Lecture Note: 1 (Units and Dimensions)

Engineering Physics

Name of the Lecturer: Abhilash Padhy

Lect. In Physics (Department of Math & Sc.), GP Gajapati

Physics is a branch of science that deals with the natural phenomena and the properties of matter and energy.

e.g. Freely falling object, dazzling of diamonds, a body moving in circle etc.

To describe the physical phenomena quantitatively, we need to define certain measurable quantities which can describe the laws of Physics invariably. These quantities are called the Physical Quantities. They can be divided into two categories

- Fundamental Physical Quantity
- Derived Physical Quantity

The fundamental physical quantities can be directly measured through some measuring procedure and doesn't need the involvement of any other physical quantity. Whereas the derived physical quantities are those which are derived from the fundamental quantities.

There are seven fundamental quantities, which are given as mass, length, time, electric current, thermodynamic temperature, luminosity, amount of substance. All the other physical quantities can be derived from these fundamental quantities. For example: volume, density, velocity, force, energy etc.

Unit

This is a standard created to measure a physical quantity and to express sets of measurements relative to each other. e.g. meter, Kg, liter etc.

Characteristics of Unit

- It should be invariable.
- It should be easily available for comparison with various measurements
- It should be non-perishable.
- It should be convenient in size.

The units of fundamental quantities are known as fundamental units and the units of derived quantities are known as derived units.

System of Units

Keeping track of the above characteristics of unit different sets of units are recognized at different places and in different areas for the same physical quantities. Some of the well-known system of units are given as

- M. K. S (or metric system)
- C. G. S (French system)
- F. P. S (British system)

In M.K.S system of units, the fundamental units of length, mass, time are 1 meter, 1 kilogram and 1 second respectively.

In C.G.S system of units the fundamental units of length, mass, time are 1 centimeter, 1 gram and 1 second respectively.

In F.P.S system of units the fundamental units of length, mass, time are 1 foot, 1 pound and 1 second respectively.

All of these above system of units defines only 3 fundamental units which may be sufficient in case of mechanics but when we deal with thermodynamics or the electricity other fundamental units will come into picture. Therefore, in 1960, a new system of seven fundamental units was decided known as the S.I system of unit (International system of Units). They are listed as below

The Fundamental Quantity	S.I unit	Symbol for the unit
length	Kilogram	kg
Mass	Meter	m
Time	Second	s
Electric current	Ampere	A
Thermodynamic temperature	Kelvin	K
luminosity	candela	cd
Amount of substance	Mole	mol

In addition to the symbols of fundamental units, the symbols of some derived units are also important. They are listed as

The Derived quantity	Unit	Symbol for the unit
Work or Energy	joule	J
Power	watt	W
Charge	coulomb	C
Magnetic flux	weber	Wb
Resistance	ohm	ohm (Ω)
Voltage	volt	V
Capacitance	farad	F
Inductance	henry	H
Conductance	siemen	S
Magnetic field intensity	tesla	T

Supplementary S.I Units

Two more fundamental units for the measurements of angle and solid angle had to be defined. These units are called ‘supplementary S.I units’.

- Unit of angle or the 2D angle: radian(rd)
- Unit of solid angle or the 3D angle: steradian(sr)

Metric Prefixes

It is not wisely to use the same units for both large and small quantities. Suppose I ask you how many kilos of rice do you have; it won't be inconvenient for you to answer the question. But suppose I ask you 'How many kilos of gold do you have?'; would it be convenient to give the answer in kilos. So for the sake of convenience, it is necessary to provide, some multiple units of standard units to measure large and small quantity. These new units can be described by adding appropriate prefixes before the standard units, which are listed in the table below

Sub-multiples			multiples		
Value	Abbreviation	prefix	prefix	Abbreviation	value
10^{-1}	d	deci	Deca	D	10^{1}
10^{-2}	c	centi	Hecta	H	10^{2}
10^{-3}	m	milli	Kilo	K	10^{3}
10^{-6}	μ	micro	Mega	M	10^{6}
10^{-9}	n	nano	Giga	G	10^{9}
10^{-12}	p	pico	Tera	T	10^{12}
10^{-15}	f	femto	Peta	P	10^{15}
10^{-18}	a	atto	Exa	E	10^{18}

Example:

1 Pm = $10^{21}\ \mu m$ (since 1Pm = 10^{15}m = $10^{15} \times 10^{6}\ \mu m = 10^{21}\ \mu m$)

Some practical units

There are three important practical units for measuring macroscopic lengths.

- Astronomical unit (A. U) : it is the mean distance of the earth from the sun. It is equal to about 1.496×10^{11} m.
- Light-year: it is the distance travelled by light, in vacuum, in one year.
  1 Light-year = $9.467 \times 10^{15}\ m$ (nearly)
- Parallactic second (Parsec): it is the distance at which a length of one astronomical unit subtends an angle of one second of arc. 1 parsec = $3.084 \times 10^{16}\ m$.

Dimensions

The dimensions of a derived physical quantity may be defined as the powers to which it's base units must be raised to represent it completely. And a dimensional formula is an expression which shows how(with what powers) and which of the fundamental units enter into the units of a physical quantity.

Steps to determine the dimensional formula of a physical quantity

i. Write the formula for the quantity, with the quantity of L.H.S of the equation.
ii. Convert all the quantities on R.H.S into the fundamental quantities mass, length and time, respectively.
iii. Substitute M, L and T for mass, length and time respectively.
iv. Collect terms of M,L and T and find their resultant powers which give the dimensions of the quantity in mass, length and time, resepectively.

Examples:

- Velocity = $\frac{displacement}{time} = \frac{[L]}{[T]} = [L\,T^{-1}] = [M^0 L^1 T^{-1}]$
- Force = mass × acceleration = $mass \times \frac{velocity}{times} = [M^1 L^1 T^{-2}]$

Principle of Homogeneity

It states that the dimensional formula of every term on the two sides of a correct relation must be same.

Uses:

i. To convert the values of a physical quantity from one system to another
Illustration: Let us convert a force of 1 newton into dyne
We know the dimensional formula for force is $[M\,L\,T^{-2}]$. In M.K.S system of units $M = M_1 = 1\,kg\,, L = L_1 = 1\,m\,, T = T_1 = 1\,s$ and similarly in C.G.S system of units $M = M_2 = 1\,g\,, L = L_2 = 1\,c.m\,, T = T_2 = 1\,s$.

According to the principle of homogeneity, $n_1\,[M_1\,L_1\,T_1^{-2}] = n_2\,[M_2\,L_2\,T_2^{-2}]$ i.e.
$$n_1 = 1\left[\frac{1\,kg}{1g}\right]\left[\frac{1m}{1\,cm}\right]\left[\frac{1\,s}{1\,s}\right]^{-2} = 10^3 \times 10^2 \times 1 = 10^5 \Rightarrow 1\,newton = 10^5\,dyne$$

ii. To check the correctness of a given relation
Illustration: Let us check the dimensional correctness of the relation $v = u + at$
L.H.S = $[v] = [L\,T^{-1}]$, R.H.S = $[u + at] = [u] + [a][t] = [L\,T^{-1}] + [L\,T^{-2}][T] = [L\,T^{-1}]$
That means the dimensions of the L.H.S and R.H.S match to each other which is expected from the principle of homogeneity; so the relation is dimensionally correct.

iii. To derive a relation between various physical quantities.
Illustration: Let us obtain an expression for centripetal force required to move to body of mass m, with velocity v in a circle of radius 'r'.
Say $F \propto m^a$, $F \propto v^b$, $F \propto r^c$ $i.e. F \propto m^a\,v^b\,r^c$
According to principle of homogeneity, the dimensions of the left and the right need to be same.
L.H.S = $[F] = [M\,L\,T^{-2}]$ and R.H.S = $[M^a][v^b][r^c] = [M^a][L\,T^{-1}]^b[L]^c = [M^a\,L^{b+c}\,T^{-b}]$
i.e. $a = 1\,, b + c = 1\,, -b = -2$; and solving these equations we would get $a = 1\,, b = 2\,, c = -1$. So we can write the formula for centripetal force as $F = k\frac{mv^2}{r}$.

Some Physical Quantities and their formula

- Pressure or Stress = $\frac{Force}{Area}$
- Impulse = $Force \times time$
- Momentum = mass × velocity
- Work or Torque = Force × distance
- Kinetic Energy = ½ m v^2
- Strain = Change in length / original length
- Frequency = 1/ Timeperiod
- Surface tension = Force / length
- Co-efficient of viscocity = force/(area × velocity gradient)
- Angle = length of arc / radius
- Angular velocity = Angle / time
- Angular acceleration = Angular velocity / time
- Angular momentum = momentum × distance
- Moment of inertia = mass × $radius^2$
- Coefficient of friction = force/ normal reaction
- Temperature ([$M^0\ L^0\ T^0\ K^1$])(fundamental quantity)
- Gas constant = Pressure × volume / temperature
- Coefficient of thermal conductivity = $Q \frac{d}{A\,(\theta_2 - \theta_1)t}$
- Electric current ([$M^0\ L^0\ T^0\ A^1$]) (fundamental quantity)
- Electric charge = Electric current × time
- Potential difference or voltage = Work / charge
- Resistance = potential difference / electric current
- Resistivity = Resistance × Area / length
- Capacitance = Charge/ potential difference
- Permittivity = $\frac{Charge^2}{Force \times distance^2}$
- Magnetic Flux density = Force / (Charge × velocity)
- Magnetic Pole = Force/ magnetic flux density
- Magnetic Permeability = Force × $distance^2$ / $Pole\ strength^2$

Try to find out the dimensional formula and the S.I units for the above physical quantities.

Lecture Note: 2 (Scalars and Vectors)

Engineering Physics

Name of the Lecturer: Abhilash Padhy

Lect. In Physics (Department of Math & Sc.), GP Gajapati

Scalar and Vector Quantities

- The physical quantities which require only the magnitude to be completely specified are known as the Scalar quantities.

  Example: Mass, length, volume, density, energy, temperature, electric charge, electric potential etc.

- The physical quantities which require magnitude as well as direction for their complete specification are known as the Vector quantities.

  Example: Displacement, velocity, acceleration, force, electric intensity, magnetic intensity, magnetic moment etc.

- Representation: A vector quantity is generally represented by the symbol for the corresponding quantity with an arrow mark over it like '$\vec{X}$'. Diagrammatically, the arrow head on a line indicates the direction of the vector quantity. Mathematically any vector can be expressed as

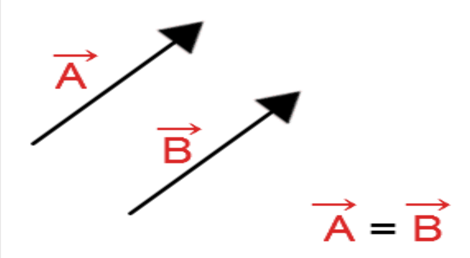
$$\vec{A} = |\vec{A}|\,\hat{a}$$

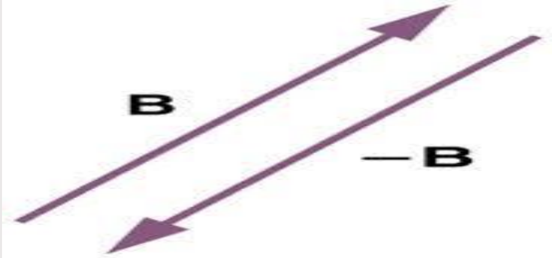
  Where $|\vec{A}|$ represents the magnitude of the vector $\vec{A}$ (*remember that the magnitude of a vector is never negative; if there is a negative sign before any vector it just indicates that the direction of the vector is opposite to that of the unit vector in right hand side*) and $\hat{a}$ stands for the unit vector along the direction of $\vec{A}$. The magnitude of this unit vector is unity as the name suggests and it signifies the direction of the quantity.

Types of Vector

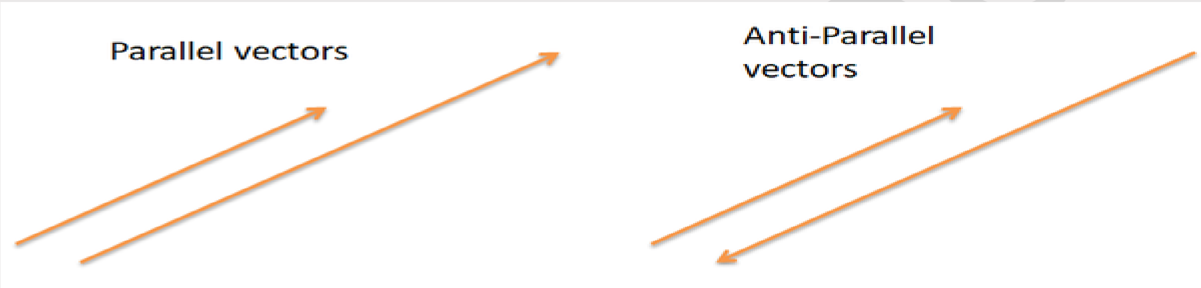
- Null vector: It is a vector having zero magnitude and arbitrary direction. It is represented by a point.

- Equal vectors: If two vectors have same magnitude and same direction then they are called as equal vectors. They can be represented by two parallel lines of equal length and the arrow mark on both pointing to the same direction.

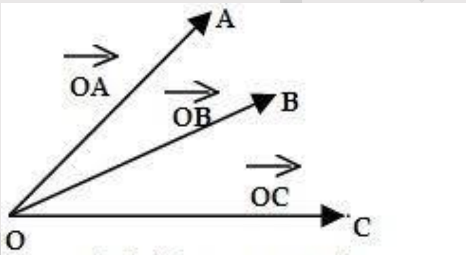




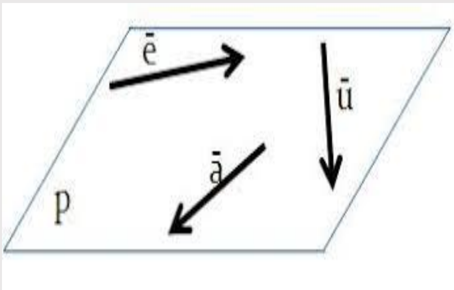
- Negative vector: A vector can be called as the negative vector of another one, if it has the same magnitude but opposite direction with respect to that vector. figure
- Parallel vectors: Two vectors are said to be parallel vectors, if both are directed along the same direction where as their magnitude may or may not be the same. So all the equal vectors can be called parallel vectors but the reverse is not true always.



- Anti-parallel vectors: Two vectors are said to be anti-parallel vectors, if their directions are opposite to each other. So all the negative vectors can be called anti-parallel vectors but the reverse is not true always.



Co-Inital vectors example



- Co-initial vectors: A number of vectors having a common initial point (or origin) are called co-initial vectors. Figure

- Co-linear vectors: Vectors having a common line of action are called collinear vectors. The Parallel and anti-parallel vectors are the two examples of co-linear vectors.
- Co-planar vectors: Vectors lying on one plane irrespective of their directions, are known as co-planar vector.

- Localized vectors: Vector whose initial point is fixed is said to be a localized or a fixed vector.
  Example: Position vector

- Non-localized vectors: Vector whose initial point is not fixed is said to be a non-localized vector or a free vector.
  Example: force, momentum, impulse etc.

Scalar Multiplication

- When a vector quantity is multiplied by a scalar quantity the resultant will be a vector quantity.
- The scalar and the magnitude of the vector quantity will get multiplied algebraically (like 2 multiplied by 3 gives 6). But what about the direction? Here we have only two possibilities. If the scalar is positive, then the direction of the vector after getting multiplied by the scalar won't change; but if the scalar is negative then the direction of the vector will be reversed. But can a scalar be negative? We know the scalars get specified by magnitude only and the magnitude is never negative. But there are certain instances, where we consider negative values of scalar. For example the temperature can take some value $-\theta\ K$.

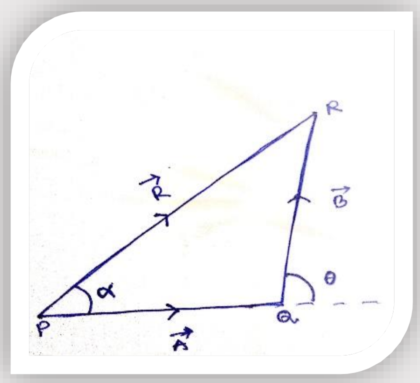
Addition of Vectors

- Though the scalars can be added algebraically but vectors are added geometrically.

  This is a special property of a vector quantity. To understand it's significance let us answer whether electric current is a scalar or a vector quantity?
  As we know the flow of charges set the direction of current, so one may think current to be a vector quantity. But do they get added geometrically? The answer is no. So we cannot categorize electric current as a vector quantity.
- The vector addition is commutative. That means, it does not matte in which order you add two vectors, the resultant will be the same. Mathematically we can write
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$
- There are basically two laws which describes how to find out the resultant of addition of two vectors. They are the
  I. Triangle law of Vector addition
  II. Parallelogram law of Vector addition

Triangle law of Vector addition

It states that, "If two vectors are represented by the two sides of a triangle, taken in the same order, then their resultant is represented by the third side of the triangle taken in opposite order." The magnitude and direction of the vectors are represented by the length of the corresponding sides and the arrowhead marked upon them respectively.



In the triangle PQR, the resultant of the addition of the vectors $\vec{A}$ (Which is represented by the side PQ) and $\vec{B}$(which is represented by the side QR) is expressed as $\vec{R}$(which is represented by the side PR), and it's magnitude and direction can be given as

$$R = \sqrt{A^2 + B^2 + 2AB\,cos\theta} \;\; (|\vec{X}| \equiv X \; for\; any\; arbitrary\; vector\; \vec{X})$$

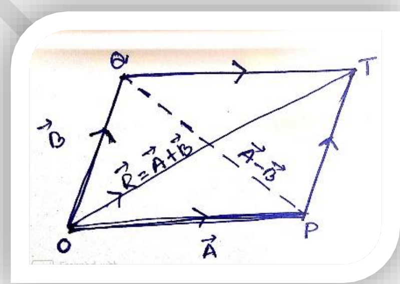
$$\alpha = \tan^{-1}(\frac{B\sin\theta}{A+B\cos\theta})$$

The angle alpha specifies the direction in which the resultant vector $\vec{c}$ is directed.

Parallelogram's law of Vector addition

It states that, "If two vectors acting simultaneously, at a point are represented, in magnitude and direction, by the two sides of a parallelogram drawn from a point, their resultant is given, in magnitude and direction, by the diagonal of the parallelogram passing through that point.

Fig.



For the parallelogram OPTQ, resultant of $\vec{A}$ and $\vec{B}$ which are expressed by the sides OP and OQ respectively, is given as

$$R = \sqrt{A^2 + B^2 + 2AB\,cos\theta}$$

$$\beta = \tan^{-1}(\frac{B\sin\theta}{A+B\cos\theta})$$

Special cases:

- Case 1: $\theta = 0^0, \cos 0 = 1, \sin 0 = 0\,, R = \sqrt{A^2 + B^2 + 2\,A\,B} = A + B,\ \beta = 0$
- Case 2: $\theta = 90^0, \cos 90 = 0, \sin 90 = 1\,, R = \sqrt{A^2 + B^2},\ \beta = \tan^{-1}\frac{B}{A}$
- Case 3: $\theta = 180^0, \cos 180 = -1, \sin 180 = 0\,, R = \sqrt{A^2 + B^2 - 2\,A\,B} = A - B,\ \beta = 0$

Subtraction of Vectors

The process of subtracting one vector from another is equivalent to adding the negative of the vector to be subtracted. That means

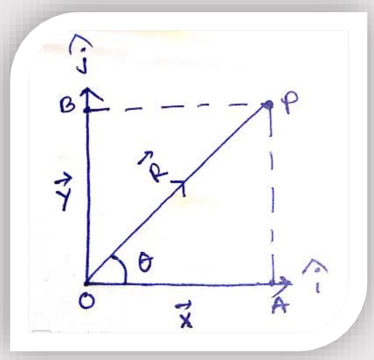
$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

In the parallelogram OPTQ, $\vec{A} - \vec{B}$ will be represented by the diagonal OP of the parallelogram with the same magnitude R as in the case of addition.

Resolution of Vectors

Resolution of vectors is the process of obtaining the component vectors which when combined, according to laws of vector addition, produce the given vector. That means it is a process just opposite to the addition of vectors.

Rectangular components: Rectangular components of a given vectors are its components of a given vectors are its components in two mutually perpendicular directions in the plane of the given vector.



The position vector $\overrightarrow{OP}$ can be resolved into its rectangular components along x- and y- directions. These component can be thought of as the projection of the position vector along the x- and y- direction. Therefore, it can be written as

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB} = x\,\hat{\imath} + y\,\hat{\jmath} = R\cos\theta\ \hat{\imath} + R\sin\theta\,\hat{\jmath}$ (R is the magnitude of $\overrightarrow{OP}$)

In general, in a three dimensional Cartesian coordinate frame any position vector can be written as

$$\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$$

Product of Vectors

There are two ways in which two vectors can be multiplied. They are given as

- The scalar product or The dot product
- The vector product or the cross product

Scalar Product

- When the resultant of product of two vectors is a scalar quantity, then the product is known as scalar product.
- This product is symbolized by a '**.**' symbol. So this is also called as dot product.
- Dot product of two vectors is defined as the product of their magnitudes and the cosine of the smaller angle between the two. Mathematically
  $\vec{A}\,.\vec{B} = A\,B\cos\theta$, where θ is the smaller angle between $\vec{A}$ and $\vec{B}$

Characteristics of dot product:

- Dot product is commutative. $\vec{A}\,.\vec{B} = \vec{B}\,.\vec{A}$
- Dot product is distributive i.e. Dot product of a vector with the sum of a number of other vectors is equal to the sum of the dot products of the vector taken with other vectors separately. $\vec{A}\,.(\vec{B} + \vec{C} + \cdots) = \vec{A}\,.\vec{B} + \vec{A}\,.\vec{C} + \cdots$
- Dot product of perpendicular vectors is Zero. Because when $\theta = 90^0$, $\cos\theta = 0$
  So $\hat{\imath}\,.\hat{\jmath} = \hat{\jmath}\,.\hat{k} = \hat{k}\,.\hat{\imath} = 0$

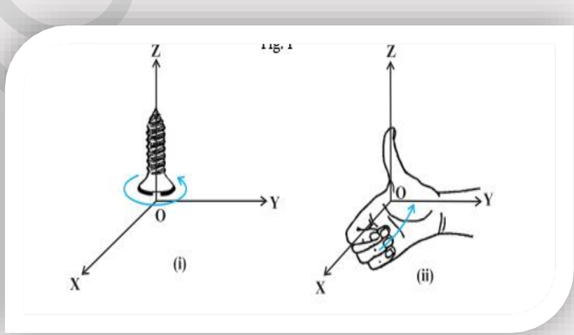
- Dot product of equal vectors gives square of their magnitude. So
  $\hat{\imath}\,.\,\hat{\imath} = \hat{\jmath}\,.\,\hat{\jmath} = \hat{k}\,.\,\hat{k} = 1$

- Dot product in terms of rectangular components: If
  $\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$ & $\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$, then
  $$\vec{A}\,.\,\vec{B} = A_x\,B_x + A_y\,B_y + A_z\,B_z$$

Vector Product

- When the resultant of product of two vectors is a vector quantity, then the product is known as vector product.
- This product is symbolized by a ' $\times$' symbol. So this is also called as cross product.
- Cross product of two vectors $\vec{A}$ and $\vec{B}$ is defined as a single vector $\vec{C}$ whose magnitude is equal to the product of their individual magnitudes and the sine of the smaller angle between them and is directed along the normal to the plane containing $\vec{A}$ and $\vec{B}$. i.e.
  $\vec{A}\ \times\ \vec{B} = \vec{C}\ =\ A\,B\sin\theta\,\hat{n}$, where $\hat{n}$ is the unit vector in a direction perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

Rules to find the direction of the resultant of cross product:



- **Right hand screw rule** : Imagine a right handed screw to be placed along the normal to the plane containing the two vectors (say $\vec{A}$ and $\vec{B}$). Rotate the cap of the screw from $\vec{A}$ to $\vec{B}$. The direction of motion of the tip of the screw gives the direction of $\vec{A}\times\vec{B}$.
- **Right hand thumb rule** : Imagine the normal to the plane containing the two vectors $\vec{A}$ and $\vec{B}$ to be held in the right hand with the thumb erect. If the fingers curl in the direction from $\vec{A}$ to $\vec{B}$, then the direction of thumb gives the direction of $\vec{A}\times\vec{B}$.

Characteristics of cross product:

- Cross product is non-commutative. i.e.
  $$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$$
- Cross product is distributive in nature. i.e.
  $$\vec{A}\times\left(\vec{B}+\vec{C}+\cdots\right) = \vec{A}\times\ \vec{B} + \vec{A}\times\ \vec{C} + \cdots$$
- Cross product of collinear or equal vectors is a null vector.
- The magnitude of the cross product of Perpendicular vectors is equal to the product of their magnitude. i.e. $|\overrightarrow{A\times B}| = AB$.
- Cross product in terms of rectangular components:

If $\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$ & $\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$, then their cross product can be determined from the determinant of following form

$$\vec{A} \times \vec{B} = det\begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix} = \hat{\imath}\,(A_yB_z - A_zB_y) - \hat{\jmath}\,(A_xB_z - A_zB_x) + \hat{k}\,(A_xB_y - A_yB_x)$$

Miscellaneous Questions

1. Show that $(\vec{A} \times \vec{B})$ is perpendicular to both $\vec{A}$ and $\vec{B}$.
2. Find the angle between two vectors $\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$ and $2\hat{\imath} + \hat{\jmath} - 4\hat{k}$.
3. Are the two vectors represented by $2\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ perpendicular to each other ?
4. A person travelled 4 k.m in the direction of east then took a sharp turning towards north and travelled 3 k.m and stopped there. Find out his net displacement.
5. Under what circumstances does the resultant of two vectors coincide with each of them.

Lecture Note: 3 (Kinematics)

Engineering Physics

Name of the Lecturer: Abhilash Padhy

Lect. In Physics (Department of Math & Sc.), GP Gajapati

Kinematics is the branch of physics which deals only with the description of motion of bodies, whereas **Dynamics** is the branch of physics which deals with the study of motion along with the cause of motion.

Rest and Motion

- A body is said to be at rest if it does not change its position with respect to its surroundings.
- A body is said to be in motion if it changes its position with respect to the surroundings.

Frame of Reference

Have you ever felt the sense of being at rest while travelling in a train whereas the world outside the train appears to move in a direction opposite to that of the train? If yes, then it would have confused you that who is moving in real? The train? Or the surrounding? Similarly, if I ask you when lying on your bed simply doing nothing, are you in motion or in rest? I guess most of you would give an answer that you are at rest. But we know the earth is moving around the sun and rotating about its own axis. And we live on the surface of earth. Then how can we say we are at rest? So from all these examples, we can understand that motion is a relative thing. We always define the motion of a body 'A' with respect to another body 'B' considering the later one as the reference point or vice versa. So to get the exact location of a body, we need to fix

i. The origin or the center of reference
ii. The frame of reference, that constitutes of a number of axes in mutually independent directions. For example: In the Cartesian coordinate frame, the location of a body is given by the set of coordinates along the three independent axis x, y, z and is expressed as (x, y, z) or by the position vector given by $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

The frame of reference can be divided into two categories

a) Inertial frame of reference
b) Non-inertial frame of reference

Inertia is the tendency of the body due to which it doesn't change its state of rest or motion without external stimulation. All the earthly frames which are at rest (this is an assumption that goes well with the physics of the nature within the confinement of earthly atmosphere) can be considered to be inertial frame of reference. Also those frame of reference which are moving with uniform velocity w.r.t the inertial frame of reference can be identified as inertial frame of reference themselves.

But if the frame is accelerating or retarding with respect to an inertial frame then they would be called as non-inertial frame of references. For example, any rotating frame of reference is an noninertial frame of reference.

Newton's laws of motion

- First law or the law of inertia: Until and unless an external force is applied on a body, it will continue its state of rest or motion. (In other words force is something that is responsible for the change of state. This provides a qualitative definition of force.)
- Second law: The force applied on a body is equivalent to the rate of change of momentum in that body and can be mathematically expressed as
  $\vec{F} = \frac{d\vec{p}}{dt} = m\,\vec{a}$ (where m is the mass and $\vec{a}$ is the acceleration)
- Third law: Every action has an equal and opposite simultaneous reaction.

Parameters of motion

The motion of a body can be described through the following parameters:

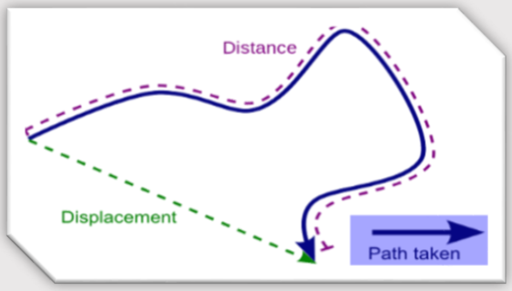
i. Distance and displacement   ii. Speed and velocity   iii. Acceleration

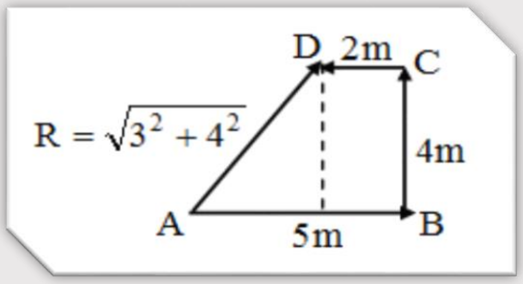
Distance and displacement:

Distance is the length of the path in which the body has travelled. It is a scalar quantity. Between any two given points, infinite number of paths are possible. For each path the distance is different.

Whereas the displacement is a vector quantity directed along the straight line path which is the shortest distance path between any two given points. Therefore, displacement is a physical quantity that is independent of the length of the path rather it depends upon the initial and final points. It is denoted as $\vec{S}$.

Example:





In the above figure, the path followed by the body is expressed by the blue curve. The length of this path is the distance travelled by the body. But it's displacement is expressed by the green line which is a straight line joining the starting point with the end point. In the second figure, a body starting his journey from the point 'A' and going to point 'B' through the path (A -> B -> C -> D) will cover a distance of 11m while it is displaced only by 5m (given by the length of 'AD'). However, if the body reaches the point 'D' following a direct path (A -> D), then it's distance and the magnitude of displacement will have the same value 5m.

Speed and velocity:

Speed of a body is defined as the distance covered by the body in one second. If 'ΔS' is the distance covered by a body in a time 'Δt', then

Average speed = $\frac{\Delta S}{\Delta t}$ and **Instantaneous speed** = $\frac{dS}{dt}$.

If the time interval Δt is chosen to be very small, i.e. $\Delta t \to 0$, the corresponding speed is called instantaneous speed.

Velocity of a body is defined as the rate of change of displacement and this is a vector quantity.

i. The average velocity: This is defined as the rate of change of displacement within a particular time interval. Mathematically

$$\vec{v}_{av} = \frac{\vec{x_2} - \vec{x_1}}{t_2 - t_1} ,$$

where $\vec{x}_1$ is the position vector of the body at time $t = t_1$ and $\vec{x}_2$ is the position vector of the body at time $t = t_2$. So $\vec{x}_2 - \vec{x}_1$ is the net displacement of the body during the time interval $t_2 - t_1$.

ii. The instantaneous velocity: If the interval is small enough to be considered as an instant, then the velocity at that instant can be called as the instantaneous velocity of that instant. Mathematically it can be expressed as

$$\overrightarrow{v_{in}} = \frac{d\vec{x}}{dt} .$$

Dimensional formula of speed or velocity: $[L\,T^{-1}]$
M.K.S or S.I unit of speed or velocity: ms^{-1}
C.G.S unit of speed or velocity: $cm\,s^{-1}$

Velocity of a body is said to be uniform if it covers equal displacements in equal intervals of time. When it covers unequal displacements in equal intervals of time, then the velocity is said to be uniform.

Acceleration:

This is defined as the rate of change of the velocity (i.e. the change in velocity in one second) and mathematically written as

$$\vec{a} = \frac{d\vec{v}}{dt} , \qquad \vec{a} = \frac{\Delta\vec{v}}{\Delta t} \textbf{ (for uniform acceleration)}$$

The acceleration is a vector quantity. Note that, a small change in direction of the velocity can cause acceleration even if the magnitude remains unchanged. The acceleration can also be expressed in terms of the displacement as

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\frac{d\vec{S}}{dt} = \frac{d^2\vec{S}}{dt^2}$$

The dimensional formula of acceleration: $[L\,T^{-2}]$
The M.K.S or S.I unit of acceleration: ms^{-2}
The C.G.S unit of acceleration: $cm\,s^{-2}$

Momentum:

Momentum is simply the product of the mass and the velocity of an object. This is a vector quantity also known as linear or translational momentum expressed by
$\vec{p} = m\vec{v}$

It's dimensional formula is given as $[M\,L\,T^{-1}]$,

The M.K.S or S.I unit of momentum is $Kg\,m\,s^{-1}$ and the C.G.S unit is $g\,cm\,s^{-1}$.

- One important thing to remember is that in an inertial frame, the total linear momentum remains conserved.

Force:

The force is defined as $\vec{F} = m\,\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$

The dimensional formula of force: $[M\,L\,T^{-2}]$

The M.K.S or S.I unit of force: $Kg\,m\,s^{-2}$ or N (Newton)

The C.G.S unit of force: $g\,cm\,s^{-2}$ or dyn (dyne)

$$\boxed{1\,N = 1\,Kg\,m\,s^{-2} = 1\,Kg \times 1\,m \times (1s)^{-2} = 1000\,g \times 100\,cm \times (1s)^{-2} = 10^5\,g\,cm\,s^{-2} = 10^5\,dyne \Rightarrow 1\,N = 10^5\,dyn}$$

Impulse:

When force is applied on a body for a certain time interval, impulse is defined as the multiplication of the force and the time interval and is expressed as

$$\mathbf{J = F\,\Delta t}$$

This is a vector quantity and its dimensional formula is given by $[M\,L\,T^{-1}]$.

- Notice that the dimensional formula of impulse matches with that of the momentum. The impulse can be visualized as the momentum transfer.
  It's S.I and C.G.S units are same as that of the momentum.

Types of motion

Basically motion can be divided into two types

i. Rectilinear motion
ii. Curvilinear motion

In case of rectilinear motion, the body moves in a straight line, where as in case of curvilinear motion, the motion of the body is not confined to a straight line only rather it moves in a curved path.

Example: Projectile motion, Circular motion, Oscillation, Wave motion

Motion in a straight line; Equations of motion

When a body moves in a straight line with uniform acceleration, the motion of the body is governed by the following equations

$$\boxed{\begin{aligned} &\vec{v} = \vec{u} + \vec{a}\,t\,, \\ &\vec{S} = \vec{u} + \frac{1}{2}\,\vec{a}\,t^2, \\ &v^2 - u^2 = 2\,\vec{a}\,.\vec{S} \qquad . \end{aligned}}$$

Where $\vec{u}$ is the initial velocity of the body (i.e. the velocity of the body at the beginning of the motion, at $t = 0$) and $\vec{v}$ is the velocity of the body at a time t. $\vec{a}$ describes the acceleration of the body and $\vec{S}$ is the displacement of the body in time t.

Lecture Note: 3 (Projectile Motion)

Engineering Physics

Name of the Lecturer: Abhilash Padhy

Lect. In Physics (Department of Math & Sc.), GP Gajapati

A body projected into the space and is no longer being propelled by fuel is called a projectile.

Examples:

i) A bullet fired from a fire.
ii) A cricket ball thrown into space.
iii) A bomb or a small bag dropped from an airplane.

Things to know about Projectile

- Every projectile experiences one signal force and that is due to gravity only.
- Horizontal velocity of a projectile remains the same throughout its flight (it may be zero also).
- No projectile ever experiences any acceleration in the horizontal direction.
- Vertical acceleration of every projectile is '- g' ms^{-2}.
- The path of projectile is parabolic except for those projected along vertical direction. In that case it's a straight line.
- The horizontal and vertical motion of a projectile are independent of each other.

Projectile motion can be of three types. They are given as

i) Projectile fired vertically upwards
ii) Projectile fired Horizontally
iii) Projectile fired at an angle θ with the horizontal

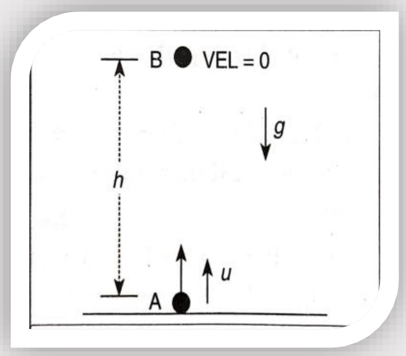
The parameters through which the projectile motion of a body can be properly understood are

i) Maximum height
ii) Time of Flight (i.e. = Time of ascent + Time of descent)
iii) Horizontal Range
iv) Equation of trajectory

The projectile can be controlled by the adjustment of

i) The initial velocity
ii) The angle of projection with the horizontal

Projectile fired vertically upwards



When a body is thrown straight up in the sky, it reaches a certain height and falls down from there back into the hand. So the motion of the body can be divided into two parts:

a) the upward motion (A to B) and

b) the downward motion (B to A)

During the upward motion:
The initial velocity at A = u,
The final velocity at B = 0,
The acceleration during the motion from A to B = $-g$,
The maximum height reached by the body = h

Similarly, during the downward motion:
The initial velocity at B = 0,
The final velocity at A = v,
The acceleration during the motion from A to B = g,
The distance covered by the body = h

- **Maximum Height**:
  It is the maximum distance travelled by the projectile in the vertical direction.
  As both the upward and downward motion are in a straight line, so from the equation of motion

  $$v^2 = u^2 + 2\,a\,S \Rightarrow 0 = u^2 + 2(-g)h$$

  $$\Rightarrow \boldsymbol{h} = \frac{\boldsymbol{u}^2}{\boldsymbol{2g}} \qquad \text{(for upward motion)}$$

  Similarly, $v^2 = 0 + 2\,g\,h \Rightarrow h = \frac{v^2}{2g}$ (for downward motion)

  That means $v = \pm u$, but we should reject the +ve sign since the velocity of body on reaching ground is in opposite direction to that at the time of projection. So we can write

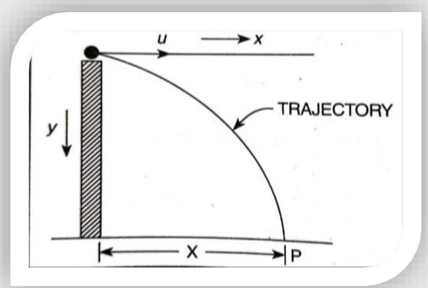
  $$\boldsymbol{v} = -\boldsymbol{u}$$

- **Time of Flight**:
  It is the total time taken by the projectile to reach the ground since projection. It is equal to the sum of time taken by the projectile to reach its maximum height since projection (i.e. the time of ascent t_a) and the time taken to reach the ground from the maximum height (i.e. the time of descent t_d).
  Applying kinematic equation,
  $v = u + a\,t \Rightarrow 0 = u - g\,t_a \Rightarrow t_a = \frac{u}{g}$ (for upward motion)
  Similarly, $v = 0 + g\,t_d \Rightarrow t_d = \frac{v}{g}$ (for downward motion)

So we get $t_a = t_d$. That means Time of Flight $\boldsymbol{T = 2\, t_a = 2\frac{u}{g}}$

➢ **Horizontal Range**:
It is the distance travelled by the projectile in the horizontal direction. Since the projection has no component of velocity along the horizontal direction, it travels no distance in the horizontal direction. i.e. **Range = zero**

Projectile fired Horizontally



Consider a body projected with a velocity 'u' in horizontal direction, from a height 'h' above the ground. Though at the point of release its initial velocity is completely in horizontal direction, but as it moves through the space its velocity have a vertical component along with the horizontal one. As there is no acceleration (i.e. no continuous force) in the horizontal direction, the horizontal component of the velocity will remain constant throughout the motion. But the vertical velocity will reach a finite value while reaching the ground which is zero at the point of release.

➢ **Time of flight:**
Here time of flight of the body is equal to the time of descent, as the body is descending only throughout the motion. So

$S = u\,t + \frac{1}{2}\,a\,t^2 \Rightarrow h = 0 + \frac{1}{2}\,g\,T^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$

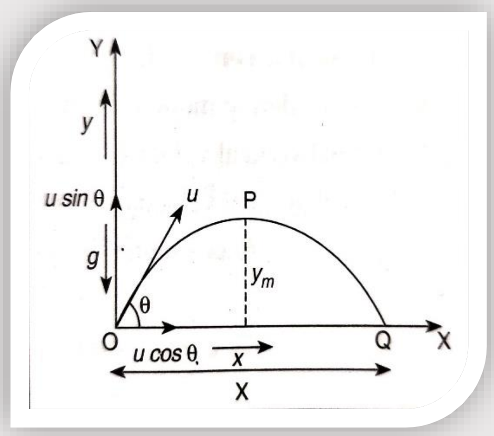
➢ **Horizontal Range:**

$$X = u\,T = u\sqrt{\frac{2h}{g}}$$

➢ **Equation of trajectory:**
It is an equation connecting the horizontal and vertical distances travelled by the projectile.
At some arbitrary time t,
In the horizontal direction $x = u\,t$
In the vertical direction $y = \frac{1}{2}\,g\,t^2 = \frac{1}{2}\,g\,\left(\frac{x}{u}\right)^2$

$$\Rightarrow x^2 = 2\frac{u^2}{g}\,y \text{ (equation of a parabola)}$$

Projectile fired at an angle θ with the horizontal



Consider a particle fired, with velocity 'u', at an angle θ with the horizontal. The projectile rises to the highest point and falls back.

The initial velocity 'u' has two components along the two perpendicular directions.

i) Horizontal component : $u\cos\theta$ (This component is uniform as there is no acceleration in the horizontal direction)

ii) Vertical component : $u\sin\theta$ (This component is non-uniform)

- **Maximum Height:**

  From kinematic equations of motion

$$v^2 = u^2 + 2\,a\,S$$

$$\Rightarrow 0 = u^2\sin^2\theta - 2\,g\,h$$

$$\Rightarrow \boldsymbol{h} = \frac{\boldsymbol{u^2\sin^2\theta}}{\boldsymbol{2g}} \qquad \text{(when the body is ascending)}$$

- **Time of flight:**

  $v = u + a\,t \Rightarrow 0 = u\sin\theta - g\,t_a \Rightarrow t_a = \frac{u\sin\theta}{g}$ (when the body is ascending)

  We know time of ascent t_a = time of descent t_b

  Total time of flight $\quad T = t_a + t_b$

$$\Rightarrow \boldsymbol{T} = 2\frac{\boldsymbol{u\sin\theta}}{\boldsymbol{g}}$$

- **Horizontal Range:**

  The velocity of the projectile in the horizontal direction is $u\cos\theta$, and it is uniform. So

  The horizontal Range $\quad X = u\cos\theta\ T = 2\frac{u\sin\theta\, u\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}$

$$\Rightarrow \boldsymbol{X} = \frac{\boldsymbol{u^2\sin 2\theta}}{\boldsymbol{g}}$$

  - The horizontal range is maximum when the angle $\theta = 45^0\ or\ \frac{\pi}{4}$
  - The maximum horizontal range is $X_{max} = \frac{u^2}{2g}$

- **Equation of trajectory:**

  At any arbitrary time, t

In the horizontal direction $x = u\cos\theta \times t \Rightarrow t = \frac{x}{u\cos\theta}$

In the vertical direction $y = ut\sin\theta - \frac{1}{2}\, g\, t^2$

$$\Rightarrow y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2}\, g\, \left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \boldsymbol{y = x\tan\theta - \frac{g}{2\, u^2\cos^2\theta}\, x^2}$$

This is the equation of trajectory of the projectile.

Lecture Note: 5 (Circular Motion)

Engineering Physics

Name of the Lecturer: Abhilash Padhy

Lect. In Physics (Department of Math & Sc.), GP Gajapati

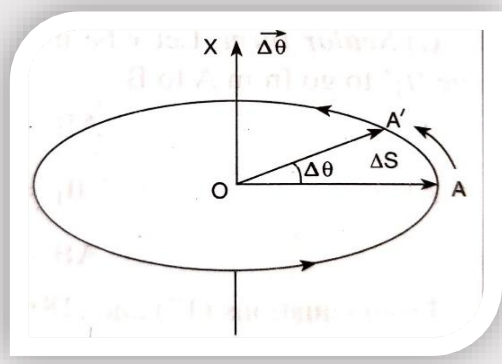
Angular displacement

Angular displacement of a particle undergoing rotational motion is defined as the angle turned by its radius vector.

A small angular displacement (i.e. a small angle subtended at center) is a vector quantity, though a large angle is considered as a scalar. Its direction is along the axis of rotation and can be understood from right hand thumb rule.

Hold the axis of rotation in your right hand with its thumb outstretched. If the fingers curl in the direction of rotation of body, the thumb gives the direction of small angular displacement.

In the figure, the angular displacement '$\Delta\theta$' of the particle rotating anticlockwise is directed upwards.



Scalar form: $\Delta\theta = \frac{\Delta S}{r}$

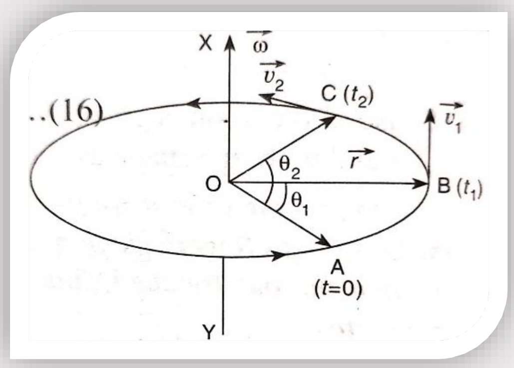
ΔS = length of the arc

r = radius of the circle

Vector form: $\boxed{\vec{\Delta S} = \vec{\Delta\theta} \times \vec{r}}$

Angular displacement is a dimensionless quantity. It can be expressed in the unit of radians.

Angular Velocity



Angular velocity of a particle, undergoing rotational motion, is defined as the rate of change of angular displacement with time.

The average angular velocity is given by $\vec{\omega}_{av} = \frac{\vec{\theta}_2 - \vec{\theta}_1}{t_2 - t_1} = \frac{\Delta\vec{\theta}}{\Delta t}$

$\vec{\theta}_1, \vec{\theta}_2$ = angular displacements of the particle at time t_1 & t_2 respectively

If the time interval Δt is very small (i.e. $\Delta t \to 0$), then this gives the instantaneous angular velocity. i.e. $\boldsymbol{\vec{\omega}_{in} = \frac{d\vec{\theta}}{dt}}$

Angular velocity is a vector quantity directed along the axis same as the angular displacement according to the right hand thumb rule.

The dimensional formula for angular velocity is $[M^0\, L^0\, T^{-1}]$ (same as that of frequency)

The S.I unit of angular velocity is $rad\ s^{-1}$.

Angular acceleration

Angular acceleration of a body is defined as the rate of change of its angular velocity with time.

The instantaneous angular acceleration is given by $\boldsymbol{\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}}$

It is also a vector quantity represented, along the axis of rotation, according to the right hand thumb rule. (same as that of the angular displacement and angular velocity)

Relation between Angular velocity and linear velocity

The linear velocity of the particle is along the tangent and always perpendicular to the radius vector.

Scalar form: $AB = v\, t_1$ and $\theta_1 = \frac{AB}{r}$. So $v = r\frac{\theta_1}{t_1} = r\,\omega$

(Linear speed = radius × angular velocity)

Vector form: Since $\vec{r}$, $\vec{\omega}$ and $\vec{v}$ are all vector quantities, the above relation indicates that there should be cross product on right hand side. Using the rule of cross-product we get

$$\boldsymbol{\vec{v} = \vec{\omega} \times \vec{r}}$$

Relation between Angular acceleration and linear acceleration

The acceleration of the rotating body has two components; one along the line of radius vector and is called radial acceleration and another is in the direction of the tangent and is denoted as the tangential acceleration.

Note that, even if the speed of the particle is uniform, the particle is accelerating as its direction is changing continuously. Similarly, though the magnitude of the radius vector is a constant here but its direction is changing continuously. So $\vec{r}$ is no longer independent of time though r is.

Scalar form:

$$\alpha = \frac{\omega_2-\omega_1}{t_2-t_1} = \frac{\frac{v_2}{r}-\frac{v_1}{r}}{t_2-t_1} = \frac{1}{r}\frac{v_2-v_1}{t_2-t_1} = \frac{a}{r}$$

$$\Rightarrow a = r\,\alpha$$

Vector form: $\vec{v} = \vec{\omega}\times\vec{r} \Rightarrow \frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times\vec{v} = \overrightarrow{a_T} + \overrightarrow{a_r}$

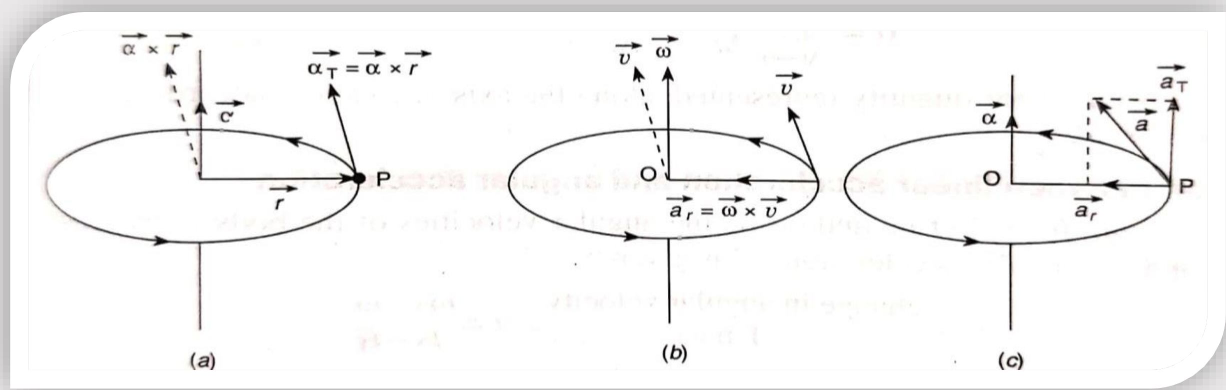
$\overrightarrow{a_T}, \overrightarrow{a_r}$ represent the tangential and radial component of the acceleration respectively.

$$\vec{r} = r\,(\cos\theta\ \hat{\imath} + \sin\theta\ \hat{\jmath})\ ,\ \vec{v} = v\cos\left(\frac{\pi}{2}+\theta\right)\hat{\imath} + v\sin\left(\frac{\pi}{2}+\theta\right)\hat{\jmath} = -v\ \sin\theta\ \hat{\imath} + v\ \cos\theta\ \hat{\jmath}$$

$$\Rightarrow \frac{d\vec{r}}{dt} = r\frac{d\theta}{dt}(-\sin\theta\ \hat{\imath} + \cos\theta\ \hat{\jmath})$$

$$= r\,\omega\,(-\sin\theta\ \hat{\imath} + \cos\theta\ \hat{\jmath}) = v\,(-\sin\theta\ \hat{\imath} + \cos\theta\ \hat{\jmath}) = \vec{v}$$

$$\boxed{\Rightarrow \frac{d\vec{r}}{dt} = \vec{v}}$$



Lecture Note: 7 (Work and Friction)

Engineering Physics

Name of the Lecturer: Abhilash Padhy

Lect. In Physics (Department of Math & Sc.), GP Gajapati

Work

Work is said to be done if a force, acting on a body, displaces the body through a certain distance and the force has some component along the displacements.

OR

Work done is defined as the dot product of force and displacement.

$$W = \vec{F}\,.\vec{S} = F\,S\cos\theta$$

- Work is a scalar quantity.
- The dimensional formula of Work is $[M\,L^2\,T^{-2}]$.
- The S.I unit of work is Joule and the C.G.S unit of work is erg.
- The conversion between Joule and erg is
  $1\,J = 1\,N \times 1\,m = 10^5 dyn \times 10^2\ c.m = 10^7 dyn\ c.m = 10^7 erg$

Special cases:

i. When the direction of force and the displacement are same
   $W = F\,S\cos 0 = F\,S$ (Work done on the system, positive work)

ii. When the direction of force and the displacement are opposite to each other
   $W = F\,S\cos\pi = -F\,S$ (Work done by the system, negative work)

iii. When the direction of force and the displacement are perpendicular to each other
   $W = F\,S\cos\frac{\pi}{2} = 0$ (zero work)

Friction

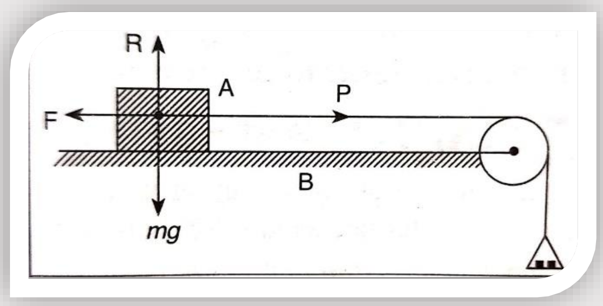
Whenever a body tends to slide over another's surface, a force called force of friction comes in to play which acts tangentially to the interface of two bodies and opposes the relative motion between the bodies.

Friction is of three types:

i. Sliding Friction
ii. Rolling Friction
iii. Fluid Friction

Sliding Friction

The force of friction which comes into play between two surfaces when one tends to slide over the other is called sliding friction.



The forces acting on the body in the figure are

i. Weight *mg* acting vertically downward.
ii. Normal reaction R acting vertically upward.
iii. Force P, due to tension in the string, in the forward direction (towards right).
iv. Force of friction F, in the backward direction (towards left).

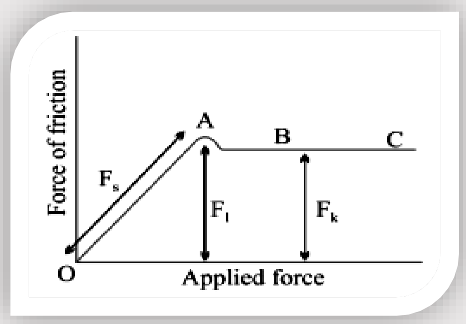
Sliding friction is of two types:

i. Static friction
ii. Dynamic friction

Static friction is the force of friction between two surfaces so long as there is no relative motion between them.

Dynamic friction is the maximum value of force of friction between two surfaces so long as there is some relative motion between them.

Limiting friction is the maximum value of force of friction between two surfaces so long as there is no relative motion between them. In other ward, this is the maximum value of the static friction.



In the figure, the frictional force (Static friction) goes on increasing along with the increase in the applied force maintaining equality (represented by the linear part of the graph i.e. OA) till it's maximum value at A (The frictional force at the peak K is the limiting friction). After that it decreases slightly and remains constant for further increase in the applied force (This is the kinetic friction represented by BC). The dynamic friction is slightly less than the limiting friction because of smaller weld effect.

Laws of Limiting Friction

i. The direction of force of friction is always opposite to the direction of motion.

ii. The force of limiting friction depends upon the nature and state of polish of the surfaces in contact and acts tangentially to the interface between the two surfaces.

iii. The magnitude of limiting friction 'F' is directly proportional to the magnitude of the normal reaction R between the two surfaces in contact, i.e.,

$$F \propto R$$

iv. The magnitude of the limiting friction between two surfaces is independent of the area and shape of the surfaces in contact so long as the normal reaction remains the same.

Coefficient of Friction

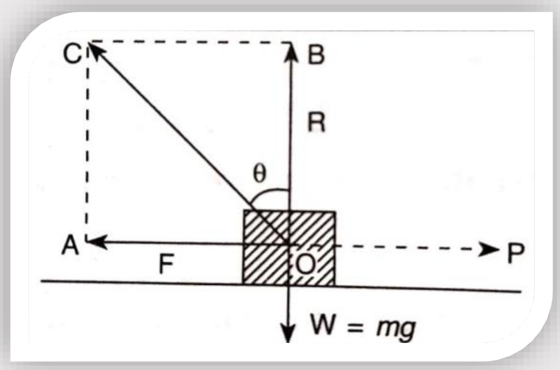
Co-efficient of friction of a pair of surfaces in contact is defined as the ratio between the force of limiting friction F to the normal reaction R. It is denoted by μ.

$$\mu = \frac{F}{R}$$

(Similarly the Co-efficient of dynamic friction can be defined as $\mu_d = \frac{dynamic\ frictional\ force}{Normal\ reaction\ force}$)

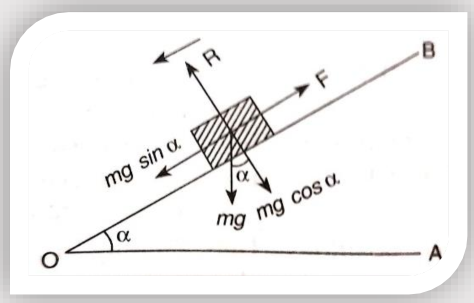
μ is dependent on the nature and state of polish of the surfaces in contact. This is a dimensionless physical quantity.

Angle of Friction & Angle of Repose



Angle of friction is the angle which the resultant of force of limiting friction and normal reaction makes with the normal reaction.

$$\mu = \tan\theta$$



Angle of repose is the angle which an inclined plane makes with the horizontal so that a body placed over it just begins to slide of its own accord.

$$mg \sin \alpha = F \text{ and } mg \cos \alpha = R \Rightarrow \mu = \frac{F}{R} = \frac{mg \sin \alpha}{mg \cos \alpha}$$

$$\Rightarrow \mu = \tan \alpha$$

Methods of Reducing Friction

Friction is usually referred as a necessary evil. Due to friction we need to work continuously to maintain the speed (evil); in the other hand without friction we won't be able to walk on earth (necessary). So for the best, we can try to reduce the friction to our requirement. Some of the methods for doing it are

- **By rubbing and polishing**, the irregularities of the surface are smoothened thereby reducing the friction.
- **By lubricants**, the irregularities can be filled avoiding their interlocking hence reducing the friction.
- **By converting sliding into rolling friction.**
- **By streamlining** (especially for reducing the fluid friction).

Lecture Note: 6 (Simple Harmonic Motion)

Engineering Physics

Name of the Lecturer: Abhilash Padhy

Lect. In Physics (Department of Math & Sc.), GP Gajapati

Periodic events and Periodic motion

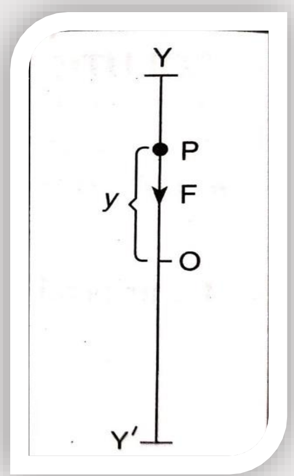
In our day to day life, we encounter a lot of events or phenomena that occur repetitively after a certain interval of time.

For example:

i) The Sun rise and the Sun set (This event repeats itself after each 24 hours)
ii) The leap year (comes in each 4 years)

These kind of events can be called as a periodic event. Similarly, when the pattern of motion (or the trajectory of motion) is repeated after a finite interval of time, then the kind of motion is said to be periodic motion. Circular motion is a good example of a periodic motion. But there are other possible types of motion which can be categorized as periodic motion. Such as: Oscillation, Wave Motion.

What is Simple Harmonic Motion (S.H.M)



A body is said to be in Simple Harmonic Motion or S.H.M, when the **restoring force** on the body is proportional to displacement of the body from the **mean position**.

(When a body moves back and forth about a fixed position known as mean position periodically, then the body is said to be under oscillation. Due to the influence of a force called restoring force the body comes back to the mean position again and again.)

Mathematically, $\boxed{\vec{F} = -k\,\vec{y}}$, where F = the restoring force and y = displacement of the body

So we can write $\vec{F} = m\,\vec{a} = -k\,\vec{y} \Rightarrow \frac{d^2\vec{y}}{dt^2} + \frac{k}{m}\,\vec{y} = 0$

So S.H.M can be defined also as

A particle is said to move in S.H.M, if its acceleration is proportional to the displacement and is always directed towards the mean position.

The solution to the above differential equation i.e. the displacement of the body can be written as $\boxed{y = r\sin(\omega\, t + \phi)}$

Here,

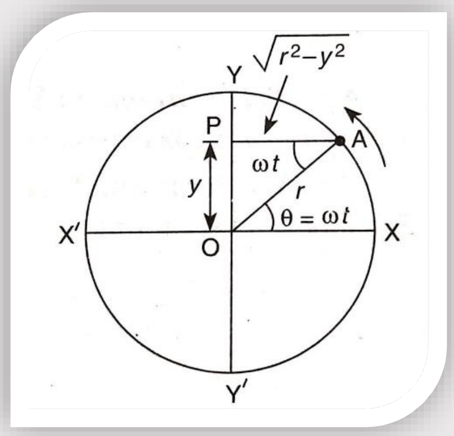
r = amplitude of S.H.M (i.e. the maximum value of displacement of the body about the mean position)

ω = angular velocity and

ϕ = Phase angle

Relation between S.H.M. and Uniform Circular Motion

Simple harmonic motion can be defined as the projection of uniform circular motion on the diameter of circle of reference, where as its center is considered to be the mean position.



Characteristics of S.H.M.

- **Displacement:**

Displacement of a particle vibrating in S.H.M., at any instant, is defined as its distance from the mean position at that instant.

OP = OA $\sin\theta \Rightarrow \boldsymbol{y = r\sin\omega\, t}$

r is the amplitude of particle, vibrating in S.H.M., defined as its maximum displacement on either side of the mean position.

Since extreme values of $\sin\omega\, t = \pm 1$ so $y = \pm r$

- **Velocity:**

$$V = \frac{dy}{dt} = \frac{d}{dt}(r\sin\omega\, t) = r\cos\omega\, t\,\frac{d}{dt}(\omega\, t) = r\,\omega\cos\omega\, t$$

$$\Rightarrow \boldsymbol{V = v\cos\omega\, t}$$

$v = r\,\omega$ is the maximum value of velocity that the particle can achieve during oscillation.

At mean position, $y = 0 \Rightarrow \sin\omega\, t = 0 \Rightarrow \omega\, t = 0 \Rightarrow \cos\omega\, t = 1$

$$\Rightarrow V = v$$

At the amplitude, $y = \pm r \Rightarrow \sin\omega\, t = \pm 1 \Rightarrow \omega\, t = \frac{\pi}{2}\ or\ \frac{3\pi}{2} \Rightarrow \cos\omega\, t = 0$

$$\Rightarrow V = 0$$

A particle vibrating in S.H.M., passes with maximum velocity through the mean position and is at rest at the extreme positions.

- **Acceleration:**

$$a = \frac{dV}{dt} = \frac{d}{dt}(v\cos\omega\, t) = v\,(-\sin\omega\, t)\frac{d}{dt}(\omega\, t) = -(r\,\omega)(\sin\omega\, t)\omega$$

$$= -(\omega^2 r)\sin\omega\, t = -\alpha\sin\omega\, t\ \ (or) - \omega^2 y$$

$$\Rightarrow \boldsymbol{a = -\omega^2 y}$$

The negative sign indicates that the acceleration of the particle is in the opposite direction to the direction of the displacement at every instant.
At mean position, $y = 0 \Rightarrow a = 0$
At the amplitude, $y = \pm r \Rightarrow a = \mp\omega^2 r$
A particle vibrating in S.H.M., has zero acceleration while passing through mean position and has maximum acceleration while at extreme positions.

- **Time Period:**

It is the time taken by the particle to complete one vibration.

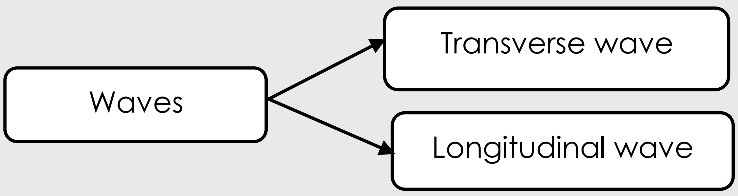
$$T = \frac{2\pi}{\omega} \text{ but we know } \omega^2 = \frac{acceleration}{displacement}$$

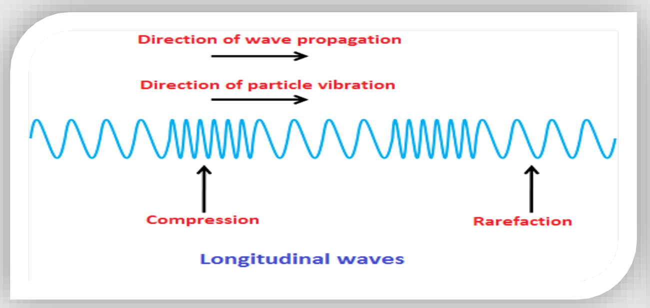
So

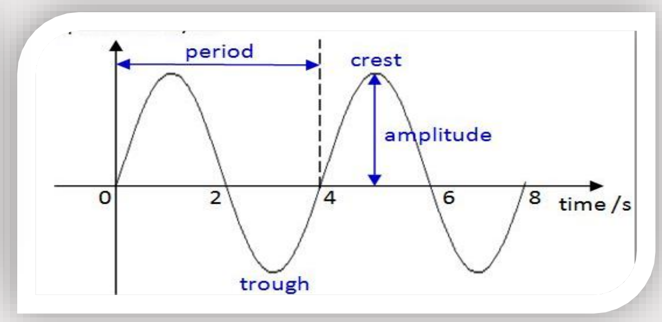
$$\boxed{\boldsymbol{T = 2\pi\sqrt{\frac{displacement}{acceleration}}}}$$

Wave motion

Wave motion is the disturbance that travels through the medium and is due to repeated periodic motion of the particles of the medium, the motion being handed over from particle to particle.







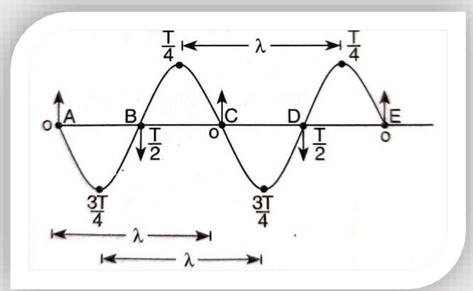
Transverse Wave	Longitudinal Wave
i. Vibrations of the particles of medium are normal to the direction of wave propagation.	i. Vibrations of the particles of medium are parallel to the direction of wave propagation.
ii. Crests and troughs are formed during its propagation.	ii. Compression and rarefactions are formed during its propagation.
iii. Causes temporary change in shape of the medium.	iii. Causes temporary change in the size of the medium.
iv. Medium through which such a wave travels possesses shear modulus and inertia.	iv. Medium through which such wave travels possess compressibility and inertia.
v. The light wave is an example of transverse wave.	v. The sound wave is an example of Longitudinal wave.

Characteristics of Wave-Motion

A wave-motion is found to possess following characteristics:

i. The disturbance that travels through the medium is due to the repeated periodic motion of the particles of the medium.
ii. Particles do not leave their mean positions but keep on vibrating to and fro, in S.H.M., about their mean positions.
iii. The velocity of wave and the velocity of particle are different from each other. The velocity of the particle is variable. It is maximum when the particle passes through the mean position and is zero at extreme position. The velocity of wave is uniform.
iv. Propagation of a wave results in formation of crests and troughs (in case of a transverse wave-motion) or in formation of compressions and rarefaction (in case of a longitudinal wave).
v. Energy is always carried in the direction of propagation of wave.

Parameters defining the Wave-Motion



- **Wave-length (λ):**
  It is defined as the distance travelled by a wave during the time particle executing S.H.M. completes one vibration.
  OR
  It is the distance between two consecutive particles executing S.H.M. in same phase. (for example the distance between two consecutive crests or troughs)
- **Wave-number ($\overline{n}$):**

Wave-number of a wave is defined as the reciprocal of the wavelength of wave. $\bar{n} = \frac{1}{\lambda}$

Its dimensional formula is $[L^{-1}]$ and the S.I. unit is m^{-1}.

- **Frequency (f):**

  Frequency is defined as the number of waves passed through a single point in one second. It is the reciprocal of the time period.

  $f = \frac{1}{T}$ dimensional formula: $[T^{-1}]$, S. I unit: s^{-1}

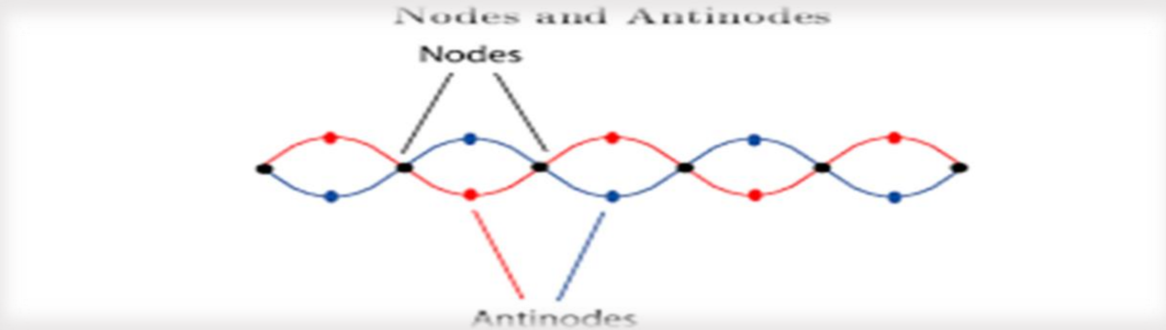
- **Velocity of Wave (v):**

  The velocity of a wave is defined by the distance travelled by the wave in one time period.

$$\boxed{v = \frac{\lambda}{T} = \lambda f}$$

Progressive Wave and the Stationary Wave

Progressive Wave	Stationary Wave
i. Each particle of the medium communicates disturbance to the next particle.	i. Disturbance is not communicated from one particle to the next. It remains fixed.
ii. Amplitude of each particle is same.	ii. Amplitudes of different particles is different. It is maximum at antinodes and is zero at nodes.
iii. Consecutive particles reach the maximum displacement positions at different times.	iii. All the particles in between consecutive nodes reach the position of maximum displacement simultaneously.
iv. There is a gradual change of phase from one particle to the next.	iv. Phase of all particles, in one segment, is same and is opposite to those in the next segment.
v. No particle is permanently at rest. Every particle comes to rest, momentarily, twice during each revolution.	v. Particles at nodes are permanently at rest.
vi. While passing through the mean position every particle acquires same maximum velocity.	vi. Maximum velocity, while they pass through their mean positions, is different for different particles at anti-nodes since they possess the biggest amplitude.



Lecture Note: 8 (Gravitation)

Engineering Physics

Name of the Lecturer: Abhilash Padhy

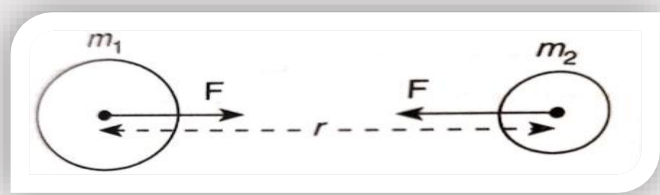
Lect. In Physics (Department of Math & Sc.), GP Gajapati

Newton's Law of Gravitation

Every particle of matter in this universe attracts every other particle with a force which varies directly as the product of the masses of two particles and inversely as the square of the distance between them.

This force is known as the force of gravitation.

For two bodies of masses m_1 and m_2 having intermediate distance 'r' between them experience a force of attraction F(i.e. the force of gravitation), such that



$$F \propto m_1 m_2 \text{ and } F \propto \frac{1}{r^2}$$

$$\Rightarrow \boxed{\boldsymbol{F = G\frac{m_1 m_2}{r^2}}}$$

Where G is a constant of proportionality named as the Gravitational constant or constant of universal gravitation.

The gravitational constant 'G' may be defined as the magnitude of force of attraction between two bodies each of unit mass and separated by a unit distance from each other.

(i.e. when $m_1 = m_2 = 1$ and $r = 1$ unit , then $G = F$)

* Dimensional formula of G: $G = \frac{F \times r^2}{(m_1 \times m_2)} = \frac{[M\,L\,T^{-2}][L^2]}{[M][M]} = [M^{-1}\,L^3\,T^{-2}]$

* S.I unit of G: $\frac{newton \times meter^2}{kg^2}$ $i.e. N\ m^2 kg^{-2}$

* C.G.S unit of G: $\frac{dyne \times cm^2}{g^2}$ $i.e. dyne\ cm^2 g^{-2}$

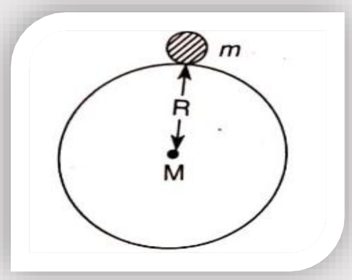
* The value of G: $\boldsymbol{G = 6.67 \times 10^{-11}\ N\ m^2 kg^{-2}}$

Acceleration due to Gravity (g)

The force of gravitation between the Earth and a body of mass m,on the surface of earth can be given as

$$F = G\frac{Mm}{R^2}\ ,$$

Where M, R are the mass and radius of the earth respectively. Here the intermediate distance between earth and the body is considered to be the distance between their corresponding center of mass (i.e. the radius of earth). From Newton's law of motion we know ($Force = mass \times\ acceleration$). From the above expression of force, it can be seen that G, M, R are constant quantities and they constitute the acceleration due to gravity given by



$$\boldsymbol{g = \frac{GM}{R^2}}$$

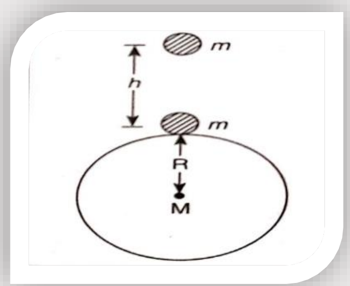
Remember that the value of $g = 9.8\ ms^{-2}$ is constant for the bodies in close proximity of the earth. This force between the body and earth is identified as the weight of the body. Weight is not to be confused with mass of the body. Mass is a fundamental physical quantity which is a conserved quantity whereas weight is a force and depends on the planet or satellite over which the body is placed.

- The dimensions and units of 'g' is same as that of acceleration.

Variation of 'g' with altitude and depth

When the body is taken to a height or depth of length comparable to the radius of the earth the acceleration due to gravity starts getting influenced (g decreases both with height and depth).

- **Variation of g with height:**



At a height 'h', the acceleration due to gravity is given by

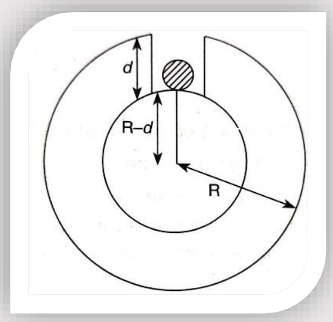
$$\boldsymbol{g' = g(1 - \frac{2h}{R})}$$

So when the body reaches a height $h = \frac{R}{2}$, $g' = 0$ the body feels weightlessness.

- **Variation of g with depth:**

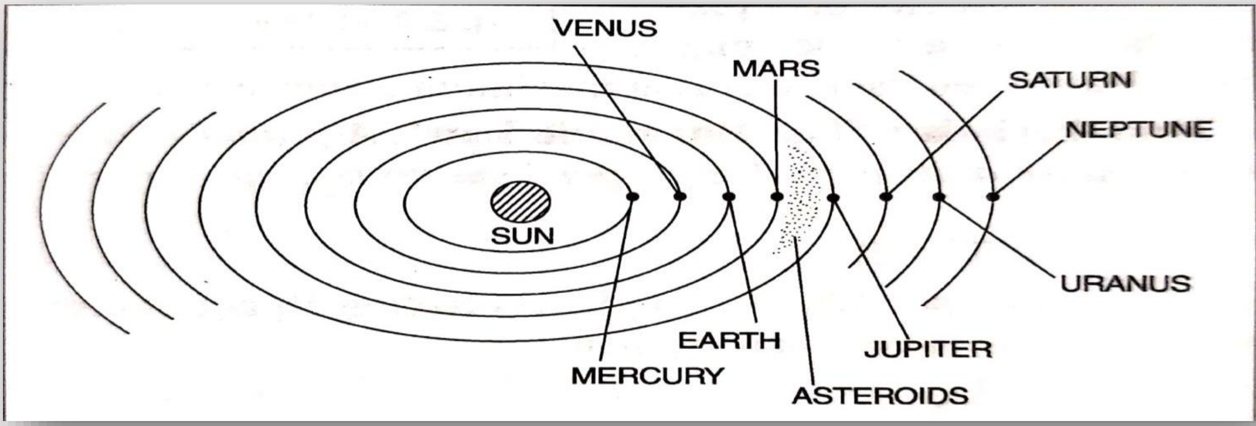
At a depth 'd', the acceleration due to gravity is given by

$$\boldsymbol{g' = g(1 - \frac{d}{R})}$$



So when the body reaches a depth $d = R$ (i.e. at at the center of the earth), $g' = 0$ the body feels weightlessness.

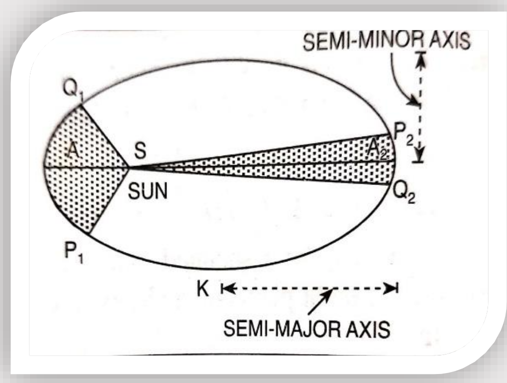
Kepler's laws of Planetary Motion



Basically the astronomical objects can be divided into three categories roughly; Stars (sources of light), Planets (illuminated by the light of the star and revolve round the sun) and Satellites (revolve round the planet). The motion of the planets follows certain laws. These laws are given by Kepler and are known as Kepler's laws of Planetary motion.

- **Law of elliptical orbits:**
  A planet moves round the sun in an elliptical orbit with sun situated at one of its foci.

- **Law of areal velocities:**
  A planet moves round the sun in such a way that its areal velocity is constant. (i.e. the line joining the planet with the sun sweeps equal areas in equal interval of time).
  Area $P_2 S\, Q_2$ = Area $P_1 S\, Q_1$



- **Law of Time Periods or The Harmonic Law:**
  A Planet moves round the sun in such a way that the square of its period is proportional to the cube of semi-major axis of its elliptical orbit.

$$\boldsymbol{T^2 \propto R^3}$$