cdot \delta n \cdot \delta z - P_n \cdot \delta x \cdot \delta z - \frac{1}{2} \cdot \rho \cdot g \cdot \delta x \cdot \delta y \cdot \delta z = 0$$

Since the volume of the fluids $\delta x \cdot \delta y \cdot \delta z$ is very small, the weight of the element is negligible in comparison with other force terms. So the above Equation becomes

$P_y = P_n$

Hence, $P_n = P_x = P_y$

Similar relation can be derived for the z-axis direction.

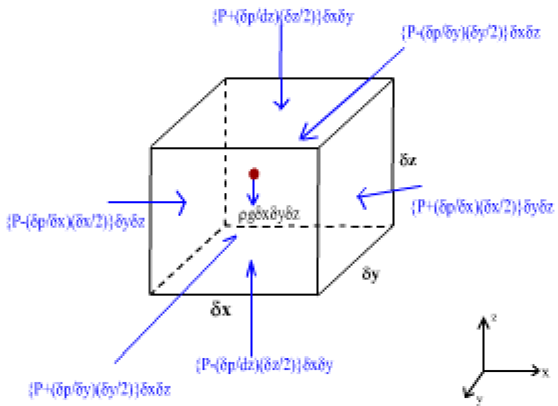
This law is valid for the cases of fluid flow where shear stresses do not exist. The cases are

a. Fluid at rest.
b. No relative motion exists between different fluid layers. For example, fluid at a constant linear acceleration in a container.
c. Ideal fluid flow where viscous force is negligible.

Basic equations of fluid statics

An equation representing pressure field $P = P(x, y, z)$ within fluid at rest is derived in this section. Since the fluid is at rest, we can define the pressure field in terms of space dimensions (x, y and z) only.

Consider a fluid element of rectangular parellopiped shape(Fig : L - 7.1) within a large fluid region which is at rest. The forces acting on the element are body and surface forces.



Body force: The body force due to gravity is

$$d\overline{F}_B = \rho \cdot g \cdot \delta x \cdot \delta y \cdot \delta z \qquad \text{L -7.1}$$

Where $\delta x \cdot \delta y \cdot \delta z$ is the volume of the element.

Surface force: The pressure at the center of the element is assumed to be *P* (x, y, z). Using Taylor series expansion the pressure at point $\left(x, y - \frac{\delta y}{2}, z\right)$ on the surface can be expressed as

$$P\left(x, y - \frac{\delta y}{2}, z\right) = P(x, y, z) + \frac{\delta p}{\delta y}\left(-\frac{\delta y}{2}\right) + \frac{1}{2!}\frac{\partial^2 p}{\partial y^2}\left(-\frac{\delta y}{2}\right)^2 + \ldots\ldots\ldots \qquad \text{L -7.2}$$

When $\delta y \to 0$, only the first two terms become significant. The above equation becomes

$$P\left(x, y - \frac{\delta y}{2}, z\right) = P(x, y, z) + \frac{\delta p}{\delta y}\left(-\frac{\delta y}{2}\right) \qquad \text{L - 7.3}$$

Similarly, pressures at the center of all the faces can be derived in terms of *P (x, y, z)* and its gradient.

Note that surface areas of the faces are very small. The center pressure of the face represents the average pressure on that face.
The surface force acting on the element in the y-direction is

$$dF_y = \left\{P + \frac{\delta P}{\delta y}\left\{-\frac{\delta y}{2}\right\}\right\}\delta x \cdot \delta y - \left\{P + \frac{\delta P}{\delta y}\left\{\frac{\delta y}{2}\right\}\right\}\delta x \cdot \delta z$$

$$= -\frac{\delta P}{\delta y} \cdot \delta x \cdot \delta y \cdot \delta z \qquad \text{L -7.4}$$

Similarly the surface forces on the other two directions (x and z) will be

$$dF_x = -\frac{\delta P}{\delta x} \cdot \delta x \cdot \delta y \cdot \delta z$$

$$dF_z = -\frac{\delta P}{\delta z} \cdot \delta x \cdot \delta y \cdot \delta z$$

The surface force which is the vectorical sum of the force scalar components

$$dF_s = -\left(\frac{\delta p}{\delta x}\hat{i} + \frac{\delta p}{\delta y}\hat{j} + \frac{\delta p}{\delta z}\hat{k}\right)(\delta x \cdot \delta y \cdot \delta z)$$

$$= -\nabla p \cdot \delta x \cdot \delta y \cdot \delta z \qquad \text{L - 7.5}$$

The total force acting on the fluid is

$$d\,\overline{F} = d\,\overline{F_S} + d\,\overline{F_B}$$

$$= \left(-\nabla p + \rho\vec{g}\right)(\delta x \cdot \delta y \cdot \delta z) \qquad \text{L - 7.6}$$

The total force per unit volume is

$$\frac{dF}{\delta x \cdot \delta y \cdot \delta z} = -\nabla p + \rho\vec{g}$$

For a static fluid, *dF=0* .

Then, $\left(-\nabla p + \rho\vec{g}\right) = 0$ L -7.7

$$\begin{bmatrix}\text{Net pressure force}\\ \text{per unit volume}\\ \text{at a point}\end{bmatrix}+\begin{bmatrix}\text{Body force}\\ \text{per unit volume}\\ \text{at a point}\end{bmatrix}=0$$

If acceleration due to gravity $\vec{g}$ is expressed as $\vec{g}=g_x\hat{i}+g_y\hat{j}+g_z\hat{k}$, the components of Eq(L- 7.8) in the x, y and z directions are

$$-\frac{\delta p}{\delta z}+\rho g_z=0$$

$$-\frac{\delta p}{\delta x}+\rho g_x=0$$

$$-\frac{\delta p}{\delta y}+\rho g_y=0$$

The above equations are the basic equation for a fluid at rest.

Simplifications of the Basic Equations

If the gravity $\vec{g}$ is aligned with one of the co-ordinate axis, for example *z*- axis, then

$$g_x=0$$

$$g_y=0$$

$$g_z=-g$$

The component equations are reduced to

$$\frac{\delta p}{\delta x}=0$$

$$\frac{\delta p}{\delta y}=0 \qquad \text{L-7.9}$$

$$\frac{\delta p}{\delta z}=-\rho g$$

Under this assumption, the pressure *P* depends on z only. Therefore, total derivative can be used instead of the partial derivative.

$$\frac{dp}{dz} = -\rho g$$

This simplification is valid under the following restrictions

a. Static fluid

b. Gravity is the only body force.

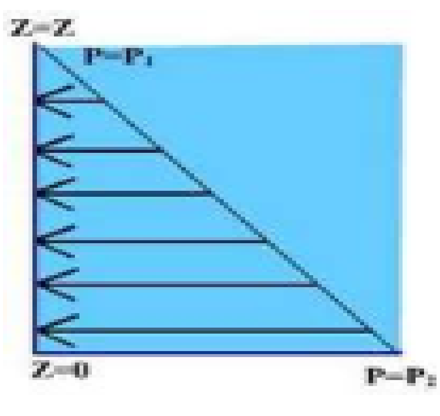
c. The z-axis is vertical and upward.

Pressure variations in an incompressible fluid at rest

In some fluid problems, fluids may be considered homogenous and incompressible *i.e* . density ρ is constant. Integrating the equation (L -7.10) with condition given in figure (Fig : L - 7.2), we have

$$\int_{P_1}^{P_2} dp = \int_{0}^{z} -\rho g \cdot dz$$

$$P_2 - P_1 = -\rho g z$$



Pressure variation in an incompressible fluid

This indicates that the pressure increases linearly from the free surface in an incompressible static fluid as illustrated by the linear distribution in the above figure.

Scales of pressure measurement

Fluid pressures can be measured with reference to any arbitrary datum. The common datum are

1. Absolute zero pressure.
2. Local atmospheric pressure

When absolute zero (complete vacuum) is used as a datum, the pressure difference is called an absolute pressure, P_{abs} .

When the pressure difference is measured either above or below local atmospheric pressure, P_{local} , as a datum, it is called the gauge pressure. Local atmospheric pressure can be measured by mercury barometer.

At sea level, under normal conditions, the atmospheric pressure is approximately 101.043 kPa.

As illustrated in figure(Fig : L -7.2),

When $P_{abs} < P_{local}$

$$P_{gauge} = P_{local} - P_{abs} \qquad \text{L - 7.12}$$

Note that if the absolute pressure is below the local pressure then the pressure difference is known as vacuum suction pressure.

Example 1 :

Convert a pressure head of 10 m of water column to kerosene of specific gravity 0.8 and carbon-tetra-chloride of specific gravity of 1.62.

Solution

Given data:

Height of water column, $h_1 = 10$ m

Specific gravity of water $s_1 = 1.0$

Specific gravity of kerosene $s_2 = 0.8$

Specific gravity of carbon-tetra-chloride, $s_3 = 1.62$

For the equivalent water head

Weight of the water column = Weight of the kerosene column.

So, $\square\ g\ h_1\ s_1 = \square\ g\ h_2\ s_2 = \square\ g\ h_3\ s_3$

Answer:- 12.5 m and 6.17 m.

Example 2

Determine (a) the gauge pressure and (b) The absolute pressure of water at a depth of 9 m from the surface.

Solution

Given data:

Depth of water = 9 m

the density of water = 998.2 kg/m^3

And acceleration due to gravity = 9.81 m/s^2

Thus the pressure at that depth due to the overlying water is P = r gh = 88.131 kN/m^2

Case a) as already discussed, gauge pressure is the pressure above the normal atmospheric pressure.

Thus, the gauge pressure at that depth = 88.131 kN/m^2

Case b) The standard atmospheric pressure is 101.213 kN/m^2

Thus, the absolute pressure as P_{abs} = 88.131+101.213 = 189.344 kN/m^2
Answer: 88.131 kN/m^2 ; 101.213 kN/m^2

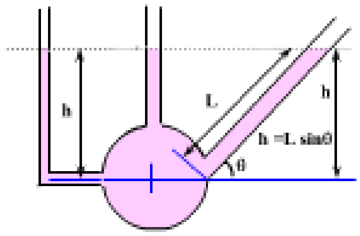
Manometers: Pressure Measuring Devices

Manometers are simple devices that employ liquid columns for measuring pressure difference between two points.

In Figure(L 8.1), some of the commonly used manometers are shown.

In all the cases, a tube is attached to a point where the pressure difference is to be measured and its other end left open to the atmosphere. If the pressure at the point P is higher than the local atmospheric pressure the liquid will rise in the tube. Since the column of the liquid in the tube is at rest, the liquid pressure P must be balanced by the hydrostatic pressure due to the column of liquid and the superimposed atmospheric pressure, P_{atm} .

$$P = \rho g h + P_{atm}$$



Simple Manometer

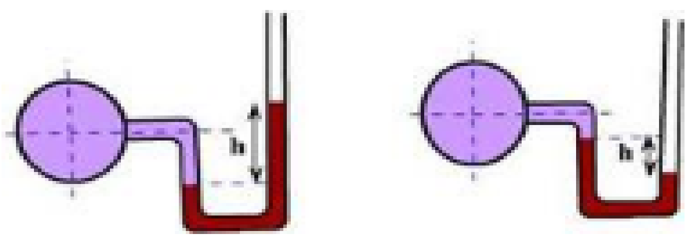
This simplest form of manometer is called a *Piezometer* . It may be inadequate if the pressure difference is either very small or large.

U - Tube Manometer

In (Fig : L -8.2), a manometer with two vertical limbs forms a U-shaped measuring tube. A liquid of different density □□ is used as a manometric fluid. We may recall the Pascal's law which states that the pressure on a horizontal plane in a continuous fluid at rest is the same. Applying this equality of pressure at points B and C on the plane gives

$$P + \rho g h = P_{atm} + \rho_1 g h_1$$

$$P - P_{atm} = \rho_1 g h_1 - \rho g h$$



U-tube Manometer

Inclined Manometer

A manometer with an inclined tube arrangement helps to amplify the pressure reading, especially in low press range. A typical arrangement of the same is shown in Fig. L-8.3.

The pressure at O is

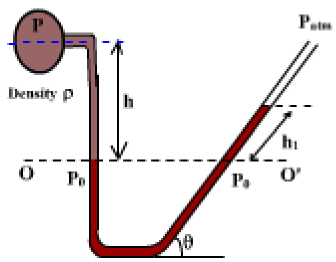
$$P_0 = P + \rho g h$$

The pressure at O is

$$P_{0'} = P_{atm} + \rho_1 g h_1 \sin\theta$$

Equating the pressures, we have

$$P_0 - P_{atm} = \rho_1 g h_1 \sin\theta - \rho g h$$



Inclined Manometer

At the same pressure difference, Equations (1) and (2) indicate that inclined tube manometer amplifies the length of measurement by $\frac{1}{\sin\theta}$, which is the primary advantage of such type of manometer.

Differential Manometers

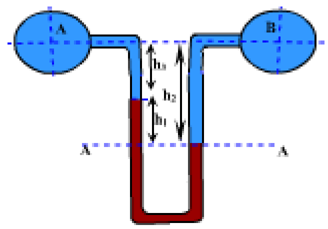
Differential Manometers measure difference of pressure between two points in a fluid system and cannot measure the actual pressures at any point in the system.

Some of the common types of differential manometers are

a. Upright U-Tube manometer
b. Inverted U-Tube manometer
c. Inclined Differential manometer
d. Micro manometer

Upright U-Tube manometer:

As shown in Fig. : L-8.4, an upright U-tube manometer is connected between points A and B. The difference of pressure between the points may be calculated by balancing pressure in a horizontal plane, the lowest interface A-A is used for this case.



Upright U-tube Manometer

$$P_A + \rho_1 g h_1 + \rho_3 g h_3 = P_B + \rho_2 g h_2$$

or

$$P_A - P_B = \rho_2 g h_2 - \rho_1 g h_1 - \rho_3 g h_3$$
$$= (\rho_2 h_2 - \rho_1 h_1 - \rho_3 h_3)\, g$$

Inverted U-Tube manometer:

The manometer fluid used in this type of manometer is lighter than the working fluids. Thus the height difference in two limbs is enhanced. This is therefore suitable for measurement of small pressure difference in liquids. For the configurations given in Fig. L-8.1.

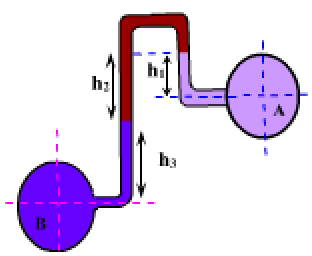


Fig. L-8.5 Inverted Manometer

$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_3 g h_3 \quad \text{Or} \quad P_A - P_B = (\rho_1 h_1 - \rho_2 h_2 - \rho_3 h_3)\, g$$

If the two points A and B are at the same level and the same fluid is used, then $P_1 = P_2 = P$ and $h_2 + h_3 = h_1$.

The above equation becomes $P_A - P_B = (\rho_1 - \rho_3) h_3 g$

Inclined Differential Manometer

In this type of manometer a narrow tube is connected to a reservoir at an inclination. The cross section of the reservoir is larger than that of the tube. Fluctuations in the reservoir may be ignored. As shown in Fig.L-8.6, the initial liquid level in both the reservoir and the tube is at o-o. The application of the differential pressure liquid level of the reservoir drops by Δh, whereas h is the rising level in the tube. Therefore

$$P_A = P_B + (h + \Delta h)\rho g$$

Since the volume of liquid displaced in the reservoir equals to the volume of liquid in the tube, we can define

$$A \cdot \Delta h = a \cdot L$$

Where 'A' and 'a' are the cross sectional areas of the reservoir and the tube respectively. Then the equation becomes $P_A - P_B = (h + \frac{a}{A}L)\rho g$

In practice, the reservoir area is much larger than that of the tube; the ratio $\frac{a}{A}$ is negligible and the above equation is reduced to $P_A - P_B = \rho g L \sin\theta$; $h = L \sin\theta$

Micro manometer:

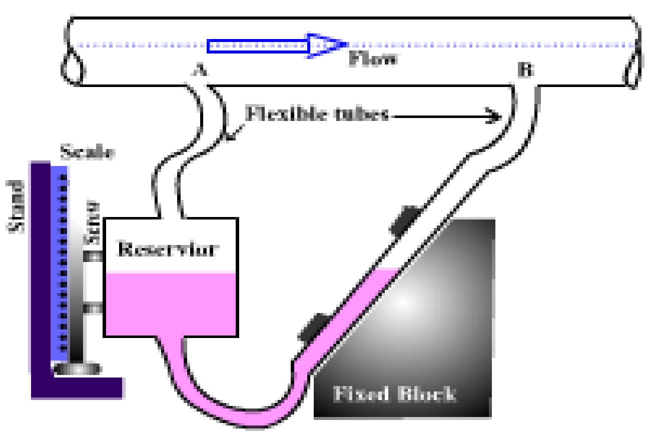


Fig. L-8.6: Micro manometer

A typical micro-manometer tube arrangement as shown in fig has a reservoir which can be moved up and down by means of micrometer screw. A flexible tube is connected between point A and the reservoir. Another flexible tube connecting point B and the other end of the reservoir is placed on an inclined surface. A reference mark 'R' is provided on the inclined portion of the tube. Before application of the pressure, the level of the reservoir is moved so as to coincide this level with the reference mark. When a pressure difference is applied, the liquid levels will be disturbed. The micrometer arrangement is then adjusted to vary the reservoir level so as to coincide with the reference. The extent of movement of the micrometer screw gives the pressure difference between the two points A and B.

Example 1:

Two pipes on the same elevation convey water and oil of specific gravity 0.88 respectively. They are connected by a U-tube manometer with the manometric liquid having a specific gravity of 1.25. If the manometric liquid in the limb connecting the water pipe is 2 m higher than the other find the pressure difference in two pipes.

Solution :

Given data:

Height difference = 2 m

Specific gravity of oil s = 0.88

Specific gravity of manometric liquid s = 1.25

Equating pressure head at section (A-A)

$$P_A + 2 \times 1.25 \rho_W \, g + (h-2) \rho_W \, g = P_B + h \times 0.88 \rho_W \, g$$

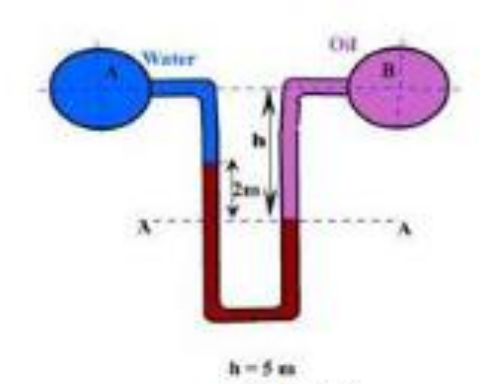


Fig. Ex1

Substituing h = 5 m and density of water 998.2 kg/m^3 we have $P_A - P_B = 10791$

Example 2:

A two liquid double column enlarged-ends manometer is used to measure pressure difference between two points. The basins are partially filled with liquid of specific gravity 0.75 and the lower portion of U-tube is filled with mercury of specific gravity 13.6. The diameter of the basin is 20 times higher than that of the U-tube. Find the pressure difference if the U-tube reading is 25 mm and the liquid in the pipe has a specific weight of 0.475 N/m^3.

Solution:

Given data: U-tube reading 25 mm

Specific gravity of liquid in the basin 0.75

Specific gravity of Mercury in the U-tube13.6

As the volume displaced is constant we have,

$$Y = 25\frac{a}{A} = 25 \times \frac{1}{20^2}$$

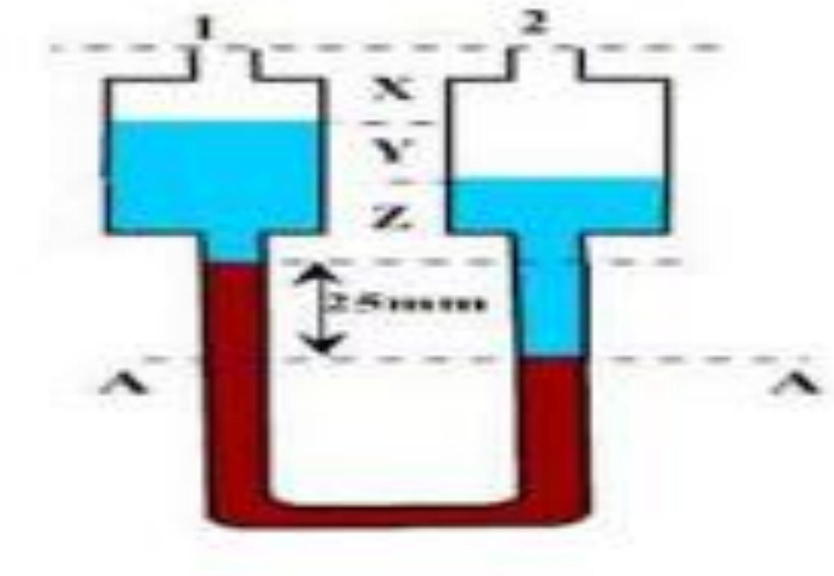


Fig. Ex 2

Equating pressure head at (A--A)

$$P_1 + X\frac{0.475}{1000}\rho_w g + (Z+Y)\rho_w g \times 0.75 + 25 \times 13.6\rho_w g$$

$$= P_2 + (X+Y)\frac{0.475}{1000}\rho_w g + (Z+25)\times 0.75\rho_w g$$

Put the value of Y while X and Z cancel out.
Answer: 31.51 kPa

Example 3:

As shown in figure water flows through pipe A and B. The pressure difference of these two points is to be measured by multiple tube manometers. Oil with specific gravity 0.88 is in the upper portion of inverted U-tube and mercury in the bottom of both bends. Determine the pressure difference.

Solution

Given data: Specific gravity of the oil in the inverted tube 0.88
Specific gravity of Mercury in the U-tube13.6

Calculate the Pressure difference between each two point as follow
$P_2 - P_1 = h \rho g = h S \rho_w g$

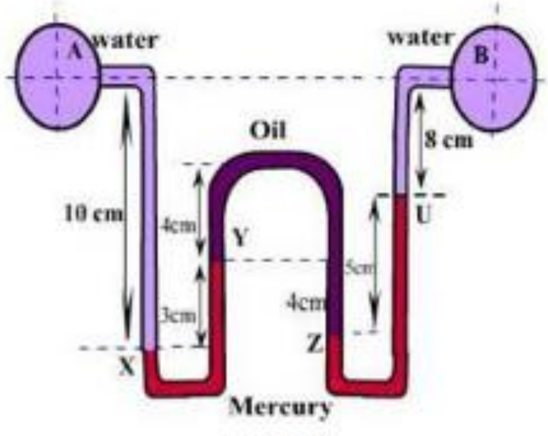


Fig. Ex3

Start from one and i.e. P_A or P_B

$$\text{Now, } P_X = P_A + 10\rho_w g$$
$$\text{Similarly, } P_Y = P_X - 3 \times 13.6\rho_w g$$
$$P_Z = P_Y + 4 \times 0.88\rho_w g$$
$$P_U = P_Z - 5 \times 13.6\rho_w g$$
$$P_B = P_U - 8\rho_w g$$

Rearranging and summing all these equations we have $P_A - P_B = 103.28 \,\square_w\, g$

Example 4:

A manometer connected to a pipe indicates a negative gauge pressure of 70 mm of mercury . What is the pressure in the pipe in N/m^2 ?

Solution :

Given data:

Manometer pressure- 70 mm of mercury (Negative gauge pressure)

A pressure of 70 mm of Mercury, P = r gh = 9.322 kN/m^2

Also we know the gauge pressure is the pressure above the atmosphere.

Thus a negative gauge pressure of 70 mm of mercury indicates the absolute pressure of

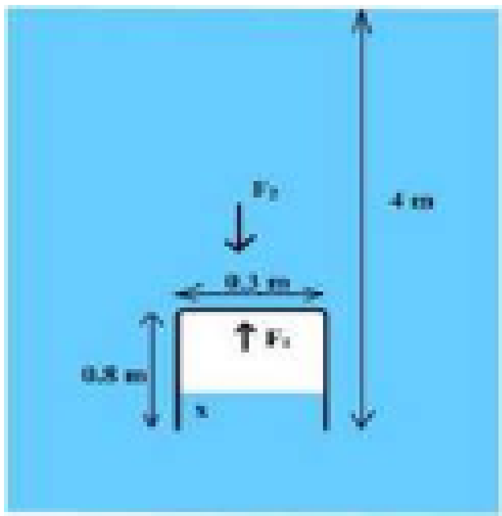
$P_{abs} = 101.213 + (-9.322) = 91.819\ kN/m^2$

Answer: $91.819\ kN/m^2$

Example 5:

An empty cylindrical bucket with negligible thickness and weight is forced with its open end first into water until its lower edge is 4m below the water level. If the diameter and length of the bucket are 0.3m and 0.8m respectively and the trapped water remains at constant temperature. What would be the force required to hold the bucket in that position atmospheric pressure being $1.03\ N/cm^2$

Solution :



Let, the water rises a height *x* in the bucket

By applying the Boyle's Law at constant temperature we have

$$p_1 \times (0.3)^2 \times \frac{\pi}{4} \times (0.8 - x) = p_{atm} \times (0.3)^2 \frac{\pi}{4} \times 0.8$$

Also, Downward pressure ion the bucket, $p_1 = p_{atm} + (4 - x) \times 9810$

Solve for, p_1 and x.

$$p_1 = 6.46 \times 10^4\ N/m^2$$

$$x = 0.610m$$

Total upward force exerted by the trapped water $F_1 = p_1 \times \frac{\pi}{4} \times 0.3^2 = 4.57 \times 10^3\ N/m^2$

Downward force due to the overlying water and the Atmospheric Pressure

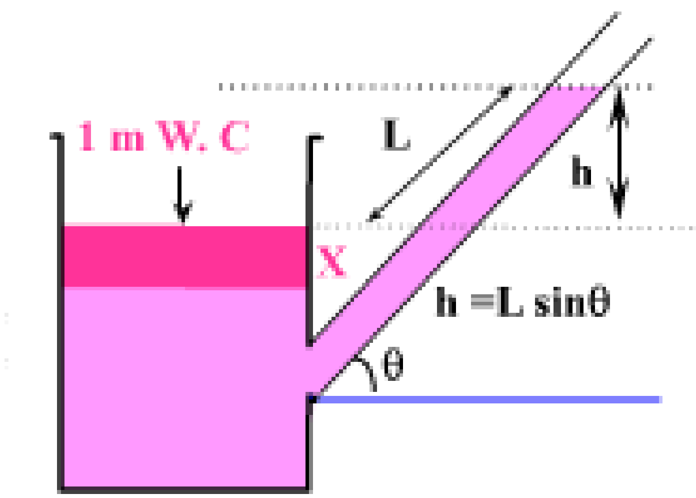
$$F_2 = [1.03\times10^4 + 9810\times(4-0.8)]\times\frac{\pi}{4}\times0.3_2$$

Answer: 1.62KN

Example 6:

A pipe connected with a tank (diameter 3 m) has an inclination of □ with the horizontal and the diameter of the pipe is 20 cm. Determine the angle ? which will give a deflection of 5 m in the pipe for a gauge pressure of 1 m water in the tank. Liquid in the tank has a specific gravity of 0.88.

Solution :



Given data:

Diameter of tank = 3 m
Diameter of tube = 20 cm
Deflection in the pipe, L = 5 m
From the figure shown
h = L sin□□□
If X m fall of liquid in the tank rises L m in the tube. (Note that the volume displaced is the same in the tank is equal to the volume displaced in the pipe)

$$x\pi\frac{3^2}{4} = L\pi\frac{0.2^2}{4}$$

$$\text{or} \quad x = \frac{0.04L}{9}.$$

Difference of head = x + h = L sin q + 0.04 L/9

And $$\left\{Lsin\theta + \frac{0.04L}{9}\right\} \times 0.88 = 1$$

Substitute L = 5m in the above equation.

Answer: □ = 12.87 0

Hydrostatic force on submerged surfaces

Introduction

Designing of any hydraulic structure, which retains a significant amount of liquid, needs to calculate the total force caused by the retaining liquid on the surface of the structure. Other critical components of the force such as the direction and the line of action need to be addressed. In this module the resultant force acting on a submerged surface is derived.

Hydrostatic force on a plane submerged surface

Shown in Fig.L-9.1 is a plane surface of arbitrary shape fully submerged in a uniform liquid. Since there can be no shear force in a static liquid, the hydrostatic force must act normal to the surface.

Consider an element of area $d\overline{A}$ on the upper surface. The pressure force acting on the element is

$$d\overline{F} = -Pd\overline{A}$$

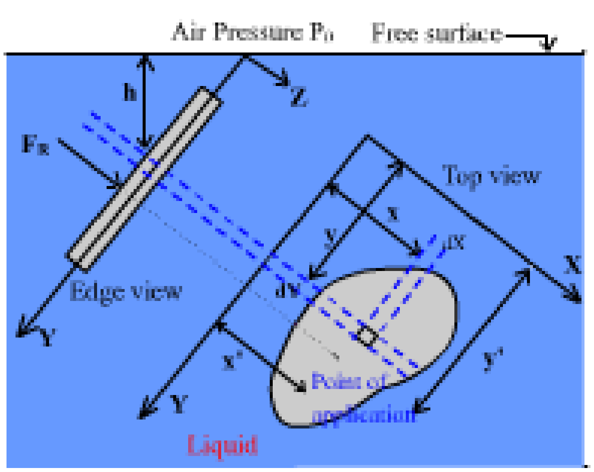


Fig : L - 9.1: Hydrostatic force and center of pressure on an inclined surface

Note that the direction of $d\bar{A}$ is normal to the surface area and the negative sign shows that the pressure force $d\bar{F}$ acts against the surface. The total hydrostatic force on the surface can be computed by integrating the infinitesimal forces over the entire surface area.

$$F = \int_A -P \cdot d\bar{A}$$

If *h* is the depth of the element, from the horizontal free surface as given in Equation (L2.9) becomes

$$\frac{dP}{dh} = \rho g = w \qquad \text{L-9.1}$$

If the fluid density ρ is constant and P 0 is the atmospheric pressure at the free surface, integration of the above equation can be carried out to determine the pressure at the element as given below

$$\begin{aligned} P &= P_0 + \int_0^h w dh \\ &= P_0 + wh \end{aligned} \qquad \text{L-9.2}$$

Total hydrostatic force acting on the surface is

$$\begin{aligned} F &= \int_A P \cdot d\bar{A} \\ &= \int_A (P_0 + wh) \cdot d\bar{A} \\ &= \int_A (P_0 + w \cdot y \sin\theta) \cdot d\bar{A} \\ &= P_0 A + w \cdot \sin\theta \int_A y \cdot d\bar{A} \end{aligned} \qquad \text{L-9.3}$$

The integral $\int_A y \cdot d\bar{A}$ is the first moment of the surface area about the x-axis.

If y_c is the y coordinate of the centroid of the area, we can express

$$\int_A y \cdot d\bar{A} = y_c \cdot A \qquad \text{L-9.4}$$

in which A is the total area of the submerged plane.

Thus

$$F = P_0 \cdot A + w \sin\theta \cdot (y_c A)$$
$$= P_c A \qquad \text{L-9.5}$$

This equation says that the total hydrostatic force on a submerged plane surface equals to the pressure at the centroid of the area times the submerged area of the surface and acts normal to it

Centre of Pressure (CP)

The point of action of total hydrostatic force on the submerged surface is called the Centre of Pressure (CP). To find the co-ordinates of CP, we know that the moment of the resultant force about any axis must be equal to the moment of distributed force about the same axis. Referring to Fig. L-9.2, we can equate the moments about the x-axis.

$$Y_{cp} F = \int_A y \cdot P \cdot dA \qquad \text{L-9.6}$$

Neglecting the atmospheric pressure ($P_0 = 0$) and substituting $F = w \sin\theta \cdot y_c A$, $P = wh$ and $h = y \sin\theta$,

We get $$Y_{cp} \cdot w \sin\theta \cdot y_c A = w \sin\theta \int_A y^2 \cdot dA$$

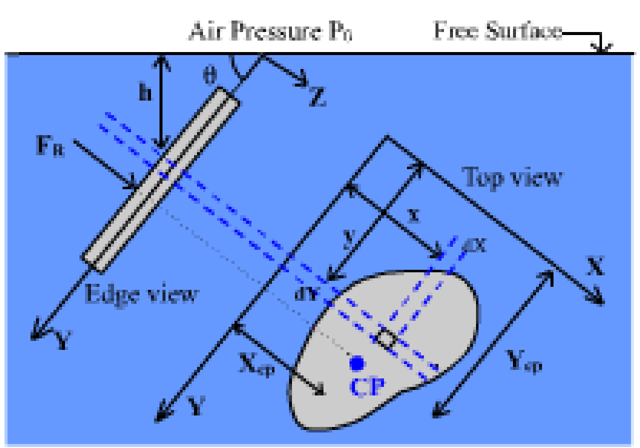


Fig. L-9.2 : Centre of pressure

We get $Y_{cp}\cdot w\sin\theta\cdot y_c A = w\sin\theta\int_A y^2\cdot dA$

$$Y_{cp} = \frac{\int_A y^2\cdot dA}{y_c A}$$

$$= \frac{\int_A y^2\cdot dA}{\int y\cdot dA}$$

$$= \frac{\text{second moment of area about '0'}}{\text{first moment of area about '0'}}$$

From parallel-axis theorem

$$I_{xx} = I_{xc} + A\cdot y_c^{\,2}$$

Where I_{xc} is the second moment of the area about the centroidal axis.

$$Y_{cp} = \frac{I_{xc} + A\cdot y_c^{\,2}}{A\cdot y_c}$$

$$= \frac{I_{xc}}{A\cdot y_c} + y_c \qquad \text{L-9.8}$$

This equation indicates that the centre of the pressure is always below the centroid of the submerged plane. Similarly, the derivation of x_{cp} can be carried out

Hydrostatic force on a Curved Submerged surface

On a curved submerged surface as shown in Fig. L-9.3, the direction of the hydrostatic pressure being normal to the surface varies from point to point. Consider an elementary area $d\bar{A}$ in the curved submerged surface in a fluid at rest. The pressure force acting on the element is

$$d\vec{F} = Pd\vec{A}$$

The total hydrostatic force can be computed as

$$\vec{F} = \int_A -Pd\vec{A}$$

Note that since the direction of the pressure varies along the curved surface, we cannot integrate the above integral as it was carried out in the previous section. The force vector $\overline{F}$ is expressed in terms of its scalar components as

$$\overline{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

in which $F_x, F_y \text{ and } F_z$ represent the scalar components of F in the x , y and z directions respectively.

For computing the component of the force in the x-direction, the dot product of the force and the unit vector ($\hat{i}$ i) gives

$$F_x = \int d\overline{F}\cdot\hat{i}$$
$$= \int_A -PdA\cdot\hat{i}$$
$$= -\int_A PdA_x$$

Where dA_x is the area projection of the curved element on a plane perpendicular to the x-axis. This integral means that each component of the force on a curved surface is equal to the force on the plane area formed by projection of the curved surface into a plane normal to the component. The magnitude of the force component in the vertical direction (z direction)

$$F_z = \int_{A_z} PdA_z$$

Since $P = P_0 + wh$ and neglecting P_0, we can write

$$F_z = \int_{A_z} wh\cdot dA_z$$
$$= \int wd\nu$$

in which is the weight of liquid above the element surface. This integral shows that the z-component of the force (vertical component) equals to the weight of liquid between the submerged surface and the free surface. The line of action of the component passes through the centre of gravity of the volume of liquid between the free surface and the submerged surface

Example 1 :

A vertical gate of 5 m height and 3 m wide closes a tunnel running full with water. The pressure at the bottom of the gate is 195 kN/m ^{2} . Determine the total pressure on the gate and position of the centre of the pressure.

Solution

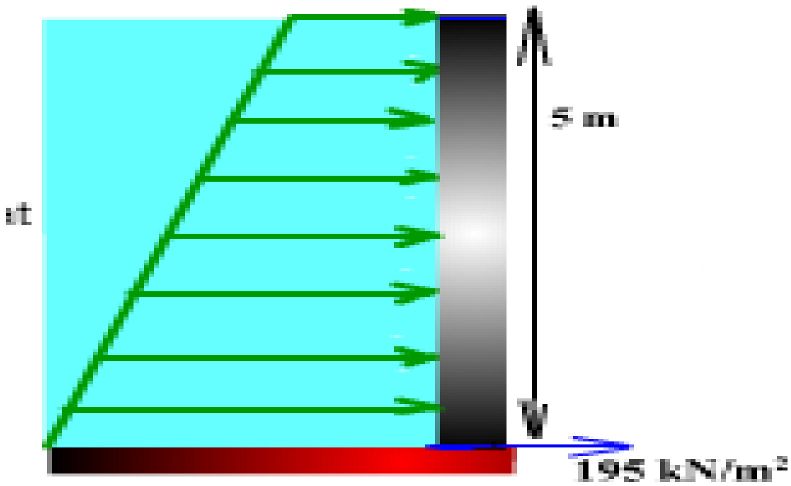


Fig. Ex1

Given data: Area of the gate = 5x3 = 15 m ^{2}

The equivalent height of water which gives a pressure intensity of 195 kN/m^2 at the bottom.

h = P/w =19.87m.

Total force $F = wA\bar{x}$.

And $\bar{x} = 19.87 - 2.5 = 17.37m$

Centre of Pressure $\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$ [$I_G = bd^3/12$]

Answer: 2.56MN and 17.49 m

Example 2 :

A vertical rectangular gate of 4m x 2m is hinged at a point 0.25 m below the centre of gravity of the gate. If the total depth of water is 7 m what horizontal force must be applied at the bottom to keep the gate closed?

Solution

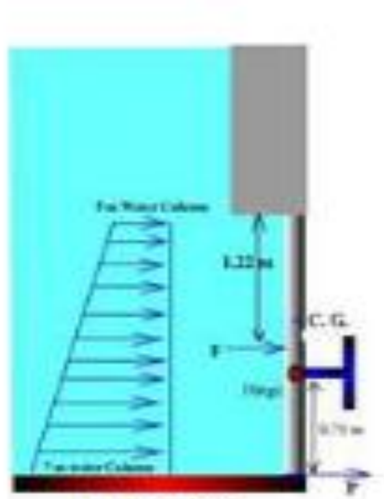


Fig. Ex2

Given data: Area of the gate = 4x2 = 8 m ^{2}

Depth of the water = 7 m

Hydrostatic force on the gate

$$F = wA\bar{x} \qquad \bar{x} = 5+1 = 6 \text{ m}$$
$$= 4.7 \times 10^5 N$$

$$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}} = 6.22m$$

Taking moments about the hinge we get, $F \times 0.03 = P \times 0.75$

Answer: 18.8 kN.

Buoyancy

Introduction

In our common experience we know that wooden objects float on water, but a small needle of iron sinks into water. This means that a fluid exerts an upward force on a body which is immersed fully or partially in it. The upward force that tends to lift the body is called the buoyant force, F_b.

The buoyant force acting on floating and submerged objects can be estimated by employing hydrostatic principle.

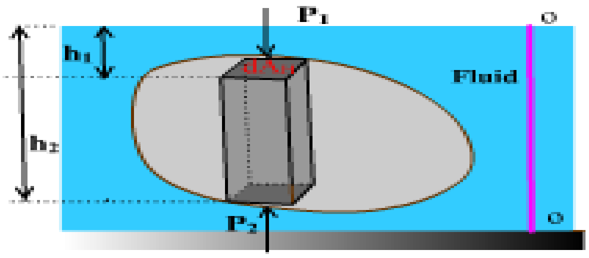


Fig L 10.1 : Buoyant force

With reference to figure(L- 10.1), consider a fluid element of area dA_H . The net upward force acting on the fluid element is

$$dF_B = (P_2 - P_1)dA_H$$
$$= w(h_2 - h_1)dA_H$$

The total upward buoyant force becomes

$$F_B = \int w(h_2 - h_1)dA_H = w(\text{volume of the body})$$ L-10.2

This result shows that the buoyant force acting on the object is equal to the weight of the fluid it displaces.

Center of Buoyancy

The line of action of the buoyant force on the object is called the center of buoyancy. To find the centre of buoyancy, moments about an axis OO can be taken and equated to the moment of the resultant forces. The equation gives the distance to the centeroid to the object volume.

The centeroid of the displaced volume of fluid is the centre of buoyancy, which, is applicable for both submerged and floating objects. This principle is known as the Archimedes principle which states:

"A body immersed in a fluid experiences a vertical buoyant force which is equal to the weight of the fluid displaced by the body and the buoyant force acts upward through the centroid of the displaced volume".

Buoyant force in a layered fluid

As shown in figure (L-10.2) an object floats at an interface between two immiscible fluids of density ρ_1 and ρ_2 .

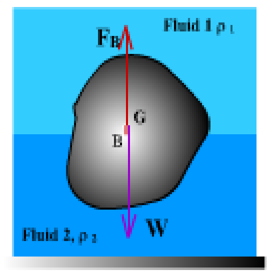


Fig. L-10.2: Buoyant force in a layered fluid

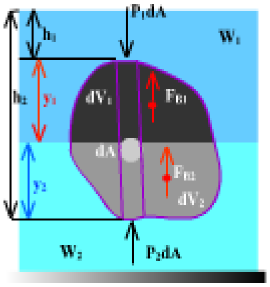


Fig. L-10.3: Element with hydrostatics forces

Considering the element shown in Figure L-10.3, the buoyant force F_B is

$$F_B = \int dF_B = \int \rho_1 g dV_1 + \int \rho_2 g dV_2$$

$$= \sum_1^n \rho_i g \left(\text{displaced volume}\right)_i \qquad \text{L-10.3}$$

where dV_1 and dV_2 are the volumes of fluid element submerged in fluid 1 and 2 respectively. The centre of buoyancy can be estimated by summing moments of the buoyant forces in each fluid volume displaced.

Buoyant force on a floating body

When a body is partially submerged in a liquid, with the remainder in contact with air (as shown in figure), the buoyant force of the body can also be computed using equation (L-10.3). Since the specific weight of the air (11.8 N/m^3) is negligible as compared with the specific weight of the liquid (for example specific weight of water is 9800 kN/m^3),we can neglect the weight of displaced air. Hence, equation (L-10.3) becomes

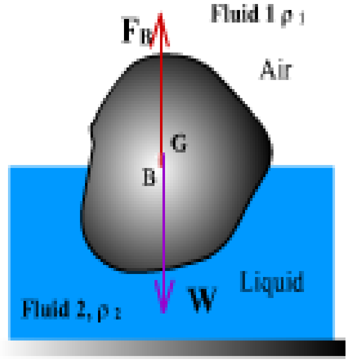


Fig. L-10.4: Partially submerged body

$F_B = \rho g$ (Displaced volume of the submerged liquid)

= The weight of the liquid displaced by the body.

The buoyant force acts at the centre of the buoyancy which coincides with the centeroid of the volume of liquid displaced.

Example 1:

A large iceberg floating in sea water is of cubical shape and its specific gravity is 0.9 If 20 cm proportion of the iceberg is above the sea surface, determine the volume of the iceberg if specific gravity of sea water is 1.025.

Solution:

Let the side of the cubical iceberg be *h*.

Total volume of the iceberg = h^3

volume of the submerged portion is = $(h - 20) \times h^2$

Now,
For flotation, weight of the iceberg = weight of the displaced water

$$(h-20)\times h^2 \times 1.025 \times w = h^3 \times 0.9 \times w$$
$$or,\ h = 164\ cm$$

The side of the iceberg is 164 cm.

Thus the volume of the iceberg is 4.41m^3

Answer: 4.41m^3

Stability

Introduction

Floating or submerged bodies such as boats, ships etc. are sometime acted upon by certain external forces. Some of the common external forces are wind and wave action, pressure due to river current, pressure due to maneuvering a floating object in a curved path, etc. These external forces cause a small displacement to the body which may overturn it. If a floating or submerged body, under action of small displacement due to any external force, is overturn and then capsized, the body is said to be in unstable. Otherwise, after imposing such a displacement the body restores its original position and this body is said to be in stable equilibrium. Therefore, in the design of the floating/submerged bodies the stability analysis is one of major criteria.

Stability of a Submerged body

Consider a body fully submerged in a fluid in the case shown in figure (Fig. L-11.1) of which the center of gravity (CG) of the body is below the centre of buoyancy. When a small angular displacement is applied a moment will generate and restore the body to its original position; the body is stable.

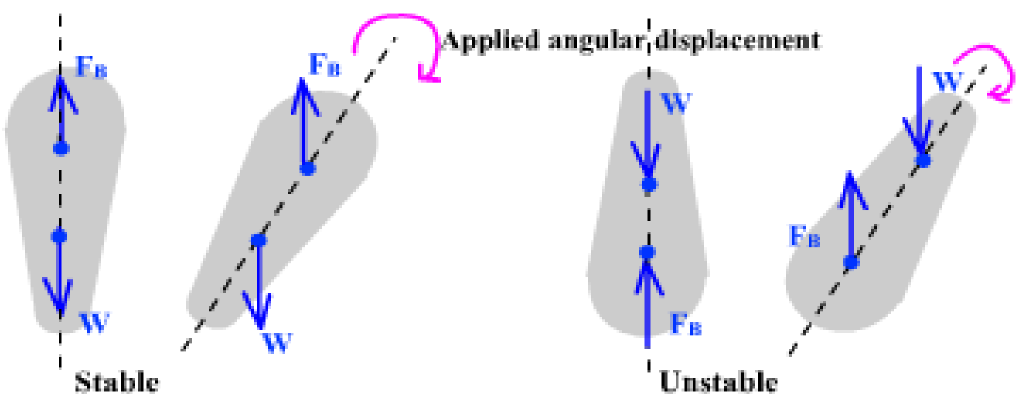


Fig. L-11.1

Fig. L-11.2

However if the CG is above the centre of buoyancy an overturning moment rotates the body away from its original position and thus the body is unstable (see Fig L-11.2). Note that as the body is fully submerged, the shape of the displaced fluid remains the same when the body is tilted. Therefore the centre of buoyancy in a submerged body remains unchanged.

Stability of a floating body

A body floating in equilibrium ($F_B = W$) is displaced through an angular displacement θ. The weight of the fluid W continues to act through G. But the shape of immersed volume of liquid changes and the centre of buoyancy relative to body moves from B to B 1 . Since the buoyant force F_B and the weight *W* are not in the same straight line, a turning movement proportional to ' $W x \theta$ ' is produced.

In figure (Fig. L-11.2) the moment is a restoring moment and makes the body stable. In figure (Fig. L-11.2) an overturning moment is produced. The point ' *M* ' at which the line of action of the new buoyant force intersects the original vertical through the *CG* of the body, is called the metacentre. The restoring moment

$$= W.x = W.\overline{GM}.\theta$$

Provided θ is small; $\sin\theta = \theta$ (in radians).

The distance *GM* is called the metacentric height. We can observe in figure that

Stable equilibrium: when *M* lies above *G* , a restoring moment is produced. Metacentric height *GM* is positive.

Unstable equilibrium: When *M* lies below *G* an overturning moment is produced and the metacentric height *GM* is negative.

Natural equilibrium: If *M* coincides with *G* neither restoring nor overturning moment is produced and *GM* is zero.

Determination of Meta-centric Height

Experimental method

The metacentric height of a floating body can be determined in an experimental set up with a movable load arrangement. Because of the movement of the load, the floating object is tilted with angle θ for its new equilibrium position. The measurement of θ is used to compute the metacentric height by equating the overturning moment and restoring moment at the new tilted position.

The overturning moment due to the movement of load *P* for a known distance, *x*, is $= P.x$

The restoring moment is $= W.\overline{GM}\theta$

For equilibrium in the tilted position, the restoring moment must equal to the overturning moment. Equating the same yields

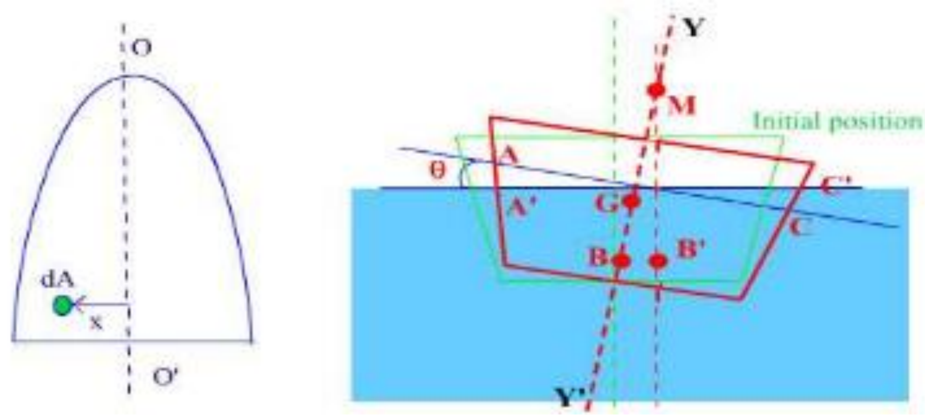
$$P.x = W.\overline{GM}.\theta$$

The metacentric height becomes

$$\overline{GM} = \frac{P.x}{W.\theta}$$

And the true metacentric height is the value of $\overline{GM}$ as $\theta \rightarrow 0$. This may be determined by plotting a graph between the calculated value of $\overline{GM}$ for various θ values and the angle θ.

Theoretical method:



For a floating object of known shape such as a ship or boat determination of meta-centric height can be calculated as follows.

The initial equilibrium position of the object has its centre of Buoyancy, B, and the original water line is AC . When the object is tilted through a small angle θ the center of buoyancy will move to new position B'. As a result, there will be change in the shape of displaced fluid. In the new position $A'C'$ is the waterline. The small wedge OCC' is submerged and the wedge OAA'

is uncovered. Since the vertical equilibrium is not disturbed, the total weight of fluid displaced remains unchanged.

Weight of wedge OAA' = Weight of wedge OCC'.

In the waterline plan a small area, *da* at a distance *x* from the axis of rotation *OO* uncover the volume of the fluid is equal to $DD'xda = x\theta da$

Integrating over the whole wedge and multiplying by the specific weight *w* of the liquid,

Weight of wedge $0AA' = \int_{OAA'} w\theta x da$

Similarly,

Weight of wedge $OCC' = \int_{OCC'} w\theta x da$

Equating Equations ( ) and (),

$$W\theta \int_{OAA'} xda = W\theta \int_{OCC} xda$$

$$\int xda = 0$$

in which, this integral represents the first moment of the area of the waterline plane about *OO* , therefore the axis *OO* must pass through the centeroid of the waterline plane.

Computation of the Meta-centric Height

Refer to Figure(), the distance $\overline{BM}$ is

$$BM = BB'/\theta$$

The distance BB' is calculated by taking moment about the centroidal axis YY'.

$$BB'wv_{A'ECCO} = \int_{AA'ECO} xwdv + \int_{OCC'} xwdv - \int_{OAA'} xwdv$$

The integral $\int_{AA'ECO} xwdv$ equals to zero, because YY' axis symmetrically divides the submerged portion $AA'ECO$.

At a distance x , $dv = Lx \tan\theta . dx$

Substituting it into the above equation gives

$$BB'V_{AECCO} = 0 + \int_{OCC} xLx\tan\theta dx - \int_{OAA'} xL(-x\tan\theta)dx$$
$$= \tan\theta \int_{waterline} x^2 dA_{waterline}$$
$$= \tan\theta\, I_O$$

Where I_0 is the second moment of area of water line plane about OO' . Thus,

$$\overline{BM} = BB'/\theta$$
$$= \frac{I_o \tan\theta}{\theta . V_{AECCO}}$$
$$= \frac{I_o}{V_{AECCO}}$$

Distance

$$\overline{BM} = \overline{GM} + \overline{BG}$$
$$\overline{GM} = \frac{I_o}{V_{submerged}} - \overline{BG}$$

Since,

Example:

A large iceberge, floating in seawater, is of cubical shape and its average specific gravity is 0.9. If a 20-cm -high proportion of the iceberg is above the surface of the water, determine the volume of the iceberg if the specific gravity of the seawater is 1.025.

Solution:

Let the side of the cubical iceberg is *h*.

Then volume of the submerged portion is = (h -20) x h^2

Total volume of the iceberg = h^3 Now,

For flotation, weight of the iceberg = weight of the displaced water

$$(h-20)\times h^2 \times 1.025 = h^3 \times 0.9$$
$$or,\ h = 164$$

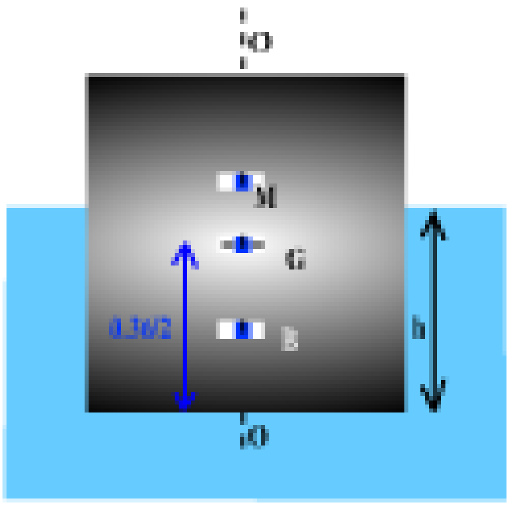
So, the side of the iceberg is 164 cm.

Thus the volume of the iceberg is $4.41m^3$

Example

A log of wood of 1296 cm 2 cross section (square) with specific gravity 0.8 floats in water. Now if one of its edges is depressed to cause the log roll, find the period of roll.

Solution



Let, *h* be the depth of immersion and *L* be the length (perpendicular to the page)

Since the section is square its dimension should be 0.36 m x 0.36 m
For flotation
Weight of water displaced = Weight of the log

$$L \times 0.1296 \times 0.8 = h \times 0.36 \times L$$

Then, *h* = 0.288 m.

$$\overline{BG} = \frac{0.36}{2} - \frac{h}{2} = 0.036$$

$$\overline{BM} = \frac{I_0}{V_{submerged}} = \frac{\frac{1}{12} \times L \times 0.36^3}{0.36 \times 0.288 \times L} \; = \; 0.0375$$

$$\overline{GM} = \left(\overline{BM} - \overline{BG}\right) = 0.0015\ m$$

Time period, $T = \frac{2}{\pi}\sqrt{\frac{K_{G^2}}{GM}}$. and we have, $K_{G^2} = \frac{0.36^2}{12}$

Answer: 5.38 second

Example

To find the metacentre of a ship of 10,000 tonnes a weight of 55 tonnes is placed at a distance of 6 m from the longitudinal centre plane to cause a heel through an angle of 3^0. What is the metacentre height? Hence find the angle of heel and its direction when the ship is moving ahead and 2.8 MW is being transmitted by a single propeller shaft at the rate of 90 rpm.

Solution

Given data: Weight of the ship, W = 10 7 kg

Angle of heel ? = 3^0

Distance of the weight X = 6 m

Weight placed *w* = 5.5 x 10 4

Meta-centric height

$$h = \frac{w.X}{W \tan\theta}$$

$$= 0.629 \text{ m}.$$

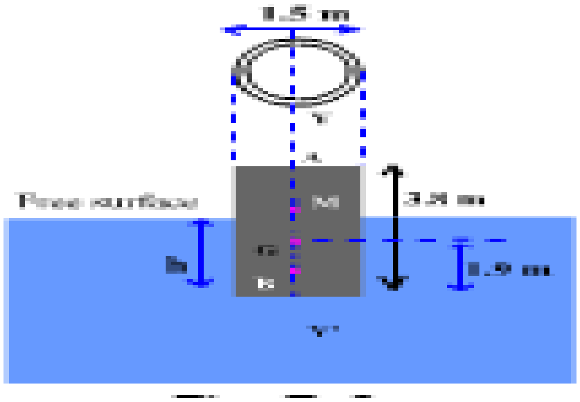
$$\text{Torque transmitted - } T = P/\omega = 2.97 \times 10^5 N - m.$$

$$\omega.h.\tan\theta' = T.$$

Answer:- 0.629 m and 0.27^0.

Example

A hollow cylinder closed in both end, of outside diameter 1.5 m and length of 3.8 m and specific weight 75 kN per cubic meter floats just in stable equilibrium condition. Find the thickness of the cylinder if the sea water has a specific weight of 10 kN per cubic meter.



Solution

Given data : Outside diameter 1.5 m

Length L = 3.8 m

Specific weight 75 kN/m^3

Let the thickness t and immersion depth *h* .

For flotation

Weight of water displaced = weight of the cylinder

$$\frac{\pi}{4}(1.5^2 \times h) \times 10 = \left[\pi\{1.5 \times t\}\, 3.8 + 2 \times \frac{\pi}{4} \times 1.5^2 \times t\right] \times 75$$

Assuming the thickness is very small compared to the diameter

h = 91 t

$$\overline{BM} = \frac{I_0}{V_{submerged}} = \frac{1.545 \times 10^{-3}}{t} \qquad \text{as we have } I_0 = \frac{\pi}{64}\, 1.5^4$$

$$\overline{BG} = \left[\frac{L}{2} - h\right] = [\frac{3.8}{2} - \frac{91}{2}t]$$

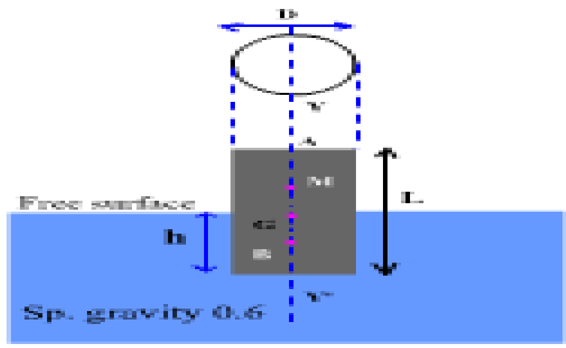
For the cylinder to be in equilibrium $\overline{BM} = \overline{BG}$

Solving for *t* we have *t* = 0.0409 *or* 0.000829m

Answer:- t = 0.83 mm

Example

A wooden cylinder of length L and diameter D is to be floated in stable equilibrium on a liquid keeping its axis vertical. What should be the relation between L and D if the specific gravity of liquid and that of the wood are 0.6 and 0.8 respectively?



Solution

Given data: Specific gravity of liquid = 0.6
Specific gravity of liquid = 0.8
If the depth of immersion is h
Weight of water displaced = weight of the cylinder

$$\frac{\pi}{4} D^2 L \times 0.6 = \frac{\pi}{4} D^2 h \times 0.8$$

The depth of immersion $h = \frac{3}{4} L$.

Height of centre of pressure from bottom x = $\frac{h}{2} = \frac{3}{8} L$

Then, $\overline{BM} = I / V = D^2 / 12L$

$$\overline{BG} = (\overline{OG} - \overline{OB}) = \frac{L}{8}$$

For Stable equilibrium

$$\overline{BM} > \overline{BG}$$

$$or \quad \frac{D^2}{12L} > \frac{L}{8}$$

Answer: L < 0.817D.

Kinematics of flow

The kinematics of fluid motion deals only with space time relationship (velocity and acceleration) without taking into account the forces ~~a acc~~ associated with them.

Types of fluid flow

① Steady and unsteady flow

→ steady flow is defined as that type of flow in which the flow characteristics such as velocity, pressure, density etc. at a point donot change with time. mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0 \quad , \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0 \quad , \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is the co-ordinates of a fixed point in fluid field.

→ unsteady flow is that type of flow in which the velocity, pressure or density etc. at a point changes with respect to time. Thus mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \quad , \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \quad , \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

③ uniform and non-uniform flow

→ uniform flow is defined as that type of flow in which velocity at any given time does not change (in magnitude as well as direction) with respect to space (i.e. in the direction of flow). Mathematically

$$\left(\frac{\partial V}{\partial s}\right)_{t_0} = 0$$

∂V is the change in velocity corresponding to a displacement ∂s in any direction s.

If the flow passage is straight and prismatic (cross-section is the same at any location across the axis of the beam), the flow will be uniform.

In uniform flow, the streamlines are straight and ~~prismati~~ parallel.

→ Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. mathematically $\left(\frac{\partial V}{\partial s}\right)_{t_0} \neq 0$

→ In real fluid flow, the velocity at solid boundary is zero (unless the boundary itself is moving) and varies across a flow section. For such cases, flow will be considered as uniform if the average velocity of flow does not vary from section to section.

(3) Laminar and Turbulent Flow

→ Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or streamline and all the streamlines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is called streamline flow or viscous flow. The flow through a small diameter tube at low velocity is generally laminar.

→ Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way resulting in formation of eddies. This eddy formation is responsible for high energy loss as compared to loss in laminar flow.

* Turbulent flow is said to be steady, if the time average velocity (also known as Temporal velocity) at a point does not change with time, even though the fluid particles moves erratically.

→ For a pipe flow, the type of the flow is determined by non-dimensional number called Reynolds Number. It is denoted by Re:

$$Re = \frac{\rho V D}{\mu}$$

$$\Rightarrow Re = \frac{VD}{\nu}$$

ρ = Density of flowing fluid.
V = mean velocity of flow in pipe
D = Diameter of pipe
μ = Dynamic viscosity of fluid.
$\nu = \frac{\mu}{\rho}$ = Kinematic viscosity of fluid.

→ If Reynold Number (R_e) < 2000 ⇒ Flow is laminar

→ If Reynold Number (R_e) > 4000 ⇒ Flow is Turbulent

→ If Reynold number is lies between 2000 and 4000, The flow is said to be Transitional (i.e) may be laminar or Turbulent.

④ compressible and incompressible Flows

→ compressible is that type of flow, in which the density of the fluid changes from point to Point (or) in other words, the density is not constant for the fluid. Mathematically, for compressible Flow

$\rho \neq$ constant

→ Incompressible is that type of flow, in which the density is constant for the fluid flow. Mathematically $\rho =$ constant.

⑤ Rotational Flow and ir-rotational Flow

→ Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own-axis.

Exp:- Forced vertex, Rotation of earth about its own axis as well as around Sun.

→ A flow is said to be ir-rotational if the fluid particles while flowing along the streamlines donot rotate about their own-axis.

Exp. Free vortex.

⑥ one, Two, and Three-Dimensional Flow

→ One dimensional flow is that type of flow in which the flow parameters such as velocity is a function of time and one space-coordinates only, say x. This space-coordinates is usually the distance measured along the flow direction. The variation of flow paramete in other two mutually perpendicular directions is assumed insignificant and hence negligible.

Hence mathematically

$$u = f(x, t), \quad v = 0, \quad w = 0$$

where u, v, w are the velocity components in x, y, z directions respectively.

If the flow is steady & one dimensional, then

$$u = f(x)$$

→ The flow is said to be two dimensional, when the flow parameters are functions of time and two space co-ordinates (x, y). The variation of flow parameters in the third direction (i.e z-direction) is assumed to be negligibly small. Hence mathematically

$u = f_1(x, y, t)$, $v = f_2(x, y, t)$, $w = 0$.

If flow is steady and 2-dimensional

$u = f_1(x, y)$, $v = f_2(x, y)$

→ In general, the fluid flow is three-dimensional in the sense that the flow characteristics such as velocity, pressure, density etc. may vary in all three mutually perpendicular direction. Here the flow parameters are functions of three space-co-ordinates (x, y, z) and time. mathematically

$u = f_1(x, y, z, t)$

$v = f_2(x, y, z, t)$

$w = f_3(x, y, z, t)$

If the flow is steady i.e. flow parameters are independent of time, Hence

$u = f_1(x, y, z)$

$v = f_2(x, y, z)$

$w = f_3(x, y, z)$

Description of flow patterns

1. stream lines
2. path lines
3. streak lines / Filament lines
4. Time line.

1. stream line

→ It is an imaginary line drawn in the flow field in such a manner that the tangent drawn at any point on this line represents the direction of the velocity vector.

→ since streamline is tangent to the velocity vector at any point, there can be no component of velocity at right angle to the stream line. In other words there can be no flow across a streamline.

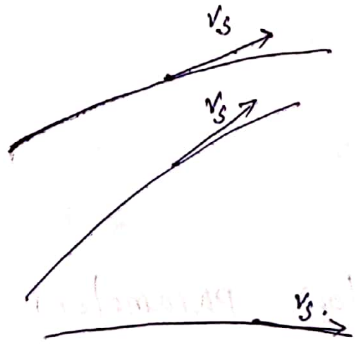
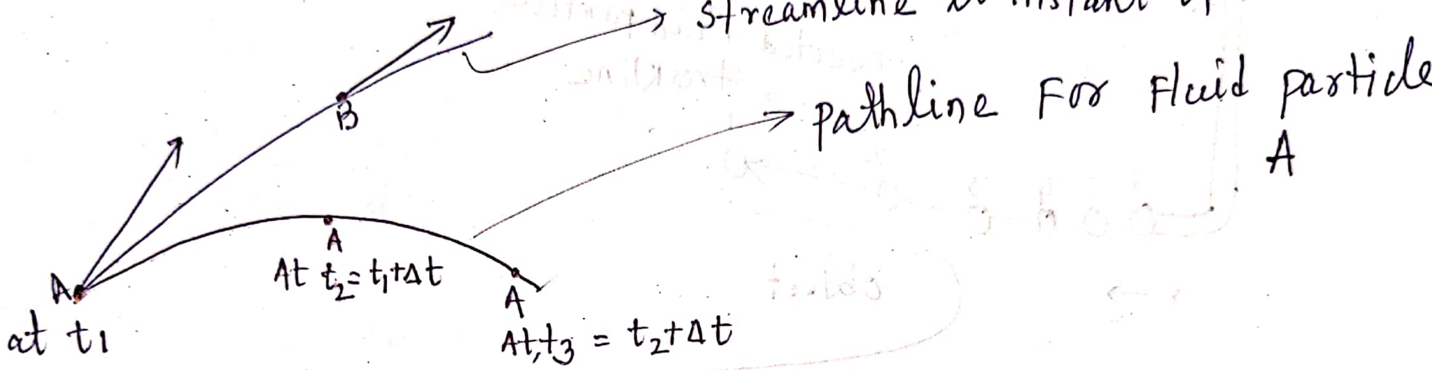


Fig: stream lines

→ In case of steady flow, since the direction of velocity of flow does not change with respect to time, Therefore the patterns of streamlines remains fixed with time.

② pathline

→ It is a line traced by a single fluid particle during its motion. (or) It is the locus of ~~the~~ a fluid particle as it moves along.



→ path line shows the direction of velocity ~~at~~ of one particle over different interval of time whereas streamline shows the direction of velocity of a numbers of fluid particles at a particular instant of time.

③ Streakline

→ It is the line formed by joining all the fluid particles that have passed a given point sequentially in the flow field over a period of time.

→ When a dye is injected in a liquid or smoke in a gas, so as to trace the subsequent motion of fluid particles passing through a fixed point, the path followed by the dye or smoke is called streakline.

→ For an example, If we insert a small tube into a flow and introduce a continuous stream of tracer fluid (dye in a water flow or smoke in an airflow), the observed pattern is a streakline.

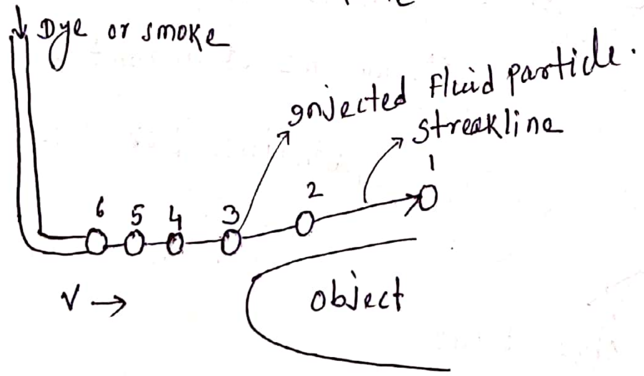


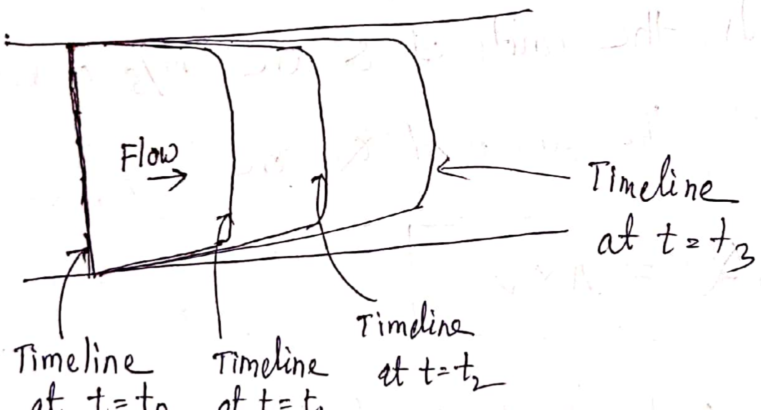
Figure shows a tracer being injected into a free stream flow containing an object. The circles represent individual injected tracer fluid particles, released at a uniform time interval. As the particles are forced out of the way by the object, they accelerate around the shoulder of the object, as indicated by the increased distance between individual tracer particle in that region. The streakline is formed by connecting all the circles into a smooth curve.

* In steady flow, stream line, path line, streakline are identical.

Time line

→ Time line is the line formed by joining number of adjacent fluid particles in the flow field at a given instant of time.

→ Time lines are formed by marking a line of fluid particles and then watching that line move (and deform) through the flow field.



This figure shows time lines in a channel flow between two parallel walls. Because of friction at the walls, the fluid velocity is zero (no-slip condition). In regions of the flow away from the walls, the marked fluid particles move at the local fluid velocity, deforming the time line.

Rate of Flow or Discharge (Q)

→ It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.

→ For an incompressible fluid (or liquid), the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.

→ For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

(i) For liquids, the units of Q are m^3/s or litre/s

(ii) For gases, the units of Q are kgf/s or Newton/s

$$\text{Discharge, } Q = A \times V$$

A = cross-sectional Area of pipe

V = Average velocity of fluid across the section.

Continuity equation for one-dimensional Flow

→ Continuity equation is based on the principle of conservation of mass.

→ According to the conservation of mass, the fluid matter can neither be created nor can be destroyed. Thus For a Fluid, Flowing through the pipe at all the cross-section, the quantity of fluid per Second is constant.

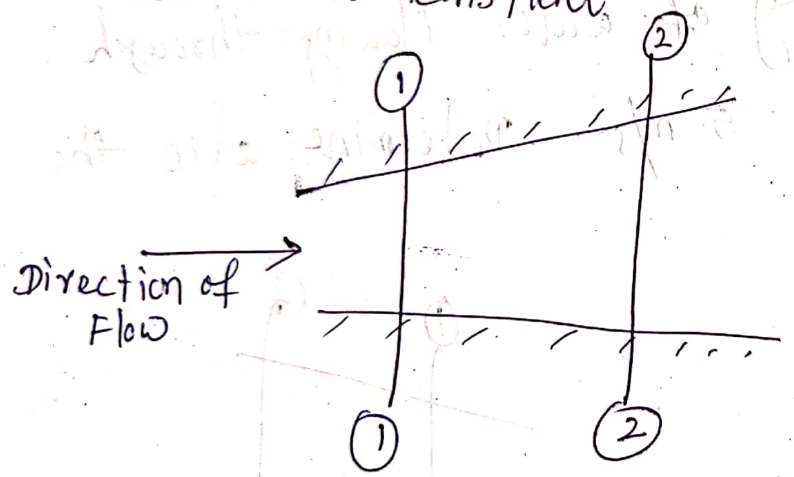


Fig:- Fluid Flowing through a pipe

Let V_1 = Average velocity at cross-section 1-1

ρ_1 = Density at section 1-1

A_1 = Area of cross-section of Pipe at 1-1.

Let V_2, ρ_2, A_2 are the corresponding values at section 2-2

Then according to conservation of mass theory. mass flow rate at section ①-① & section ②-② are equal.

$\therefore \dot{m}_1 = \dot{m}_2$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

This eqn is applicable for both compressible and incompressible Fluids and is called continuity equation.

→ If the Fluid is incompressible ($\rho_1 = \rho_2$)

$$\Rightarrow A_1 V_1 = A_2 V_2$$

⇒ This equation shows that if any section, the flow area decreases, the velocity must increase.

Q-1

The diameters of a pipe at the section 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.

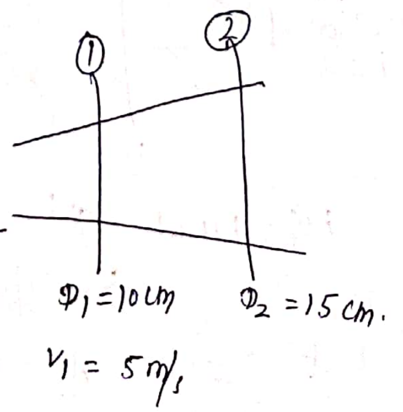
Solution:

At section-1

$D_1 = 10\text{ cm} = 0.1\text{ m}$

$A_1 = \frac{\pi}{4}(D_1)^2 = \frac{\pi}{4} \times (0.1)^2 = 0.007854\text{ m}^2$

$V_1 = 5\text{ m/s}$



At section-2

$D_2 = 15\text{ cm} = 0.15\text{ m}$

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$A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767 \ m^2$

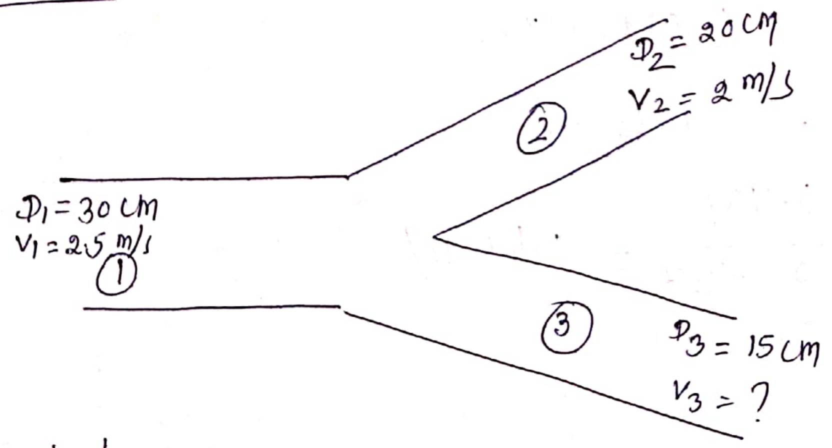
Discharge, $Q = A_1 V_1 = 0.007854 \times 5 = 0.03927 \ m^3/s$

we know $A_1 V_1 = A_2 V_2$

$\Rightarrow V_2 = \frac{A_1 V_1}{A_2} = \frac{0.007854 \times 5}{0.01767} = 2.22 \ m/s$

Q-2 A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.

Solution



(i) Find Q_1

(ii) Find V_3.

section 1

$D_1 = 30 \ cm = 0.3 m$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.07068 \ m^2$

$V_1 = 2.5 \ m/s$

Section-2

$D_2 = 20\ cm = 0.2\ m$

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314\ m^2$

$V_2 = 2\ m/s$

Section-3

$D_3 = 15\ cm = 0.15\ m$

$A_3 = \frac{\pi}{4} D_3^2 = \frac{\pi}{4} \times (0.15)^2 = 0.01767\ m^2$

$Q_1 = A_1 V_1 = 0.07068 \times 2.5 = 0.1767\ m^3/sec$ (Answer)

$Q_2 = A_2 V_2 = 0.0314 \times 2 = 0.0628\ m^3/s$

$Q_1 = Q_2 + Q_3$

$\Rightarrow Q_3 = Q_1 - Q_2$

$Q_3 = 0.1767 - 0.0628 = 0.1139\ m^3/s$

$Q_3 = A_3 V_3 = 0.01767 \times V_3$

$\Rightarrow 0.1139 = 0.01767 \times V_3$

$\Rightarrow V_3 = \frac{0.1139}{0.01767} = 6.44\ m/s.$

In Kinematics we have studied the velocity and acceleration at a point in a Fluid Flow without taking into consideration of the forces causing the Flow. But In dynamics we will study the fluid motion with the forces causing the Flow.

Equations of Motions

According to Newton's second law of motion, the net Force (F_x) acting on a Fluid element in the direction of x is equal to mass 'm' of the fluid element multiplied by the acceleration (a_x) in the x-direction.

mathematically

$$F_x = ma_x$$

In the fluid Flow following forces are present

(i) F_g → gravity Force
(ii) F_p → Pressure Force
(iii) F_v → Force due to viscosity
(iv) F_t → Force due to turbulence
(v) F_c → Force due to compressibility

$$\therefore \boxed{F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x}$$

If the Force due to compressibility, turbulence and viscosity are negligible, then the resulting net Force

$$\boxed{F_x = F_g + F_p}$$

This is known as Euler's equation of motion.

Bernoulli's Theorem

Statement

It states that for a steady, ideal Flow of an incompressible Fluid, the sum of pressure energy, Kinetic energy and potential energy per unit mass at every section of the flow remains constant.

mathematically

$$\boxed{\frac{P}{\rho} + \frac{V^2}{2} + gZ = \text{constant}}$$

$\frac{P}{\rho}$ = pressure energy per unit mass of the Fluid.

$\frac{V^2}{2}$ = Kinetic energy per unit mass of the Fluid.

gZ = Potential energy/Datum energy per unit mass of the fluid.

we can also write

$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant}}$$

, where every term represent energy per unit weight of the fluid.

Assumption

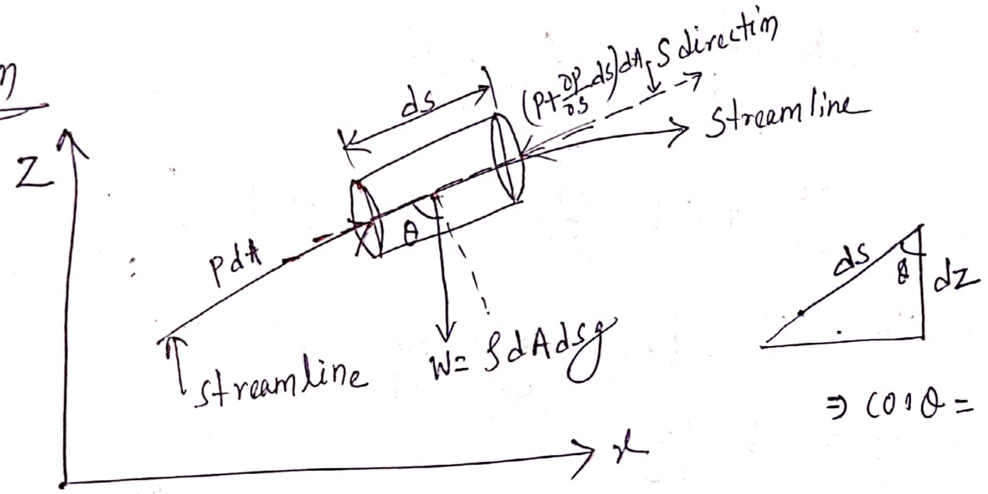
→ The Fluid is ideal (i.e. viscosity is zero)

→ The Flow is steady

→ The Flow is incompressible

→ The Flow is irrotational

Derivation



$\Rightarrow \cos\theta = \frac{dz}{ds}$.

consider a streamline in which flow is taking place in S-direction. Taking a cylindrical element having cross-section dA and length ds. The Force acting on the cylindrical element are

1. Pressure Force pdA in the direction of flow

2. pressure force $\left(p+\frac{\partial p}{\partial s}.ds\right)$ opposite to the direction of flow.

3. weight of element $= mg = \rho vg = \rho dA ds g$

Let θ is the angle between the direction of flow and the line of action of the weight of the element

The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element x acceleration in the direction S.

$\therefore \; pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho\, dA\, ds\, g \cos\theta = \rho\, dA\, ds \times a_s$ —— (1)

where a_s = acceleration in the direction of s.

$a_s = \frac{dv}{dt}$, where v is a Function of s, t

$= \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\partial t} \cdot \frac{dt}{dt}$

$= \frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial s} \quad \left(\because \frac{ds}{dt} = v\right)$

when the flow is steady $\frac{\partial v}{\partial t} = 0$.

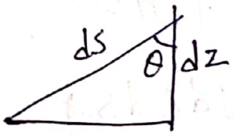
$\therefore \; a_s = v \frac{\partial v}{\partial s}$.

substituting the value of a_s in eq^n (1).

$p\,dA - p\,dA - \frac{\partial p}{\partial s} ds\,dA - \rho\, dA\, ds\, g \cos\theta = \rho\, dA\, ds \cdot v \frac{\partial v}{\partial s}$

Taking all the terms of left side into right side and dividing each term by $\rho\, dA\, ds$

$\frac{1}{\rho} \frac{\partial p}{\partial s} + g \cos\theta + v \frac{\partial v}{\partial s} = 0$



$\cos\theta = \frac{dz}{ds}$

$\therefore \; \frac{1}{\rho} \frac{\partial p}{\partial s} + g \frac{dz}{ds} + v \frac{\partial v}{\partial s} = 0$

$\therefore \; \boxed{\frac{\partial p}{\rho} + g\,dz + v\,\partial v = 0}$ → Euler's equation of motion.

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Bernoulli's equation is obtained by integrating Euler's equation of motion.

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = ~~constant~~ \int derivative\ (constant)$$

$$\Rightarrow \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{Constant}$$

$$\Rightarrow \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\Rightarrow \boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}} \Rightarrow \text{Bernoulli's equation.}$$

Here $\frac{p}{\rho g}$ = pressure energy per unit weight of fluid (or) pressure head.

$\frac{v^2}{2g}$ = Kinetic energy per unit weight (or) Kinetic head.

z = potential energy per unit weight (or) potential head.

Q-3

water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section which is 5 m above the datum line.

Soln

Diameter of pipe, $D = 5\text{ cm} = 0.05\text{ m}$

Pressure, $p = 29.43\text{ N/cm}^2 = 29.43 \times 10^4\text{ N/m}^2$

velocity $v = 2.0\text{ m/s}$

Total head = pressure head + kinetic head + datum head.

Pressure head $= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30\text{ m}$.

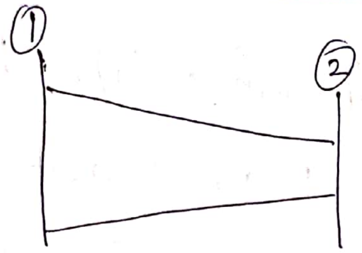
Kinetic head $= \frac{v^2}{2g} = \frac{2^2}{2 \times 9.81} = 0.204\text{ m}$

∴ Total head $= \frac{p}{\rho g} + \frac{v^2}{2g} + z$

$= 30 + 0.204 + 5 = 35.204\text{ m}$ (Ans)

Q-4 A pipe through which water is flowing, is having diameters, 20 cm and 10 cm at the cross section ① & ② respectively. The velocity of water at section 1 is given 4 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.

Solution



$D_1 = 20\,cm = 0.2\,m$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314\, m^2$

$V_1 = 4\, m/s$

$D_2 = 10\,cm = 0.1\,m$

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times 0.1^2 = 0.00785\, m^2$

(i) velocity head at section ①

$$= \frac{V_1^2}{2g} = \frac{4^2}{2 \times 9.81} = 0.815\, m$$

(ii) velocity head at section ②

$A_1 V_1 = A_2 V_2$

$$\Rightarrow V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314 \times 4}{0.00785} = 16\, m/s$$

$$\frac{v^2}{2g} = \frac{16^2}{2 \times 9.81} = 13.047\, m$$

(iii) Rate of discharge, $Q = A_1 V_1$ or $A_2 V_2$

$\therefore Q = A_1 V_1$

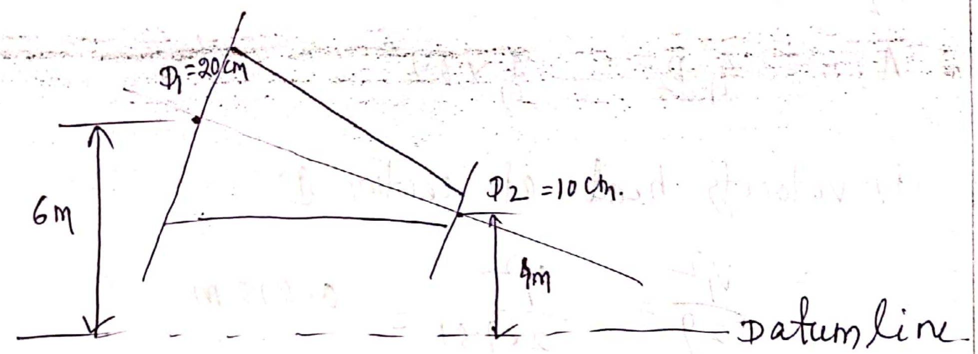
$= 0.0314 \times 4 = 0.1256\, m^3/s$

$= 0.1256 \times 1000$ litre/s

$= 125.6$ litre/s

Q-5 The water is flowing through a pipe having diameter 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litre/s. The section 1 is 6m above datum and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm², Find the intensity of pressure at section 2.

Soln



At section 1, $D_1 = 20\ cm = 0.2\ m$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314\ m^2$$

$$P_1 = 39.24\ N/cm^2 = 39.24 \times 10^4\ N/m^2$$

$$Z_1 = 6\ m.$$

At section 2, $D_2 = 10\ cm = 0.1\ m$

$$A_2 = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.1^2 = 0.00785\ m^2$$

$$Z_2 = 4m.$$

Rate of flow = 35 litre/s = $35 \times 10^{-3}\ m^3/s = 0.035\ m^3/s$

$$Q = A_1 V_1 = A_2 V_2$$

$$\Rightarrow V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114\ m/s$$

similarly $V_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456\ m/s.$

Applying Bernoulli's equation at section 1 & 2, we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{(39.24 \times 10^4)}{1000 \times 9.81} + \frac{1.114^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{4.456^2}{2 \times 9.81} + 4$$

$$46.063 = \frac{P_2}{9810} + 5.012$$

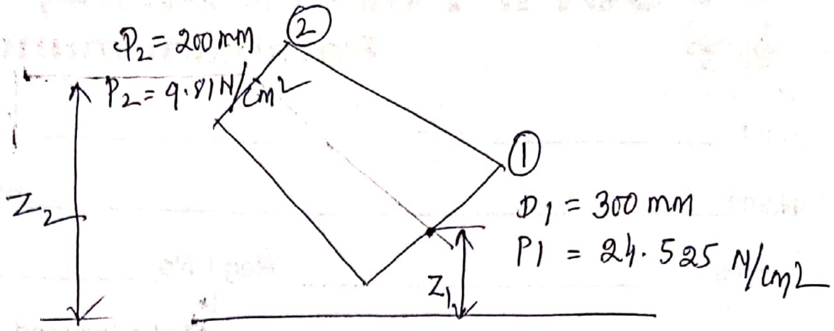
$$\therefore P_2 = 40.27 \times 10^4\ N/m^2 \quad \text{(Ans)}$$

Q-6 Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 40 litre/s.

Sol

$\underline{Sol^n}$

(ii)



$D_1 = 300$ mm $= 0.3$ m

$P_1 = 24.525$ N/cm² $= 24.525 \times 10^4$ N/m²

$D_2 = 200$ mm $= 0.2$ m

$P_2 = 9.81$ N/cm² $= 9.81 \times 10^4$ N/m²

Rate of flow, $Q = 40$ litre/s $= 40 \times 10^{-3}$ m³/sec $= 0.04$ m³/s

$A_1 V_1 = A_2 V_2 =$ rate of flow, $Q = 0.04$ m³/s

$$V_1 = \frac{Q}{A_1} = \frac{0.04}{\frac{\pi}{4} D_1^2} = \frac{0.04}{\frac{\pi}{4} \times 0.3^2} = 0.5658 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{\frac{\pi}{4} D_2^2} = \frac{0.04}{\frac{\pi}{4} \times 0.2^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's eqn at (1)&(2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{(24.525 \times 10^4)}{1000 \times 9.81} + \frac{0.5658^2}{2 \times 9.81} + Z_1 = \frac{(9.81 \times 10^4)}{1000 \times 9.81} + \frac{1.274^2}{2 \times 9.81} + Z_2$$

$25 + 0.32 + Z_1 = 10 + 1.623 + Z_2$

$\therefore Z_2 - Z_1 = 25.32 - 11.623 = 13.697 \approx 13.70$ m. (Ans)

Application of Bernoulli's Equation

Bernoulli's equations is applied in all problems of incompressible fluid flow where energy considerations are involved. Bernoulli's equation is applied to following measuring device.

① venturimeter
② orifice meter
③ pitot tube.

Venturimeter

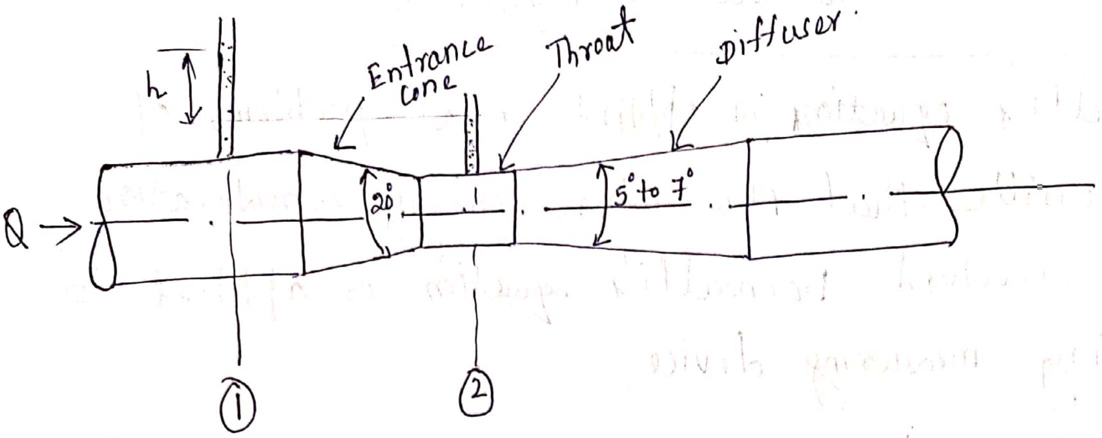
→ A venturimeter is a device for measuring rate of flow in a pipe line. It is based on the Bernoulli's principle, that is when the velocity head increases, there is a corresponding reduction in the piezometic head. i.e (pressure head + datum head).

→ venturimeter consists of 3 parts

(i) a short converging part, having entrance cone of angle 20°.
(ii) a cylindrical portion of short length, known as Throat
(iii) a diverging part, known as diffuser of cone angle 5° to 7°.

Let d_1 = diameter at inlet or at section ①
p_1 = pressure at section ①
v_1 = velocity of fluid at section ①
a_1 = area of section ① $= \frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are the corresponding values at section ②.



Applying Bernoulli's equation at section ① and ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As pipe is horizontal, $Z_1 = Z_2$

$$\therefore \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \text{——— (1)}$$

But $\left(\frac{P_1 - P_2}{\rho g}\right)$ is the difference of pressure head at section ① & ② and it is equal to h.

$$\therefore \frac{P_1 - P_2}{\rho g} = h \quad \text{——— (2)}$$

putting eqn ② in eqn ①

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \text{——— (3)}$$

Now applying continuity eqn at section ① & ②.

$$A_1V_1 = A_2V_2$$

$$\Rightarrow V_1 = \frac{A_2 \cdot V_2}{A_1} \quad \text{——— (4)}$$

Substituting eqn (4) in eqn (3).

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{A_2 V_2}{A_1}\right)^2}{2g} \Rightarrow \frac{V_2^2}{2g} - \frac{A_2^2 V_2^2}{A_1^2} \times \frac{1}{2g}$$

$$\Rightarrow h = \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$\Rightarrow h = \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$\therefore V_2^2 = 2gh\left[\frac{a_1^2}{a_1^2 - a_2^2}\right]$$

$$\Rightarrow V_2 = \sqrt{2gh} \times \sqrt{\frac{a_1^2}{a_1^2 - a_2^2}}$$

$$\therefore V_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh} \quad \text{——— (5)}$$

$\therefore Q = a_2 V_2$

$$\Rightarrow Q = a_2 \cdot \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh}$$

$$\Rightarrow Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{—— (6)}$$

This equation is the discharge under ideal condition and is called Theoritical discharge. Actual discharge is less than the theoritical discharge.

$$Q_{act.} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

where C_d = coefficient of discharge of venturimeter and it is always less than 1.

h = Difference in height of Piezometer between Section ① and ② in terms of flowing Fluid.

Note

→ Instead of Piezoometer, if U-tube differential manometer is inserted between section ① & section ②. 'h' is calculated by following formula.

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

→ when a manometer contains a liquid which is heavier than the liquid flowing through it.

x = Difference in height of manometric fluid in both the limbs.

S_h = specific gravity of heavier liquid

S_o = specific gravity of liquid flowing through the pipe.

→ If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$ → use either ratio of specific gravity or density)

S_l = ~~spd~~ specific gravity of lighter liquid.

→ Discharge does not depend upon the orientation of the venturimeter. It may be kept horizontal, inclined at an angle, or even vertical, but the discharge remains same.

Q.7 A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$

Sol:

$d_1 = 30 \text{ cm} = 0.3 \text{ m}$

Area of inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.07 \text{ m}^2$

$d_2 = 15 \text{ cm} = 0.15 \text{ m}$

Area of Throat, $a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.15)^2 = 0.0176 \text{ m}^2$

$C_d = 0.98$

Reading of differential mercury manometer $x = 20\,cm = 0.2\,m$

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

$$h = 0.2\left[\frac{13600}{1000} - 1\right] = 0.2 \times 12.6 = 2.52\,m \text{ of water}$$

Discharge, $Q = C_d \cdot \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

$$= 0.98 \times \frac{0.07 \times 0.0176}{\sqrt{0.07^2 - 0.0176^2}} \times \sqrt{2 \times 9.81 \times 2.52}$$

$$= 0.1253\ m^3/sec.$$

$$= 0.1253 \times 10^3 \text{ litre/s}$$

$Q = 125.3$ litre/s (solution).

Q-8
An oil of specific gravity 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. calculate the discharge of oil through the horizontal venturimeter. $C_d = 0.98$

Solution

$d_1 = 20\,cm = 0.2\,m$

$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314\ m^2$

$d_2 = 10\,cm = 0.1\,m$

$a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times 0.1^2 = 0.00785\ m^2$

$C_d = 0.98$

Reading of differential manometer, $x = 25\ cm = 0.25\ m$

sp. gravity of oil = 0.8

sp. gravity of mercury = 13.6

$\therefore h = x\left[\frac{S_h}{S_o} - 1\right] = 0.25\left[\frac{13.6}{0.8} - 1\right] = 4$ m of oil

$$Q = C_d.\ \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

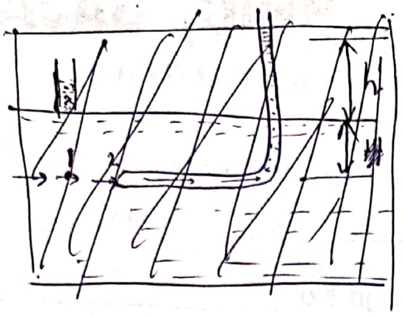
$$= 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{(0.0314)^2 - (0.00785)^2}} \times \sqrt{2 \times 9.81 \times 4}$$

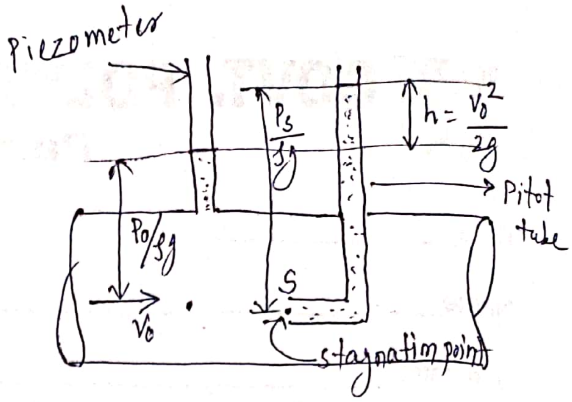
$= 0.07\ m^3/s$

$= 0.07 \times 10^3$ litre/s

= 70 litre/s.

Pitot tube





→ It is a device used for velocity measurement. It, infact, measures the stagnation pressure at any point flow.

→ The pitot-tube consists of a tube having a 90° bend of shorter length, opened at its both ends. The bent leg is directed upstream, so that a stagnation point (where velocity = 0) is created immediately ahead of its inlet.

→ It is based upon the principle that, if the velocity of flow at a point becomes zero, the pressure is increased due to conversion of kinetic energy into pressure energy. Thus the liquid rises up in the tube

→ A Piezometer installed on the pipe boundary indicates the static pressure. The difference betn stagnation pressure and static pressure is the dynamic pressure

Applying Bernoulli's eqn between 'o' & 's'

$$\frac{P_0}{\rho g} + z_0 + \frac{V_0^2}{2g} = \frac{P_s}{\rho g} + z_s + \frac{V_s^2}{2g}$$

As z_0 & z_s are in same level and $V_s = 0$

$$\frac{P_0}{\rho g} = \frac{P_s}{\rho g} + \frac{V_0^2}{2g} \qquad \frac{P_0}{\rho g} + \frac{V_0^2}{2g} = \frac{P_s}{\rho g}$$

$$\frac{P_s - P_0}{\rho g} = \frac{V_0^2}{2g}$$

$$V_0^2 = 2\left(\frac{P_s - P_0}{\rho}\right)$$

$$V_0 = \sqrt{2\left(\frac{P_s - P_0}{\rho}\right)}$$

$$\frac{P_s}{\rho g} - \frac{P_0}{\rho g} = h$$

$$\Rightarrow \frac{P_s - P_0}{\rho} = gh$$

$$\boxed{V_0 = \sqrt{2gh}}$$

This is theoritical velocity

Actual velocity / $\boxed{V_{act} = C_v \sqrt{2gh}}$

C_v = co-efficient of pitot tube.

Q-9 A pitot static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity of the pipe is 0.8 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60 mm of water. Take the co-efficient of pitot tube as $C_v = 0.98$

Soln

$d = 300$ mm $= 0.3$ m

$h = 60$ mm of water $= 0.06$ m of water.

$C_v = 0.98$

Mean velocity, $\bar{V} = 0.8 \times$ central velocity central velocity $= C_v\sqrt{2gh} = 0.98\sqrt{2\times9.81\times0.06}$

$= 1.063$ m/s

$\bar{V} = 0.80 \times 1.063 = 0.8504$ m/s

$Q = A\bar{V} = \frac{\pi}{4}d^2 \times \bar{V}$

$= \frac{\pi}{4}\times(0.3)^2\times0.8504 = 0.06$ m^3/s

Q-10 Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot-tube is 100 mm. Take co-efficient of pitot tube 0.98 and specific gravity of oil = 0.8.

Solution

$x = 100$ mm $= 0.1$ m

$h = x\left[\frac{S_M}{S_o} - 1\right] = 0.1\left[\frac{13.6}{0.8} - 1\right] = 1.6$ m of oil

velocity of flow $= C_v\sqrt{2gh} = 0.98\sqrt{2\times9.81\times1.6} = 5.49$ m/s

Q-11 A pitot static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6m and static pressure head is 5m. calculate velocity of flow assuming co-efficient of tube = 0.98.

Solution Stagnation pressure head, $h_s = 6$ m

static pressure head, $h_t = 5$ m

$\therefore h = h_s - h_t = 6 - 5 = 1$ m

$\therefore$ velocity of flow $= C_v\sqrt{2gh} = 0.98\sqrt{2\times9.81\times1} = 4.34$ m/s.

Limitation of Bernoulli's equation

→ It is applicable only to steady Flow.

→ Negligible viscous effects

→ Bernoulli's equation is not applicable in a flow section that involve pump, turbine, Fan or impeller since such devices disrupt the streamlines.

→ Applicable only for incompressible Flow.

→ Bernoulli's equations should not be used for flow sections that involve significant temperature + change such as heating and cooling section.

→ Bernoulli's equation is applicable along a streamline.

Orifice, notches, and weirs

→ Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which the fluid flows.

→ Orifices are used for the measurement of rate of flow of fluid.

→ The main features of orifice flow is that most of the potential energy of the fluid is converted into the kinetic energy of the free jet passed through the orifice.

Classification of orifices

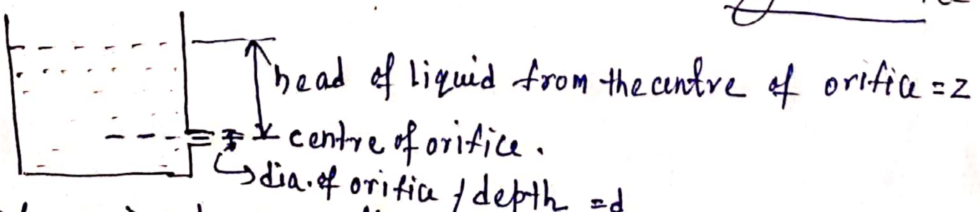
The orifices are classified on the basis of their size, shape, nature of discharge, shape of the upstream edge.

① According to size of orifice.

The orifice are classified as "small orifice" or "large orifice" depending upon the size of orifice and the head of liquid from the centre of orifice.

→ If the head of liquid from the centre of orifice is more than five times the depth of orifice, the orifice is called small orifice.

→ If the head of liquid is less than five times the depth of orifice, it is known as Large orifice.



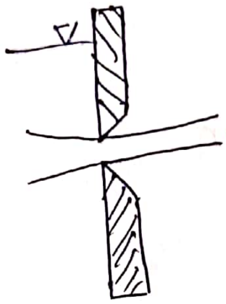
If $z > 5d$ ⇒ small orifice.
$z < 5d$ ⇒ Large orifice.

② According to the shape of orifice / cross-sectional Area of orifice.

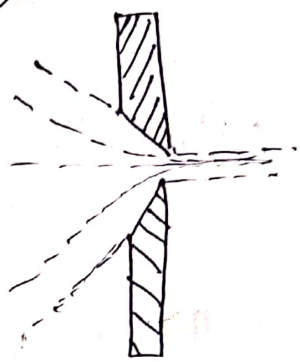
(i) circular orifice.
(ii) Triangular orifice
(iii) Rectangular orifice
(iv) square orifice.

③ According to the shape of upstream edge of orifice

(i) Sharp edged orifice
(ii) Bell mouthed orifice



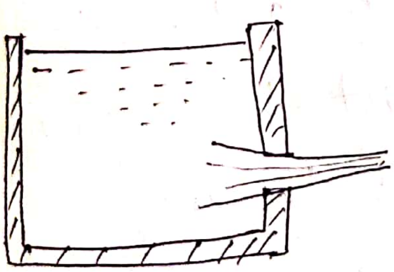
Sharp edged orifice.



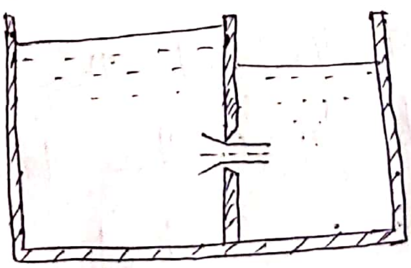
Bell mouthed orifice.

④ According to the nature of discharge.

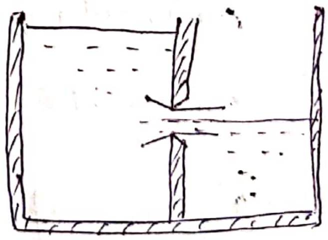
(i) free discharging orifices
(ii) Drowned / submerged orifice. — Fully submerged orifice / partially submerged orifice



free discharging orifice.

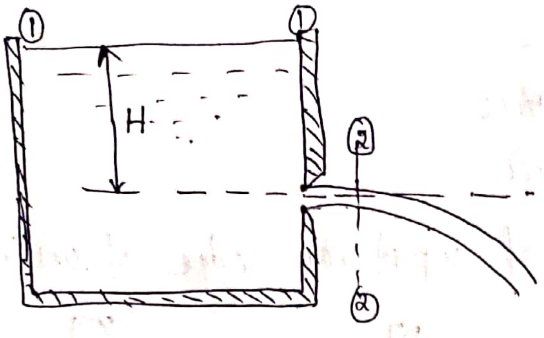


Fully submerged orifice.

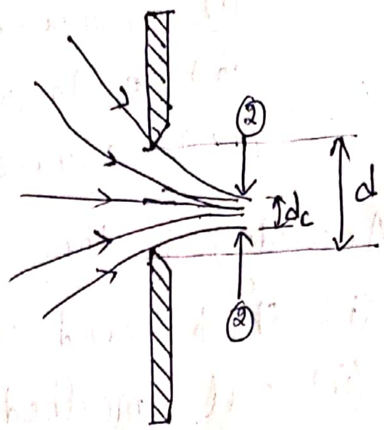


partially submerged orifice.

Flow through an orifice



a) Fluid through an orifice.



b) Flow pattern in the vicinity of the orifice.

consider a large tank filled with a liquid with a small orifice located in the wall at a large depth 'H' below the free surface. The liquid flows out through the orifice into the atmosphere. As the liquid approaches the orifice it tends to contract due to inability of streamlines to take a sharp turn and results in contraction of the jet. The fluid jet tends to contract owing to inability of the streamlines to take a sharp turn at the opening. The contraction of the jet is limited to a distance of about one half to one diameter downstream from the opening.

The cross-section where the contraction is greatest or where the area of the jet is minimum is called Vena-contracta. At this section, the streamlines are straight and parallel to each other and perpendicular to the plane of orifice and the pressure is atmospheric. Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.

The head on the orifice 'H' is measured from the centre of orifice to the free surface. Assuming the head on the orifice to be held constant, the Bernoulli's eqn is applied betn section (1)-(1) ~~section (2)(2)~~ on the free surface and the centre of vena contracta (2)-(2), neglecting losses

$$H + 0 + 0 = 0 + 0 + \frac{V^2}{2g}$$

(Here datum is taken as the horizontal plane passing through the orifice centre)

$$\boxed{V = \sqrt{2gH}}$$

This is Theoritical velocity. because the loses between the two sections were neglected. Actual velocity will be less than this value.

Hydraulic co-efficients

The hydraulic coefficients are
① co-efficient of velocity (C_v)
② co-efficient of contraction (C_c)
(3) co-efficient of discharge (C_d)

① coefficient of velocity (C_v)

→ It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and theoritical velocity of jet.

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoritical velocity}}$$

$$= \frac{V_{act.}}{V_{th}} = \frac{V_{act.}}{\sqrt{2gH}}$$

→ Because of loss of energy in friction, the actual velocity is less than theoritical velocity.
The value of C_v varies from 0.95 to 0.99 for different orifices, depending on shape, size of orifice and on the head under which flow takes place.
→ Generally the value of $C_v = 0.98$ is taken for sharp edged orifice.

② co-efficient of contraction. (C_c)

It is defined as the ratio between area of the jet at vena-contracta to the area of the orifice. It is denoted as (C_c).

a = area of orifice
a_c = area of jet at vena contracta.

$$C_c = \frac{\text{area of jet at vena contracta}}{\text{area of orifice}}$$

$$= \frac{a_c}{a}$$

→ The value of C_c varies from 0.61 to 0.69 depending on shape, size and head of liquid under which the flow takes place.
→ In general the value of C_c is taken as 0.64.

③ **co-efficient of discharge (C_d)**

→ It is defined as the ratio of the actual discharge from an orifice to the theoritical discharge from the orifice. It is denoted by (C_d).

Q = actual discharge.
Q_{th} = Theoritial discharge

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual Area} \times \text{Actual velocity}}{\text{Theoritical Area} \times \text{Theoritical velocity}}$$

$$= \frac{\text{Actual Area}}{\text{Theoritical Area}} \times \frac{\text{Actual velocity}}{\text{Theoritical velocity}}$$

$$\boxed{C_d = C_c \times C_v}$$

→ The value of C_d varies between 0.61 to 0.65. but generally the value of C_d is taken as 0.62.

Q-1 The head of water over an orifice of diameter 40 mm is 10m. Find the actual discharge and actual velocity of the jet at vena-contracta. $C_d = 0.6$, $C_v = 0.98$

Sol^n

$H = 10m$

$d = 40mm = 0.04m$

$\therefore$ Area $= \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.04)^2 = 1.256 \times 10^{-3} m^2 = 0.001256 m^2$

$C_d = 0.6$

$C_v = 0.98$

(i) Given $C_d = 0.6$

$\Rightarrow \frac{\text{Actual discharge}}{\text{Theoritical discharge}} = 0.6$

but Theoritical discharge = Area of orifice × $V_{theoritical}$

$= 0.001256 \times \sqrt{2gH}$

$= 0.001256 \times \sqrt{2 \times 9.81 \times 10}$

$= 0.01758 \ m^3/s$

$\therefore$ Actual discharge = 0.6 × Theoritical discharge.

$= 0.6 \times 0.01758$

$= 0.01054 \ m^3/s$

(ii) Given $C_v = 0.98$

$\frac{\text{Actual velocity}}{\text{Theoritical velocity}} = 0.98$

Actual velocity $= 0.98 \times V_{th}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 10}$

$= 0.98 \times 14 = 13.72 \ m/s$

Q.2 The head of water over the centre of an orifice of diameter 20 mm is 1m. The actual discharge through the orifice is 0.85 litre/s. Find the coefficient of discharge.

Soln

$d = 20\ mm = 0.02\ m$

Area $= \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.02)^2 = 0.000314\ m^2$

Head $= H = 1\ m$

Actual discharge $= 0.85$ litre/sec.

we know 1 litre $= 10^{-3}\ m^3/sec.$

$\therefore 0.85$ litre/sec $= 0.85 \times 10^{-3}\ m^3/s = 0.00085\ m^3/s$

$\therefore Q_{act} = 0.85 \times 10^{-3}\ m^3/s = 0.00085\ m^3/s$

Theoritical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1} = 4.429\ m/s$

Theoritical discharge = Area of orifice $\times V_{th}$

$= 0.000314 \times 4.429 = 0.00139\ m^3/s$

$\therefore$ coefficient of discharge $= C_d = \frac{Q_{Act}}{Q_{th}} = \frac{0.00085}{0.00139} = 0.61$

Notches and weirs

Notch: A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It is defined as an opening provided in one side of a tank or a reservoir, with upstream liquid level below the top edge of the opening.

→ A notch may have only bottom edge and sides.

→ A notch is usually made up of metallic plate.

Weir :

→ A weir is a concrete or masonary structure, placed in an open channel over which the flow occures. It is simply an obstruction in the channel that causes the liquid to rise behind the weir and then flows over it. By measuring the height of upstream liquid surface, the rate of flow is determined.

Nappe or vein

The jet of liquid or water flowing through a notch or over a weir is called nappe or vein.

Crest or sill

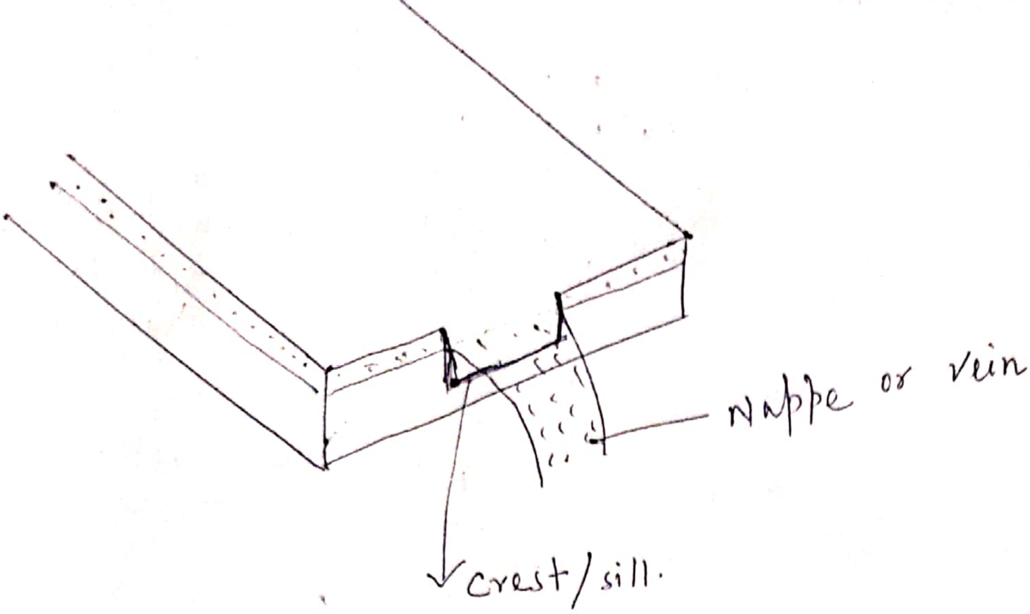
The bottom edge of a notch or a top of a weir over which water flows is known as sill or crest.

Classification of Notches

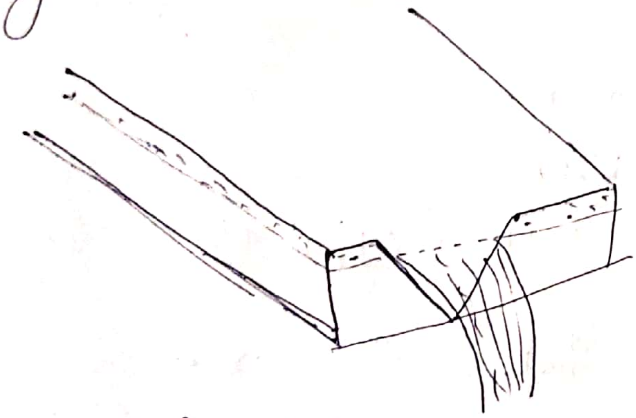
① According to the shape of the opening

a) Rectangular Notch

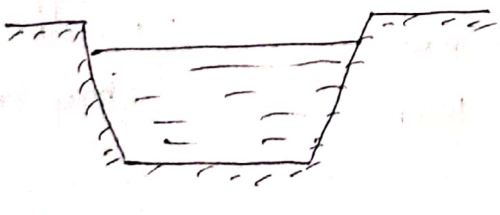
↳ Notch cross-section is rectangular in shape.



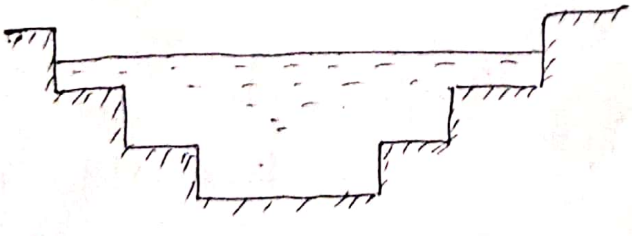
b) Triangular Notch. (Notch cross-section is Triangular)



(c) Trapezoidal Notch.



(d) Stepped Notch

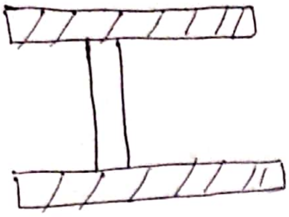


(2) According to the effect of the sides on the nappe.

a) Notch without end contraction.

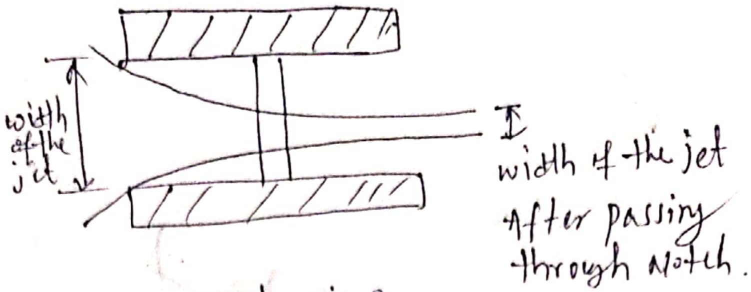
b) Notch with end contraction.

↳ width of the jet contracts as it is passes through Notches.



Top view

without end contraction



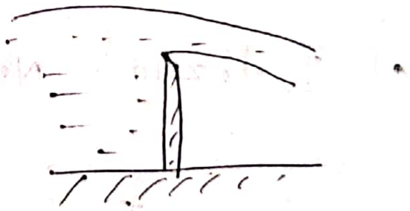
Top view.

with end contraction

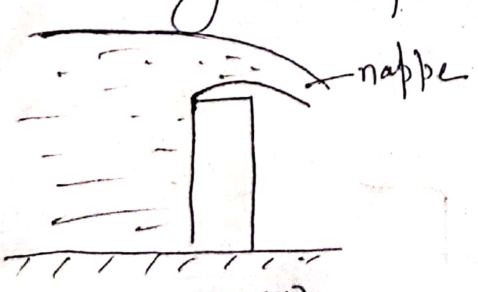
Classification of weirs

① According to the shape of the opening
  a) Rectangular weir
  b) Triangular weir
  c) Trapezoidal weir (cippoletti weir)

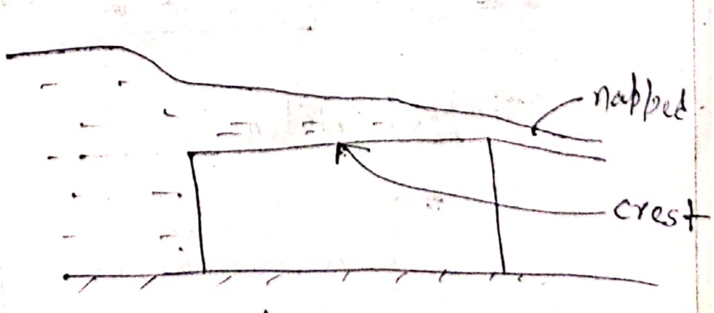
② According to the shape of the crest
  (i) Sharp crested weir
  (ii) Broad crested weir
  (iii) Narrow-crested weir
  (iv) ogee-shaped weir.



Sharp crested weir



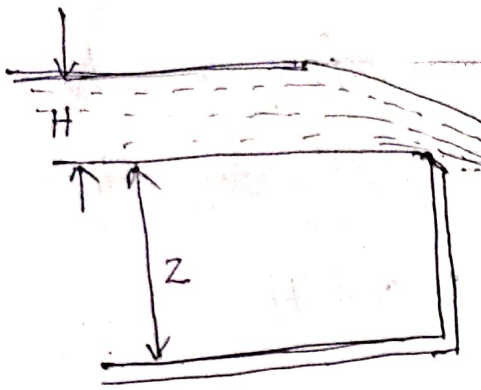
~~Narrow~~ narrow ~~Sharp~~ crested weir.



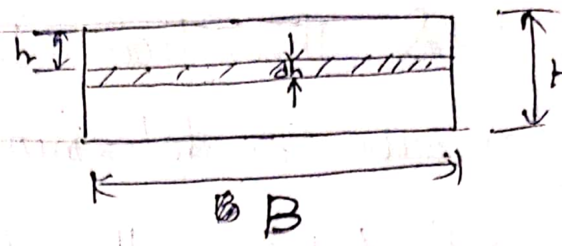
Broad crested weir.

③ According to the effect of sides on the emerging nappe
  a) weir without end contraction
  b) weir with end contraction.

Discharge over a Rectangular notch / weir



Flow over sharp crested weir.



Cross-section of nappe at crest

Consider a rectangular Notch or weir provided in a channel carrying water as shown in the figure.

H = Head of water over crest.

B = width of the notch / weir

The rate of the flow is determined by measuring the head 'H' over the weir crest, at a distance upstream at least four times the maximum head to be used.

For Finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and ~~Length L~~ width B at a depth 'h' from the free surface of water.

The area of the strip = Bdh

Theoritical velocity of water flowing through the strip = $\sqrt{2gh}$

The discharge dQ, through strip is

$(dQ)_{theoritical}$ = Area of strip × Theoritical velocity

$= Bdh \times \sqrt{2gh}$

∴ The total theoritical discharge for the whole notch or weir is determined by integrating the above eqn between the limit 0 and H.

$$Q_{th} = \int_0^H B\,dh \times \sqrt{2gh}$$

$$= B\sqrt{2g}\int_0^H \sqrt{h}\,dh$$

$$= B\sqrt{2g}\left[\frac{h^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)}\right]_0^H$$

$$= B\sqrt{2g}\left[\frac{H^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$\boxed{Q_{th} = \frac{2}{3}B\sqrt{2g}\,H^{\frac{3}{2}}}$$

~~But~~. But the actual discharge will be decreased slightly by fluid friction.

$\therefore Q_{act.} = C_d \times Q_{th}$ C_d = coefficient of discharge

$$\boxed{Q_{act.} = \frac{2}{3} C_d \cdot B \sqrt{2g}\, H^{3/2}}$$

↳ Expression for discharge over a rectangular weir or notch is same.

Q-3

Find the discharge of water flowing over a rectangular Notch of 2 m length when the constant head over the notch is 300 mm. Take $C_d = 0.6$

Soln

$H = 300\ mm = 0.3\ m$

$C_d = 0.6$

$B = 2\ m$

$Q_{act.} = \frac{2}{3} \times C_d \times B \sqrt{2g}\ H^{3/2}$

$= \frac{2}{3} \times 0.6 \times 2 \times \sqrt{2 \times 9.81} \times (0.3)^{3/2}$

$= 0.582\ m^3/s$

Q-4

The head of water over a rectangular notch is 900 mm. The discharge is 300 litre/s. Find the length of the notch. ; $C_d = 0.62$

Solution

$H = 900\ mm = 0.9\ m$

$Q_{act} = 300\ litre/s = 300 \times 10^{-3}\ m^3/s = 0.3\ m^3/s$

$C_d = 0.62$

Length of the noth = width of the notch = B = ?

$$Q = \frac{2}{3} C_d B \sqrt{2g} \times H^{3/2}$$

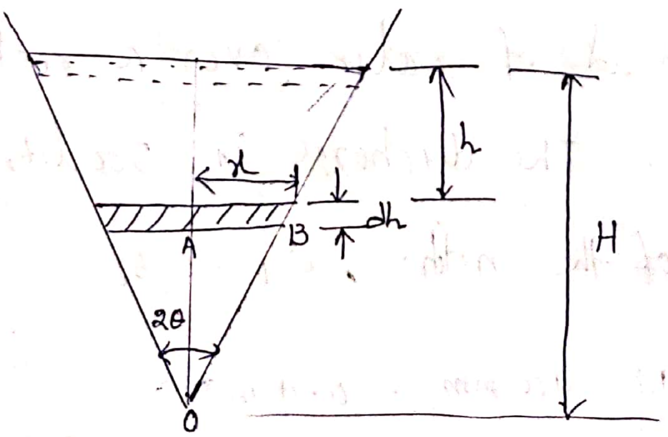
$$0.3 = \frac{2}{3} \times 0.62 \times B \sqrt{2 \times 9.81} \times (0.9)^{3/2}$$

$\Rightarrow B = 0.192\ m = 192\ mm$

Discharge over a tringular Notch / weir

→ The Triangular weir is particularly useful where the discharge varies over a large range and desired accuracy level is high for both small and large discharge.

→ For measuring small discharges accurately and to avoid Surface tension effects which are associated with flow at low heads and to obtain higher measurable heads, a triangular weir is prefered over rectangular weir.



Triangular weir / V-notched weir

H = Head of water above V-notch

2θ = vertex angle of V-notch

consider a horizontal strip of water of thickness 'dh' at a depth 'h' from the free surface of water as shown

$$\tan\theta = \frac{AB}{OA} = \frac{x}{H-h}$$

$$\Rightarrow x = (H-h)\tan\theta$$

Area of strip $= 2x \cdot dh = 2(H-h)\tan\theta \cdot dh$

Theoritical velocity of water through strip $= \sqrt{2gh}$

$\therefore$ Discharge, dQ, through the strip is

$$dQ_{\text{theoritical}} = \text{Area of strip} \times \text{Theoritical velocity}$$

$$= 2(H-h)\tan\theta \cdot dh \times \sqrt{2gh}$$

$\therefore$ The total discharge flowing through the V-notched weir may be obtained by integrating dQ betn the limits $h=0$ and $h=H$

$\therefore$ Total discharge $Q = \int_0^H 2(H-h)\tan\theta \cdot dh \cdot \sqrt{2gh}$

$= 2\tan\theta\sqrt{2g}\int_0^H (H-h)\cdot\sqrt{h}\; dh$

$= 2\tan\theta\sqrt{2g}\int_0^H \left(Hh^{\frac{1}{2}} - h^{\frac{3}{2}}\right) dh$

$= 2\tan\theta\sqrt{2g}\left[\frac{Hh^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{h^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}\right]_0^H$

$= 2\tan\theta\sqrt{2g}\left[\frac{2}{3}\cdot H\cdot H^{\frac{3}{2}} - \frac{2}{5}H^{\frac{5}{2}}\right]$

$= 2\tan\theta\sqrt{2g}\left[\frac{2}{3}H^{\frac{5}{2}} - \frac{2}{5}H^{\frac{5}{2}}\right]$

$= 2\tan\theta\sqrt{2g}\left[\frac{4}{15}H^{\frac{5}{2}}\right]$

$Q_{th} = \frac{8}{15}\tan\theta\sqrt{2g}\,H^{\frac{5}{2}}$

$\Rightarrow Q_{Actual} = C_d \times Q_{theoritical}$

$\therefore$ $Q_{Actual} = \frac{8}{15}\times C_d \tan\theta\sqrt{2g}\,H^{\frac{5}{2}}$

$\therefore$ For rectangular notch $Q \propto H^{\frac{3}{2}}$

but for Triangular notch $Q \propto H^{\frac{5}{2}}$

Scanned with CamScanner

Q-5 Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. Assume $C_d = 0.6$

solution

$2\theta = 60° \Rightarrow \theta = 30°$

$H = 0.3$ m

$C_d = 0.6$

$$Q = \frac{8}{15} \times C_d \times \tan\theta \sqrt{2g} \; H^{\frac{5}{2}}$$

$$= \frac{8}{15} \times 0.6 \times \tan 30 \sqrt{2 \times 9.81} \times (0.3)^{\frac{5}{2}}$$

$$= 0.04 \; m^3/s$$

- Difference between notch & weir

Notch	Weir
→ It is an opening provided in one side of a tank or reservoir with free surface of liquid present below the top edge of the opening.	→ It is a structure which obstructs the flow in an open channel.
→ notch is normally made of a metallic plate	→ weir is made of concrete or masonry
→ notches are smaller size than weir.	→ weirs are comparatively of larger size
→ It is used to measure discharge in small channels.	→ It is used to measure discharge in large bodies like rivers.

Flow through pipes

→ The term pipe, duct and conduit are usually used interchangeably for flow sections. In general, Flow sections of circular cross-section are refered to as pipes (especially when the fluid is a liquid) and Flow sections of non-circular cross-section as ducts (especially when the fluid is a gas). small diameter pipes are usually referd to as tubes.

→ most Fluids, especially liquids, are transported in circular pipes. This is because pipes with circular cross-section can withstand large pressure differences between the inside and outside without undergoing significant ~~distor~~ distortion.

→ So a pipe is a colosed conduit which is used for carrying fluids under pressure. As the pipe carry Fluids under pressure, they always run full.

→ The Fluid Flowing in a pipe is always subjected to resistance due to shear ~~stress~~ Forces between the fluid particles and the boundry walls of the pipe & the fluid particles themselves (This is due to viscosity)

The resistance to Flow of fluid due to above reason is known as frictional resistance. Because of this there always occures some loss of energy in the direction of flow. This loss depends upon whether the flow is laminar or turbulent.

→ When the Reynold number is less than 2000, for pipe flow, the flow is known as laminar flow whereas when the Reynold number is more than 4000, the flow is known as Turbulent flow.

→ In this chapter, the turbulent flow of fluids through pipes running full will be considered. If the pipes are partially full, as in case of sewer lines, the pressure inside the pipe is same and aqual to atmosphere. Then the flow of fluid in the pipe is not under pressure. This case will be taken in the chapter of flow of water through open channel (Not in the syllabus). Here we will consider flow of fluids through pipes under pressure only.

Loss of energy in pipes

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of the fluid is lost. This loss of energy is classified as.

Energy losses

Major energy losses

This is due to Friction & it is calculated by the following

a) Parcy-weisbach Formula
b) chezy's Formula.

Minor energy losses

This is due to

a) Sudden expansion of pipe
b) Sudden contraction of pipe
(c) Bend in pipe
(d) Pipe Fittings etc.
(e) An obstruction in pipe

Loss of energy (or head) due to Friction

a) Darcy - Weisbach Formula :

$$h_f = \frac{4 f L V^2}{2gd}$$

where h_f = loss of head due to Friction.

f = coefficient of friction.

L = Length of pipe

V = mean velocity of flow

d = diameter of pipe.

$f = \frac{16}{Re}$ for $Re < 2000$ → If flow is laminar.

$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

↳ Reynold's Number.

$f = \frac{0.079}{Re^{1/4}}$ If $Re > 4000$ → If the flow is Turbulent.

b) Chezy's Formula

$$V = C\sqrt{mi}$$

C = Chezy's constant $= \sqrt{\frac{\rho g}{f'}}$ → f' = Frictional resistance per unit wetted area per unit velocity

m = hydraulic mean depth or hydraulic radius.

$= \frac{A}{P} = \frac{\text{Area of Flow / Area of cross-section of pipe}}{\text{wetted perimeter of pipe}}$

hydraulic mean depth for circular pipe, $m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$

i = loss of head per unit length of pipe

$= \frac{h_f}{L}$ where h_f = Loss of head due to Friction.
L = Length of pipe.

Q-1 Find the head lost due to Friction in a pipe of diameter 300 mm and length 50 m through which water is flowing at a velocity of 3 m/s using (i) Darcy Formula (ii) Chezy's Formula For which C = 60.

Take ν for water = 0.01 stoke

Solution

$d = 300\ mm = 0.3\ m$

$L = 50\ m$

$V = 3\ m/s$

$C = 60$

Kinematic viscosity, $\nu = 0.01$ stoke $= 0.01\ cm^2/s = 0.01 \times 10^{-4}\ m^2/s$

(i) Darcy Formula $h_f = \frac{4fLV^2}{2gd}$

but f = coefficient of friction which is a Function of Reynold's Number

$R_e = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{3 \times 0.3}{0.01 \times 10^{-4}} = 9 \times 10^5$

$\therefore R_e > 4000$ ⇒ Hence the Flow is turbulent

$\therefore f = \frac{0.079}{R_e^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256$

$\therefore$ head lost, $h_f = \dfrac{4fLv^2}{2gd}$

$= \dfrac{4 \times 0.00256 \times 50 \times 3^2}{2 \times 9.81 \times 0.3}$

$= 0.7828 \, m$

(ii) By chezy's Formula $V = C\sqrt{mi}$

$V = 3 \, m/s$

$C = 60$ (given)

$m = \dfrac{d}{4}$ (For circular pipe

$= \dfrac{0.3}{4} = 0.075 \, m$

$\therefore \quad V = C\sqrt{mi}$

$3 = 60\sqrt{0.075 \times i}$

$\Rightarrow i = 0.0333$

But we know $i = \dfrac{h_f}{L} \Rightarrow h_f = i \times L = 0.0333 \times 50$

$= 1.665 \, m.$

Q-2 Find the diameter of a pipe of Length 2000 m, when the rate of flow of water through the pipe is 200 litres/s and the head lost due to Friction is 4m. $C = 50$

Solution

$L = 2000 \, m$

$Q = 200 \, lit/s = 200 \times 10^{-3} \, m^3/s = 0.2 \, m^3/s$

$h_f = 4m$

$C = 50$

$d = ?$

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.2}{\frac{\pi}{4}d^2} = \frac{0.2 \times 4}{\pi d^2}$$

$$m = \frac{d}{4}$$

$$i = \frac{h_f}{L} = \frac{4}{2000} = 0.002$$

we know $V = C\sqrt{mi}$

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times 0.002}$$

$$\Rightarrow \frac{0.2 \times 4}{\pi \times 50} = d^2 \times \sqrt{\frac{d}{4} \times 0.002}$$

$$= \frac{d^5}{4} \times 0.002$$

$$\Rightarrow \left[\frac{0.2 \times 4}{\pi \times 50}\right]^2 \frac{\times 4}{\times 0.002} = d^5$$

$$\Rightarrow 0.05187 = d^5$$

$$\Rightarrow d = (0.05187)^{1/5} = 0.553 \text{ m} = \underline{553 \text{ mm}}.$$

Q-3

An oil of speific gravity 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 lit/s. Find the head lost due to Friction and power required to maintain the flow for a length of 1000 m. $\nu = 0.29$ stokes.

Solution

$S = 0.7 \Rightarrow \rho = 700 \text{ Kg/m}^3$

$d = 300 \text{mm} = 0.3 \text{ m}$

$Q = 500 \text{ lit/s} = 500 \times 10^{-3} \text{ m}^3/\text{s} = 0.5 \text{ m}^3/\text{s}$

$L = 1000$, $\nu = 0.29$ stokes $= 0.29 \times 10^{-4}\ m^2/s$

$$V = \frac{Q}{Area} = \frac{0.5}{\frac{\pi}{4} d^2} = \frac{0.5}{\frac{\pi}{4} \times 0.3^2} = 7.073\ m/s$$

Reynold Number, $Re = \frac{VD}{\nu} = \frac{7.073 \times 0.3}{0.29 \times 10^{-4}} = 7.316 \times 10^4$

As $Re > 4000$, the Flow is Turbulent

$$\therefore f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(7.316 \times 10^4)^{1/4}} = 0.0048$$

h_f = head lost due to Friction $= \frac{4fLv^2}{2gd}$

$$= \frac{4 \times 0.0048 \times 1000 \times 7.073^2}{2 \times 9.81 \times 0.3}$$

$$= \underline{163.18m}.$$

Power Required $= \frac{\rho g Q h_f}{1000}$ KW

$$= \frac{700 \times 9.81 \times 0.5 \times 163.18}{1000}$$

$$= \underline{560.28\ KW}$$

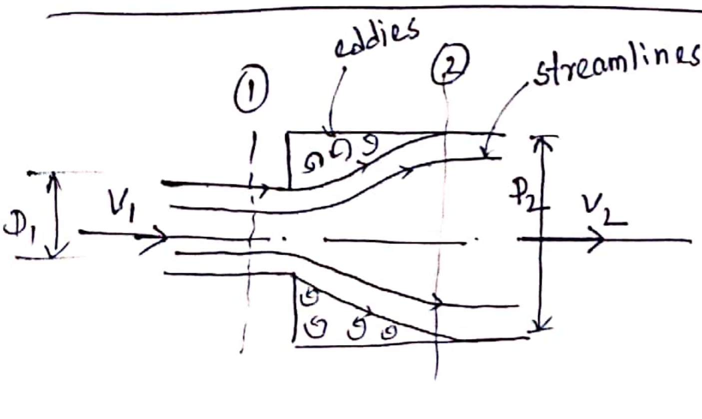
Minor Energy (Head) losses

The loss of head or energy due to friction in a pipe is known as major loss while loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy include

(1) Loss of head due to sudden enlargement.
(2) Loss of head due to sudden contraction.
(3) Loss of head at the entrance to a pipe.
(4) Loss of head at the exit of a pipe.
(5) Loss of head due to an obstruction in a pipe
(6) Loss of head due to bend in the pipe
(7) Loss of head in various pipe Fittings.

In case of long pipe, the above losses are small as compared to the loss of head due to Friction and hence they are called minor losses. But in case of a short pipe, these losses are comparable with the loss of head due to friction and Hence they can not be neglected.

(1) Loss of head due to sudden enlargement



consider a liquid flowing through two pipes of different sizes having Diameter D_1 & D_2 and causing sudden enlargement of pipe section from diameter 'D_1' to 'D_2'. The fluid flowing through the small pipe is unable to take sharp turn at the corner and follows the path shown by the streamlines. Due to the abrupt change of the boundry, the flow separates from the boundry and turbulent eddies are formed in the corners, thus results in dissipation of energy in the form of heat. The lost hydraulic energy is thus converted into the thermal energy (raising slightly the fluid temperature) which inturn lost to the surrounding medium.

There is a rise in pressure from section (1) to section (2) at the expense of velocity in accordance with the Bernoulli principle. The rise in pressure is not equal to the drop in velocity as some head loss occures due to sudden enlargement. Down stream of section (2), the outer streamlines merge with the pipe boundry. Section (2) is located downstream of Section (1) at a distance of about 8 times the larger diameter.

∴ Loss of head due to sudden enlargement, $h_e = \frac{(V_1 - V_2)^2}{2g}$

V_1 = velocity of flow at section (1)-(1)

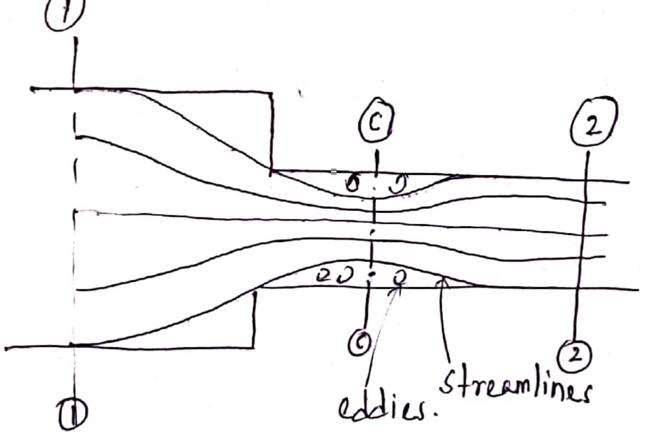
V_2 = velocity of flow at section (2-2)

From continuity equation $A_1 V_1 = A_2 V_2$

$$\therefore h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

$$\text{(or)}\ h_e = \frac{V_2^2}{2g}\left(\frac{A_2}{A_1} - 1\right)^2$$

② Loss of head due to sudden contraction



→ If the flow in the pipe system is reversed, the flow from the large pipe would enter the pipe with smaller diameter causing the flow Area to be suddenly reduced from A_1 to A_2.

→ As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at a section c-c. This section c-c is called vena-contracta. After section c-c a sudden enlargement of the area takes place.

→ Eddies are formed between the pipe wall and the vena contracta and it is these eddies which cause practically all the dissipation of energy. Between the vena-contracta and the downstream section (2), where the flow has again become practically uniform and the flow pattern is similar to that which occures at a sudden enlargement.

→ The loss of head due to sudden contraction is actually due to sudden enlargement from vena contracta to smaller pipe.

A_c = Area of flow at section - c-c

V_c = velocity of flow at section - c-c

A_2 = Area of flow at section 2-2

V_2 = velocity of flow at section 2-2

h_c = Loss of head due to sudden contraction.

$$h_c = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2$$

From continuity equation $A_c V_c = A_2 V_2 \Rightarrow \frac{V_c}{V_2} = \frac{A_2}{A_c}$

but co-efficient of contraction $C_c = \frac{A_c}{A_2}$

$$\therefore \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{C_c}$$

$$\therefore h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = K\left(\frac{V_2^2}{2g}\right) \quad \text{where } K = \left(\frac{1}{C_c} - 1\right)^2$$

If the value of C_c is not given, then $K = 0.5$ (assumed)

$$\therefore h_c = 0.5\left(\frac{V_2^2}{2g}\right)$$

Q-4 Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litre/s.

Solution

This is the case of "Loss of head due to sudden enlargement."

$D_1 = 200\ mm = 0.2\ m \Rightarrow A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.2)^2 = 0.03141\ m^2$

$D_2 = 400\ mm = 0.4\ m \Rightarrow A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564\ m^2$

$Q = 250\ litre/s = 250 \times 10^{-3}\ m^3/s = 0.25\ m^3/s$

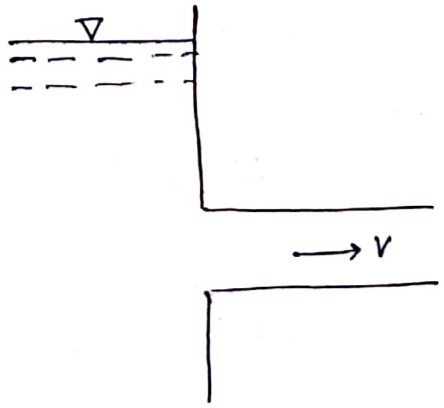
$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96\ m/s$

$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99\ m/s$

∴ Loss of head, due to sudden expansion,

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81} = 1.816\ m \text{ of water.}$$

③ Loss of head at the entrance of a pipe



→ This is the loss of energy which occures when a liquid enters a pipe which is connected to a large tank or reservoir.

→ This loss is similar to the loss of head due to sudden contraction.

→ This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance.

→ The value of loss of head at the entrance (inlet) of a pipe with sharp cornered entrance

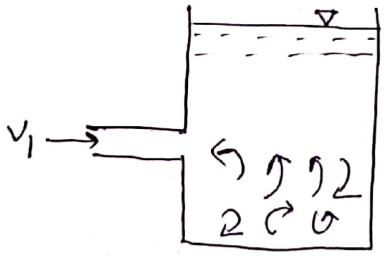
$$= 0.5 \frac{V^2}{2g}$$

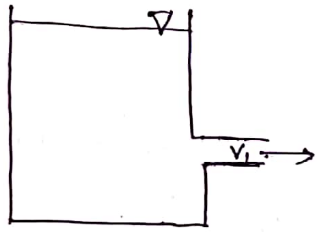
∴ This loss is denoted by h_i.

$$\boxed{h_i = 0.5 \frac{V^2}{2g}}$$

V = velocity of liquid in the pipe

(4) Loss of head at the exit of pipe / Exit losses





→ This is the loss of head (or energy) due to the velocity of liquid at the outlet of the pipe which is dissipated / disappeared in the form of free jet (if outlet of the pipe is free) or it is lost in the tank or reservoir (if the outlet of the pipe is connected to the tank or reservoir).

→ This loss is denoted by (h_o).

→ ~~ho~~ This case is similar to loss of head due to sudden enlargement.

$$h_o = \frac{(V_1 - V_2)^2}{2g} \quad \text{or} \quad \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

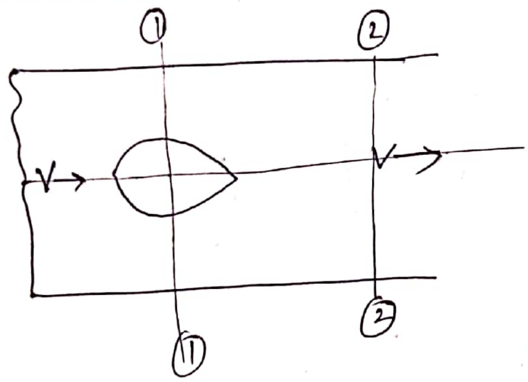
when a pipe discharges into large reservoir, $A_2 \to \infty$

$$\therefore h_o = \frac{V_1^2}{2g}\left[1 - \frac{A_1}{\infty}\right]^2$$

$$\therefore \boxed{h_o = \frac{V_1^2}{2g}}$$

V_1 = velocity of the jet at the exit of the pipe.

⑤ Loss of head due to an obstruction in a pipe



whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place.

a = maximum area of obstruction
A = Area of pipe
v = velocity of liquid in pipe.
(A-a) = Area of flow of liquid at section 1-1

→ As the liquid flows and passes through section 1-1, a vena-contracta is formed beyond section 1-1, After which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform and equal to velocity, v in the pipe.

→ This situation is similar to the flow of liquid through sudden enlargement.

V_c = velocity of liquid at vena-contracta

→ Loss of head due to ~~sudden~~ obstruction

= Loss of head due to enlargement from vena contracta to section-(2-2)

$$\Rightarrow h_{ob} = \frac{(V_c - V)^2}{2g}$$

(6) Loss of head due to bend in pipe

When there is any bend in pipe, the velocity of flow changes, due to which separation of the flow from the boundary and also formation of eddies takes place. Thus energy is lost. Loss of head in pipes due to bend is expressed as.

$$h_b = \frac{KV^2}{2g}$$

h_b = Loss of head due to bend.
V = velocity of flow
K = coefficient of bend.

The value of K depends on
(i) Angle of bend (ii) Radius of curvature of bend (iii) Diameter of pipe

⑦ Loss of head in various pipe Fittings

The loss of energy caused by various pipe fittings such as valves, elbows, bends and couplings etc occure because of their rough and irregular interior surfaces which produce excessive large scale turbulence.

$$\text{Loss of head} = \frac{KV^2}{2g}$$

K = co-efficient of pipe Fittings

V = mean velocity in the pipes.

Exp

Name of pipe Fitting	Loss co-efficient (K_L)
45° elbow	0.4
90° elbow (short radius)	0.9
(Med. Radius)	0.75
(long Radius)	0.6

Total Energy line (or) Energy grade line (or) Total head line.

→ It is defined as the line which is obtained by joining the tops of all vertical co-ordinates showing the sum of datum head, pressure head and velocity head from an arbitarily assumed horizontal datum.

→ It is also defined as the line which gives sum of pressure head, datum head and Kinetic head of a Flowing Fluid in a pipe with respect to some reference line.

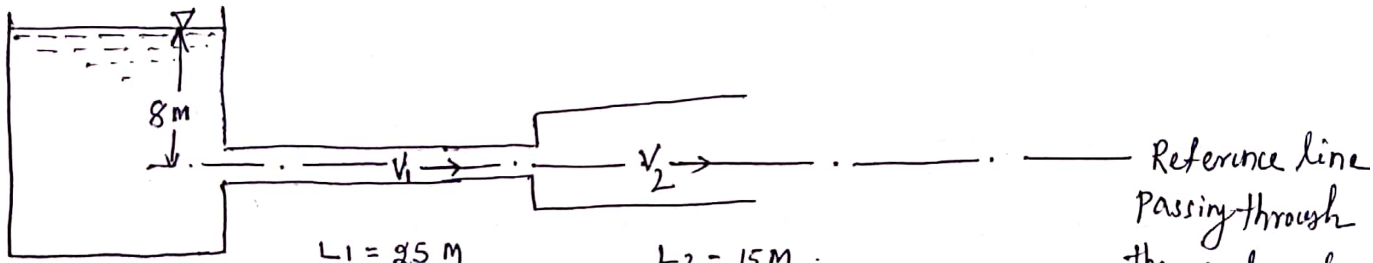
$$\therefore \quad \frac{P}{\rho g} + Z + \frac{V^2}{2g}$$

Datum head (Z); Pressure head ($\frac{P}{\rho g}$); velocity head ($\frac{V^2}{2g}$).

Hydraulic gradient line

→ It is the line which gives the sum of pressure head $\left(\frac{P}{\rho g}\right)$ and datum head (Z) of a flowing fluid in a pipe with respect to some reference line. (or) It is the line which is obtained by joining the top of all vertical ordinates showing the pressure head $\left(\frac{P}{\rho g}\right)$ and datum head (Z) from an arbitarily assumed horizonal datum.

Q



$f = 0.01$

$L_1 = 25$ m	$L_2 = 15$ m
$d_1 = 0.15$ m	$d_2 = 0.3$ m
$V_1 = 4.452$ m/s	$V_2 = 1.113$ m/s

Draw hydraulic gradient line and total gradient line.

Solution

The various head losses are

h_i = Loss of head at the entrance of a pipe 1 from large reservoir

$$= 0.5\frac{V_1^2}{2g} = 0.5 \times \frac{4.452^2}{2 \times 9.81} = 0.5 \text{ m}$$

Loss of head due to Friction in pipe 1 =

$$h_{f_1} = \frac{4fL_1V_1^2}{2gd_1} = \frac{4\times0.01\times25\times(4.452)^2}{2\times9.81\times0.15} = 6.73\ m$$

Loss of head due to sudden enlargement from pipe 1 to pipe 2 =

$$h_e = \frac{(V_1-V_2)^2}{2g} = \frac{(4.452-1.11)^2}{2\times9.81} = 0.568\ m$$

Loss of head due to Friction in pipe 2 =

$$h_{f_2} = \frac{4fL_2V_2^2}{2gd_2} = \frac{4\times0.01\times15\times(1.113)^2}{2\times9.81\times0.3}$$

Loss of head at the exit of the pipe 2 =

$$h_o = \frac{V_2^2}{2g} = \frac{1.113^2}{2\times9.81} = 0.063\ m.$$

$$\frac{V_1^2}{2g} = \frac{4.452^2}{2\times9.81} = 1\ m.$$

Total Energy line

(i) point A lies on free surface of water.

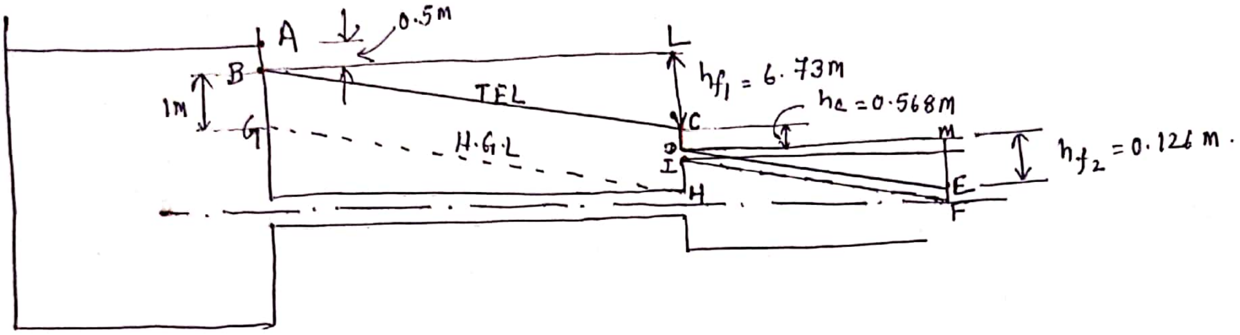
(ii) Take AB = h_i = 0.5 m.

(iii) From B, draw a horizontal line Take BL = L_1. From L, draw a vertical line downward. Cut the line LC = h_{f_1} = 6.73 m. Join the point B to C.

(iv) Take a line CD, vertically downward equal to h_e = 0.568 m.

(v) From D, draw DM horizontal and From point F which is lying on the centre of the pipe, draw a vertical line in the upward direction, meeting at M. From M, take a distance ME = h_{f_2} = 0.126 m. Join DE.

(vi) Then the line ABCDE, represents the total energy line



Hydraulic gradient line

(i) From B, take $BG = \frac{V_1^2}{2g} = 1$ m.

(ii) Draw GH, parallel to the line BC.

(iii) From F, Draw a line FI parallel to the line DE.

(iv) Joint the point H and I.

(v) The line GHIF represents the hydraulic grade line.

Impact of Jets

→ When momentum of a flowing fluid is changed by a body, a dynamic force is exerted upon it. This dynamic force exerted by the fluid jet is equal to the rate of change of momentum in the desired direction. This force is obtained from Newton's second law of motion or from impulse momentum equation.

$$F_x = \dot{m}\,\Delta V_x$$

$\dot{m}$ = mass flow rate of fluid striking the body.

ΔV_x = change of velocity of fluid in x-direction.

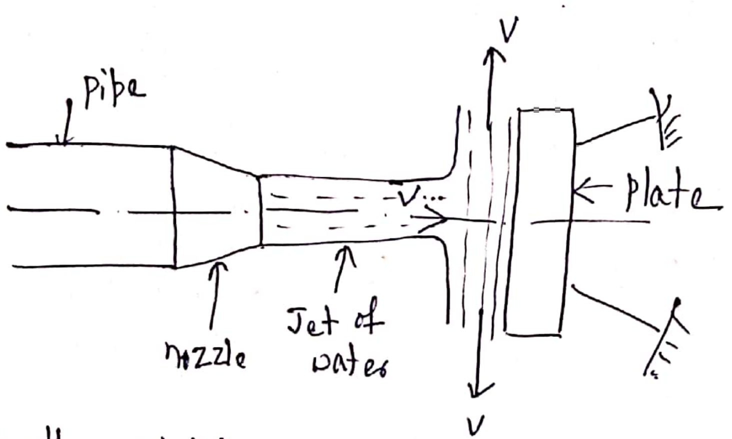
F_x = Force exerted by the jet in x-direction.

→ Impact of jet means the force exerted by the jet on a plate/body which may be stationary or moving.

case-I

Force exerted by the jet on a stationary vertical plate

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate.



V = velocity of the jet

d = diameter of the jet

a = area of cross-section of the jet $= \frac{\pi}{4} d^2$

Let the fluid jet area, 'a' strike the plate with a velocity V. After the impact, the jet is divided into two parts. It is assumed that since the plate is smooth, the frictional resistance offered is negligible. The velocity of the jet after the impact, can, therefore, be assumed unchanged.

→ The Jet after striking the plate, will move along the plate. But the plate is at right angle to the jet. Hence the jet, after striking, will get deflected through 90°. Hence the Component of the velocity of jet, in the direction of the jet, after striking will be zero.

∴ Force exerted by the jet on the plate in the direction of oncoming Jet, F_x = Rate of change of momentum in the direction of Force.

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{mass} \times \text{initial velocity}) - (\text{mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{mass}}{\text{Time}} [\text{initial velocity} - \text{Final velocity}]$$

$$= \dot{m} [\text{velocity of the jet before striking} - \text{Final velocity of the jet after striking}]$$

$$= \rho a v [v - 0]$$

$$\boxed{F_x = \rho a v^2}$$

ρ = density of fluid striking the plate.

Force exerted by the jet on the plate perpendicular to the direction of oncoming jet, $F_y =$ ~~m[0 − (v~~ $0 - [\frac{\dot{m}}{2} v + (-\frac{\dot{m}}{2} v)]$

$$\boxed{F_y = 0}$$

Case-II

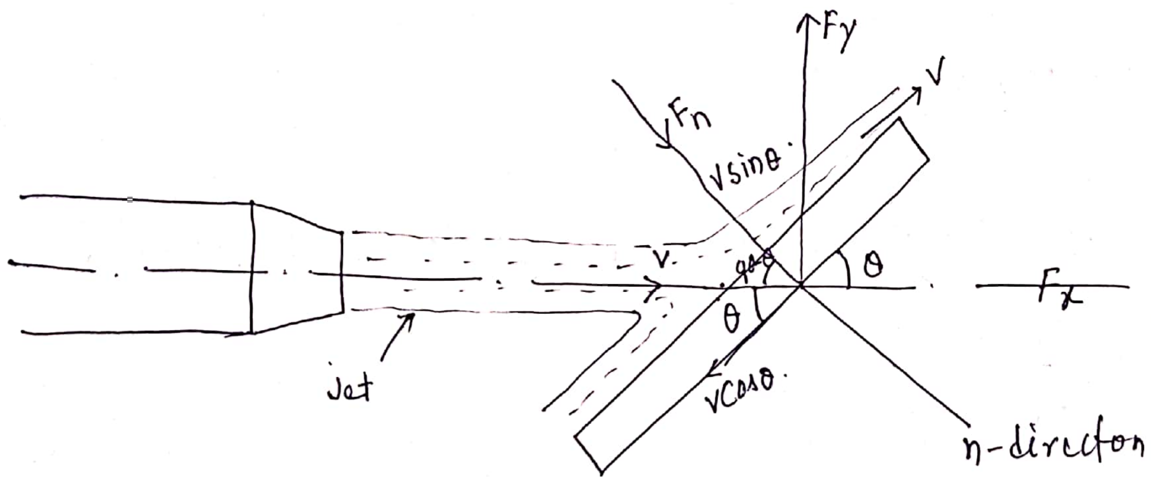
Force exerted by the jet on a stationary inclined Flat plate.

Let a jet of water, coming out from the nozzle, strikes on inclined flat plate.

v = velocity of the jet in the direction of x

θ = Acute angle between the plate and oncoming jet.

∴ mass of the jet striking the plate per sec. $= \dot{m} = \rho a v$



→ If the plate is smooth and if it is assumed that there is no loss of energy due to impact of the jet, then the jet after striking the plate will move with the ~~same~~ velocity equal to initial velocity v.

∴ Force exerted by the jet on the plate in the direction normal to plate, (F_n) = mass of the jet striking per second × ΔV_n

$F_n = \Big[$ mass of jet striking per second $\times$ (Initial velocity of the jet before striking in n-direction − Final velocity of the jet After striking in n-direction) $\Big]$

$= \rho a V [V \sin\theta - 0]$

$= \rho a V^2 \sin\theta$

This Force can be resolved into two components, one in the direction of oncoming jet and other perpendicular to the direction of oncoming jet.

$\therefore F_x$ = Component of F_n in the direction of Flow

$= F_n \cos(90-\theta)$

$= \rho a V^2 \sin\theta \cdot \sin\theta$

$$\boxed{F_x = \rho a V^2 \sin^2\theta}$$

$\therefore F_y$ = Component of F_n Perpendicular to the direction of flow

$= F_n \sin(90-\theta)$

$$\boxed{F_y = \rho a V^2 \sin\theta \cdot \cos\theta}$$

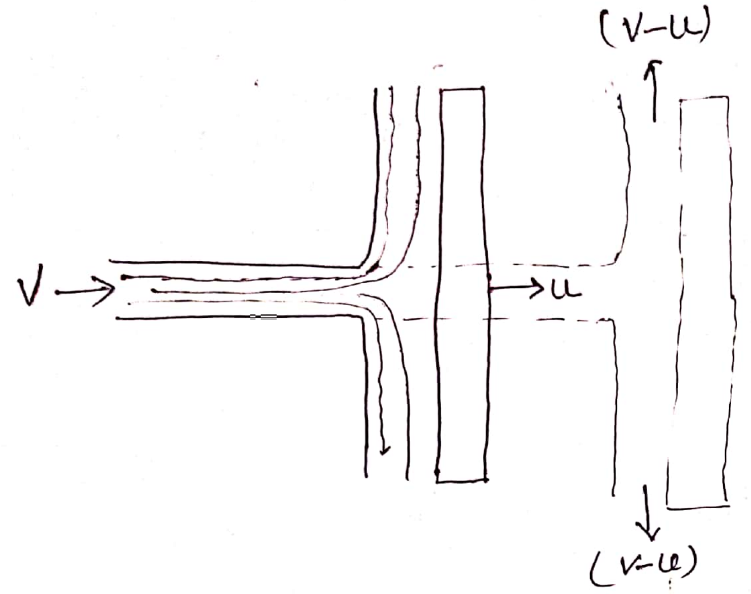
→ If $\theta = 90°$ (Earlier case) 1st case, $F_x = \rho a V^2 (\sin 90)^2 = \rho a V^2$

$F_y = \rho a V^2 \sin 90 \cdot \cos 90 = 0$

→ workdone per second by the jet on the plate is zero as the plate is stationary.

Case-III

Force exerted by the jet on moving vertical flat plates.



Consider a jet of water striking a smooth vertical flat plate moving with a uniform velocity away from the jet.

V = velocity of the jet (absolute)
a = cross-sectional Area of jet
u = velocity of the flat plate.

→ In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity which is equal to the absolute velocity of jet of water minus the velocity of the plate.

Hence relative velocity of the jet with respect to plate

$$= V-u$$

mass of water striking the plate per sec.

$$= \rho \times \text{Area of jet} \times \text{velocity with which jet strikes the plate}$$

$$= \rho a (V-u)$$

∴ Force exerted by the jet on the moving plate in the direction of jet,

$= F_x$ = mass of water striking per sec. × [Initial velocity with which water strikes − Final velocity]

$= \rho a (V-u) [(V-u) - 0]$

$$\Rightarrow F_x = \rho a (V-u)^2$$

workdone per. second by the jet on the plate

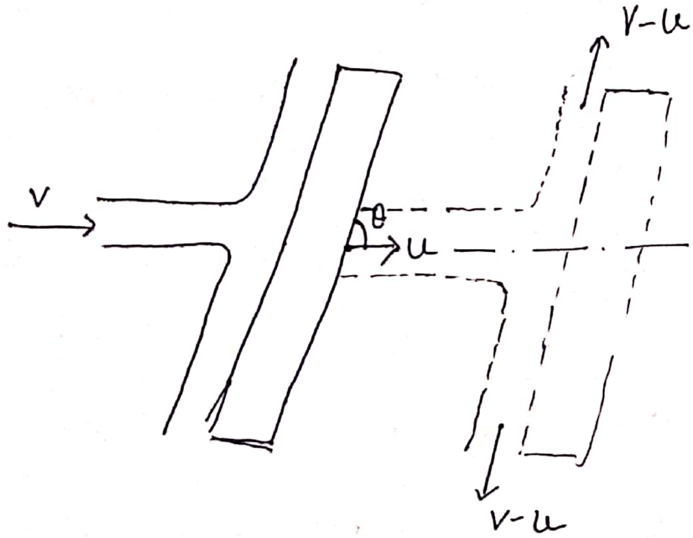
$= \text{Force} \times \frac{\text{Distance travelled in the direction of force}}{\text{Time}}$

$= F_x \times u$

$$\text{workdone} = \rho a (V-u)^2 \cdot u$$

case-IV

Force exerted by the jet on the inclined flat plate moving in the direction of the jet.



Let a jet of water strikes an inclined plate, which is moving with a uniform velocity in the direction of the jet.

V = absolute velocity of jet of water

u = velocity of the plate in the direction of jet

a = cross-sectional area of jet

θ = Angle between jet and plate.

Relative velocity of jet of water $= (V-u)$

∴ The velocity with which jet strikes $= (V-u)$

Mass of water striking per second $= \rho a (V-u)$

If the plate is smooth and loss of energy due to impact of jet is assumed zero, the jet of water will leave the inclined plate with a velocity equal to $(V-u)$

∴ The force exerted by the jet of water on the plate in the direction normal to the plate,

F_n = mass of jet striking per second × [Initial velocity in the normal direction with which jet strikes − Final velocity]

$$= \rho a (V-u) \left[(V-u) \sin\theta - 0 \right]$$

$$= \rho a (V-u)^2 \sin\theta .$$

This normal force, F_n is resolved into two component namely F_x (in the direction of jet) and F_y (perpendicular to the direction of jet).

$\therefore F_x = F_n \sin\theta = \rho a (V-u)^2 \sin^2\theta$

$F_y = F_n \cos\theta = \rho a (V-u)^2 \sin\theta . \cos\theta$

workdone per second by the jet on the plate

$= F_x \times$ distance per second in the direction of x

$= F_x . u$

$= \rho a (V-u)^2 \sin^2\theta . u$

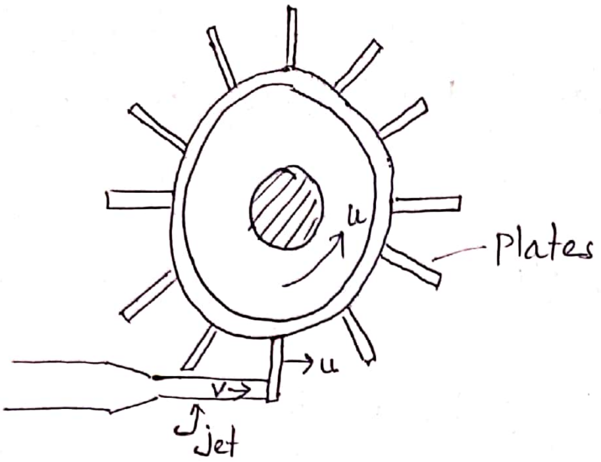
$= \rho a (V-u)^2 . u \sin^2\theta$

If $\theta = 90°$, $F_x = \rho a (V-u)^2 \Rightarrow$ same as III case.

Case-V

Force exerted by a jet of water on series of vanes.

The Force exerted by a jet of water on a single moving plate (which may be Flat or curved) is not practically feasible. This case is only a thoritical one. In actual practice, a certain number of evenly spaced Flat plates are mounted on the circumference of a wheel. The jet strikes a plate and due to the Force exerted by the jet on the plate, the wheel starts movings and the 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd plate. Thus each plate appears successively before the jet and jet exerts force on each plate. The wheel starts moving at a constant speed.



v = velocity of jet
d = diameter of jet
a = cross-sectional Area of jet
$= \frac{\pi}{4} d^2$
u = velocity of vane/plate.

→ The number and location of plates is so arranged that no portion of jet goes waste without doing work on the plate. Thus entire Fluid mass issuing from the nozzle is considered to strike the plate.

→ ∴ The mass of water per second striking the series of the Plate = $\rho a v$

→ velocity with jet strikes the plate $= (v - u)$

→ After striking, the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate = zero.

∴ The force exerted by the Jet in the direction of motion of Plate,

F_x = mass per second × [Initial velocity − Final velocity]

$= \rho a v [(v-u) - 0]$

$$F_x = \rho a v (v-u).$$

→ Workdone by the jet on the series of plates per second

$= \text{Force} \times \text{Distance per second in the direction of force}$

$= F_x \cdot u$

$= \rho a V (V-u) \cdot u$

→ Kinetic energy of the jet per second $= \frac{1}{2} m V^2$

$= \frac{1}{2} \rho a V \cdot V^2 = \frac{1}{2} \rho a V^3$

→ Efficiency of the jet $= \dfrac{\text{Workdone per second by the Jet}}{\text{Kinetic energy of the jet}}$

$$= \frac{\rho a V \cdot (V-u) \cdot u}{\frac{\rho a V^3}{2}} = \frac{2(V-u) \cdot u}{V^2}$$

$$\therefore \boxed{\eta = \frac{2u(V-u)}{V^2}}$$

Condition for maximum efficiency

For a given jet velocity (V), the efficiency will be maximum when $\dfrac{d\eta}{du} = 0$

$$\frac{d}{du}\left[\frac{2 \cdot u \cdot (V-u)}{V^2}\right] = 0$$

$$\Rightarrow \frac{d}{du}\left[\frac{2uV - 2u^2}{V^2}\right] = 0 \quad \Rightarrow 2V - 4u = 0$$

$$\Rightarrow V = 2u$$

or $u = \frac{V}{2}$

maximum efficiency

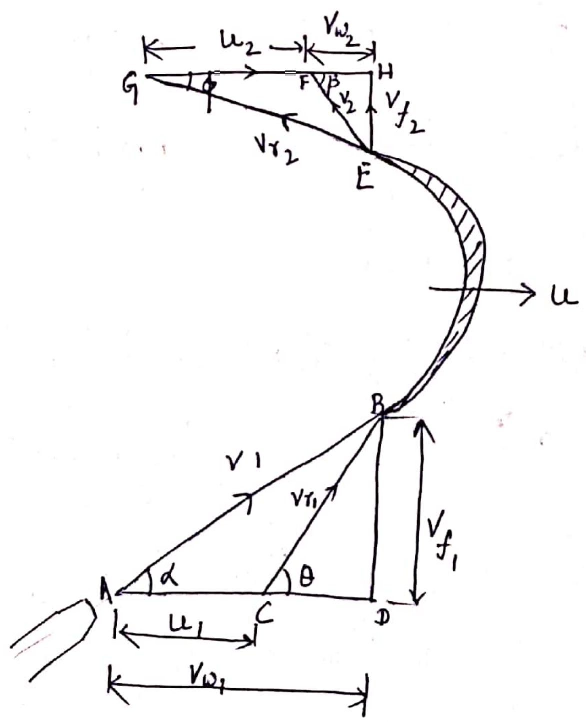
Substituting the value of $V = 2u$, in $\eta = \frac{2u(V-u)}{V^2}$

$$\Rightarrow \eta_{max} = \frac{2u(2u-u)}{4u^2} = \frac{2u \cdot u}{4u^2} = \frac{2u^2}{4u^2} = \frac{1}{2} \text{ or } 50\%$$

$\therefore$ $\eta_{max} = 50\%$

Case-VI

Force exerted by a jet of water on an un-symmetrical moving curved plate when jet strikes tangentially at one of the tips.



→ A Jet of water striking a moving curved plate (also called vane) tangentially, at one of its tips. As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as the plate is moving, the velocity with which jet of water strikes the plate is equal to the relative velocity of the jet with respect to the plate. As the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet and velocity of the plate at inlet.

V_1 = velocity of the jet at inlet

u_1 = velocity of the plate (vane) at inlet.

V_{r1} = relative velocity of the jet with respect to plate at inlet.

α = angle between the direction of the jet and direction of motion of the plate at inlet also called guide blade angle / Nozzle angle.

θ = Angle made by relative velocity (V_{r1}) with the direction of motion of the plate at inlet.

V_{w1} = The component of the velocity of the jet (V_1), in the direction of motion and is called velocity of whirl at inlet.

V_{f1} = The component of the velocity of the jet (V_1). perpendicular to the direction of the motion of the vane and it is known as velocity of flow at inlet.

V_2 = velocity of the jet at the outlet of vane.

u_2 = velocity of the vane at outlet.

V_{r_2} = Relative velocity of the jet with respect to the vane at outlet.

ϕ = Angle made by the relative velocity (V_{r_2}) with the direction of motion of the vane at outlet and also called vane angle at outlet.

β = Angle made by the velocity (V_2) with the direction of motion of the vane at outlet.

V_{w_2} = component of the velocity of the jet (V_2) in the direction of motion of vane and is called velocity of whirl at outlet.

V_{f_2} = component of the velocity of the jet (V_2), perpendicular to the direction of the motion of the vane at outlet and is known as velocity of flow at outlet.

The triangles ABD and EGH are called the velocity triangles at inlet and outlet.

→ If vane ~~is smooth and~~ is having velocity in the direction of motion at inlet and outlet are equal

$\therefore \boxed{u_1 = u_2 = u}$

→ If the vane surface is assumed to be very smooth, the loss of energy due to friction is zero. The water will be gliding over the surface of the vane with a relative velocity V_{r_1} and will come out of the vane with a relative velocity V_{r_2}. This means that $\boxed{V_{r_1} = V_{r_2}}$.

→ But in practice, however smooth may be the blade surface, there is always some friction which reduces the magnitude of V_{r_2}. The effect of blade friction is expressed by the ratio

$$K = \frac{V_{r_2}}{V_{r_1}}, \quad \text{where } K = \text{blade velocity coefficient}$$

→ Mass of water striking vane per sec. $= \rho a V_{r_1}$

a = area of jet of water, V_{r_1} = relative velocity of jet at inlet.

→ Force exerted by the jet in the direction of motion,

F_x = Mass of water striking per sec. × [Initial velocity with which jet strikes in the direction of motion − Final velocity of jet in the direction of motion.]

→ Initial velocity with which jet strikes the vane $= V_{r_1}$

The component of this velocity in the direction of motion

$= V_{r_1} \cos\theta$

$= (V_{w_1} - u_1)$ (From Figure $V_{r_1}\cos\theta = V_{w_1} - u_1$)

Similarly the final velocity of the jet in the direction of motion

$= -V_{r_2} \cos\phi$

$= -(u_2 + V_{w_2})$ (From Fig. $u_2 + V_{w_2} = V_{r_2}\cos\phi$)

⊖ sign indicates the component of V_{r_2} in the direction of motion is in the opposite direction.

→ $\therefore F_x = \rho a V_{r_1} \{(V_{w_1} - u_1) - (-[u_2 + V_{w_2}])\}$

$= \rho a V_{r_1} [V_{w_1} - u_1 + u_2 + V_{w_2}]$ $(\because u_1 = u_2)$

$$\boxed{F_x = \rho a V_{r_1} [V_{w_1} + V_{w_2}]}$$

→ This equation is true only when the angle β is acute angle.

If $\beta = 90°$, $\Rightarrow V_{w_2} = 0$

If $\beta > 90°$ (obtuse angle), $F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}]$

∴ In general $$F_x = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}]$$

→ workdone per sec. on the vane by the jet

= Force x Distance per sec. in the direction of force

= $F_x \times u$

$$= \rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u$$

→ workdone per sec. per unit weight of Fluid striking per sec.

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\text{weight of Fluid striking/sec.}}$$

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\rho a V_{r_1} \cdot g}$$

$$= \frac{[V_{w_1} \pm V_{w_2}] u}{g}$$ (unit = $\frac{Nm}{N}$)

→ workdone per sec. per unit mass of fluid striking per sec.

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\text{mass of fluid striking per sec.}} = \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) u}{\rho a V_{r_1}}$$

$$= (V_{w_1} \pm V_{w_2}) u$$

→ Efficiency of the jet

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{workdone per sec. on the vane}}{\text{initial Kinetic energy per sec. of the jet}}$$

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\frac{1}{2} m V^2}$$

where $m = \rho a V_1$ $\quad$ V_1 = initial velocity of the jet

$$\therefore \eta = \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\frac{1}{2} (\rho a V_1) V_1^2}$$

Exp-1 Find the Force exerted by a jet of water of diameter 75 mm on a stationary Flat plate, when the jet strikes the plate normally with a velocity 20 m/s.

Sol^n

Jet strikes normally & it is a stationary Flat plate. So it is a case - I type problem.

$d = 75\ mm = 0.075\ m$

$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.075)^2 = 0.004417\ m^2$

$V = 20\ m/s$.

$F_x = \rho a V^2 = 1000 \times 0.004417 \times 20^2 = 1766.8$ Newton.

Exp-2 Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and head of water at the centre nozzle is 100m. Find the force exerted by the jet of water on a fixed vertical plate. The coefficient of velocity = 0.95.

Solution

$d = 100mm = 0.1 m$

Head of water, $H = 100 m$

$C_v = 0.95$

Area of nozzle, $a = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 0.1^2 = 0.007854 \ m^2$

Theoritical velocity of water $= \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.294 \ m/s$

Actual velocity of water $= C_v\sqrt{2gH} = 0.95 \times 44.294 = 42.08 \ m/s$

$F = \rho a v^2 = 1000 \times 0.007854 \times (42.08)^2 = 13907.2 N = 13.9 KN$

Exp-3 A jet of water of diameter 75 mm moving with a velocity 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate
(ii) in the direction of the jet.

Solution

$d = 75 mm = 0.075 m$

$a = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times (0.075)^2 = 0.004417 \ m^2$

$V = 25 \ m/s$

$\theta = 60°$

(i) $F_n = \rho a v^2 \sin\theta$

$= 1000 \times 0.004417 \times 25^2 \times \sin 60$

$= 2390.7$ N

(ii) $F_x = \rho a v^2 \sin^2\theta$

$= 1000 \times 0.004417 \times 25^2 \times (\sin 60)^2$

$= 2070.4$ N.

Exp-4

A jet of water of diameter 50 mm strikes a Fixed plate in such a way that the angle between the plate and jet is 30°. The force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.

Solution

$d = 50$ mm $= 0.05$ m

$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.05)^2 = 0.001963\ m^2$

$\theta = 30°$

$F_x = 1471.5$ N

we know $F_x = \rho a v^2 \sin^2\theta$

$\Rightarrow 1471.5 = 1000 \times 0.001963 \times V^2 \times (\sin 30)^2$

$\Rightarrow V = 54.77$ m/s

$\therefore$ Discharge, Q = Area × velocity

$= 0.001963 \times 54.77$

$= 0.1075\ m^3/s$

$= 107.5$ litres/s.

Exp-5

A jet of water of diameter 10 cm strikes a flat plate normally with a velocity of 15 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find
(i) Force exerted by the jet on the plate
(ii) Workdone by the jet on the plate per second.
(iii) Efficiency

Solution

$d = 10\ cm = 0.1\ m$

$Area = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.1)^2 = 0.007854\ m^2$

$V = 15\ m/s$

$u = 6\ m/s$

(i) $F_x = \rho a (V-u)^2 = 1000 \times 0.007854 \times (15-6)^2 = 636.17\ N$

(ii) $workdone = F_x \cdot u = 636.17 \times 6 = 3817.02\ Nm/s$

(iii) $Efficiency = \frac{output}{input} = \frac{workdone/sec}{input\ energy\ of\ the\ jet\ per\ sec}$

input energy of the jet per sec. $= \frac{1}{2} \dot{m} V^2$

$= \frac{1}{2} \times \rho a V \cdot V^2 = \frac{1}{2} \rho a V^3 = \frac{1}{2} \times 1000 \times 0.007854 \times 15^3$

$= 13253.6\ Nm/s$

$\therefore \eta = \frac{3817.02}{13253.6} = 0.288 = 28.8\%$

Exp-6 A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate. Also determine power and efficiency of the jet when the plate is moving with a velocity of 15 m/s.

Solution

$d = 7.5\ cm = 0.075\ m$

$a = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times (0.075)^2 = 0.004417\ m^2$

$\theta = 90° - 45° = 45°$

$V = 30\ m/s$

$u = 15\ m/s$

$F_n = \rho a (V-u)^2 \sin\theta = 1000 \times 0.004417 \times (30-15)^2 \times \sin 45°$

$= 702.74 N.$

workdone/sec. = power $= F_x \cdot u$

$= F_n \sin\theta \cdot u$

$= 702.74 \times \sin 45° \times 15$

$= 7453.5\ Nm/s$

$= 7453.5\ J/s = 7453.5\ watt = 7.453\ KW$

Efficiency =

$\eta = \frac{output}{input} = \frac{workdone/sec.}{K.E/sec.} = \frac{7453.5}{\frac{1}{2} \times (\rho a V) \cdot V^2} = \frac{7453.5}{\frac{1}{2}\rho a V^3}$

$= \frac{7453.5}{\frac{1}{2} \times 1000 \times 0.004417 \times 30^3}$

$= 0.1249 \approx 0.125 = 12.5\%$

Exp-7

A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane at outlet. calculate

(i) vane angles, so that the water enters and leaves the vane without shock.

(ii) workdone per sec. per unit weight of water striking the vane per second.

Solution

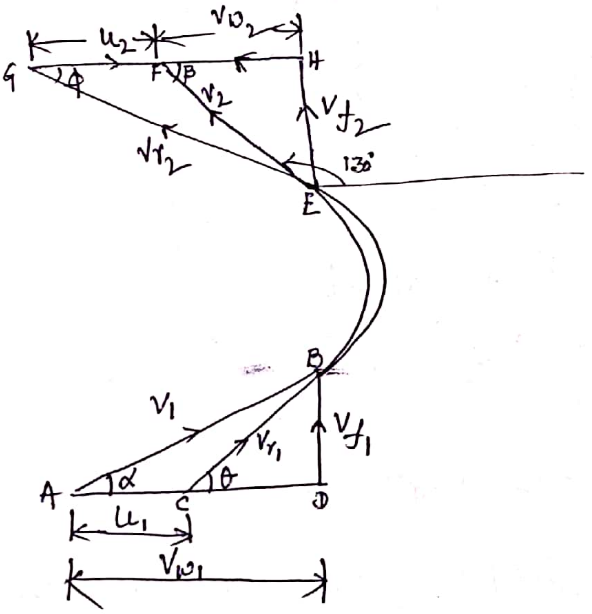
$V_1 = 20$ m/s

$u_1 = 10$ m/s.

$\alpha = 20°$

$\beta = 180 - 130 = 50°$

$u_1 = u_2 = 10$ m/s.



water enters and leaves the vane without shock (i.e.) $V_{r_1} = V_{r_2}$

→ we have to Find out θ, ϕ, workdone per unit weight of water.

from Δ ABD

$V_{f_1} = V_1 \sin\alpha = 20 \sin 20 = 6.84$ m/s

$V_{w_1} = V_1 \cos\alpha = 20 \cos 20 = 18.794$ m/s.

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{6.84}{18.794 - 10} = 0.7778$$

$\therefore \theta = 37.875°$

$$\sin\theta = \frac{V_{f1}}{V_{r_1}} \Rightarrow V_{r_1} = \frac{V_{f1}}{\sin\theta} = \frac{6.84}{\sin 37.875^\circ} = 11.14.$$

$V_{r_1} = V_{r_2} = 11.14 \text{ m/s}.$

From ΔEFG, Applying sine rule.

$$\frac{V_{r_2}}{\sin 130^\circ} = \frac{u_2}{\sin(\beta - \phi)}$$

$$\Rightarrow \frac{11.14}{\sin 130^\circ} = \frac{10}{\sin(50-\phi)}$$

$$\Rightarrow \sin(50-\phi) = \frac{10}{11.14} \times \sin 130^\circ$$

$$\Rightarrow \sin(50-\phi) = 0.6876$$

$$\Rightarrow 50 - \phi = \sin^{-1}(0.6876)$$

$$\Rightarrow 50 - \phi = 43.44$$

$$\Rightarrow \phi = 6.56^\circ$$

$$u_2 + V_{w_2} = V_{r_2} \cos\phi$$

$$V_{w_2} = V_{r_2}\cos\phi - u_2 = 11.14 \cos(6.56) - 10 = 11.067 - 10 = 1.067 \text{ m/s}.$$

workdone per unit weight of water striking per second

$$= \frac{[V_{w_1} + V_{w_2}] \times u}{g} \qquad (+\text{ sign is taken as } \beta < 90^\circ \text{ acute angle})$$

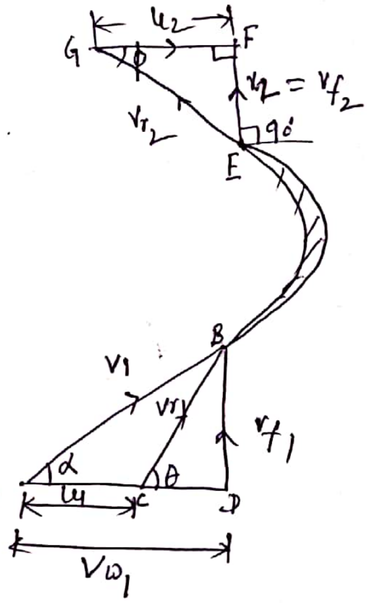
$$= \frac{[18.794 + 1.067] \times 10}{9.81}$$

$$= 20.24 \ \frac{Nm}{N}$$

Exp-8

A Jet of water having a velocity of 40 m/s strikes a curved vane, which is moving with a velocity of 20 m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet and outlet and determine the vane angles at inlet and outlet so that the water enters and leaves the vane without shock.

Solution



$V_1 = 40$ m/s.

$u_1 = 20$ m/s. $= u_2$

$\alpha = 30°$

Angle made by leaving jet $= 90°$

$\therefore \beta = 180 - 90° = 90°$

Find out, $\theta, \phi = ?$

water enters and leaves the vane without shock $\Rightarrow V_{r_1} = V_{r_2}$

$V_{f_1} = V_1 \sin\alpha = 40 \sin 30 = 20$ m/s

$V_{w_1} = V_1 \cos\alpha = 40 \cos 30 = 34.64$ m/s

$\tan\theta = \dfrac{V_{f_1}}{V_{w_1} - u_1} \Rightarrow \tan\theta = \dfrac{20}{34.64 - 20} = \dfrac{20}{14.64} = 1.366$

$\Rightarrow \boxed{\theta = \tan^{-1}(1.366) = 53.79°}$

$\sin\theta = \dfrac{V_{f_1}}{V_{r_1}} \Rightarrow V_{r_1} = \dfrac{V_{f_1}}{\sin\theta} = \dfrac{20}{\sin 53.79°} = 24.78$ m/s.

From Δ EFG

$$\cos\phi = \frac{u_2}{V_{r_2}} = \frac{20}{24.78} = 0.8071$$

$$\phi = \cos^{-1}(0.8071)$$

$$\boxed{\phi = 36.18^\circ}$$